Corporate Finance: Capital structure and PMC

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Brander and Lewis, AER 1986

"Oligopoly and Financial Structure:
The Limited Liability Effect"

Cournot duopoly with differentiated products

- Two firms produce differentiated products for which the inverse demand functions are

$$p_1 = A - q_1 - \gamma q_2 \qquad p_2 = A - q_2 - \gamma q_1$$

- γ reflects the “degree of product differentiation”
  - $\gamma = 1$ the products are perfect substitutes
  - $0 < \gamma < 1$ the products are imperfect substitutes
  - $\gamma = 0$ the products are unrelated
  - $\gamma < 0$ the products are complements

- Both firms have a constant marginal cost k

- The profit functions:

$$\pi_1 = (p_1 - k)q_1, \qquad \pi_2 = (p_2 - k)q_2$$

The Nash equilibrium

- F.o.c for firm 1:

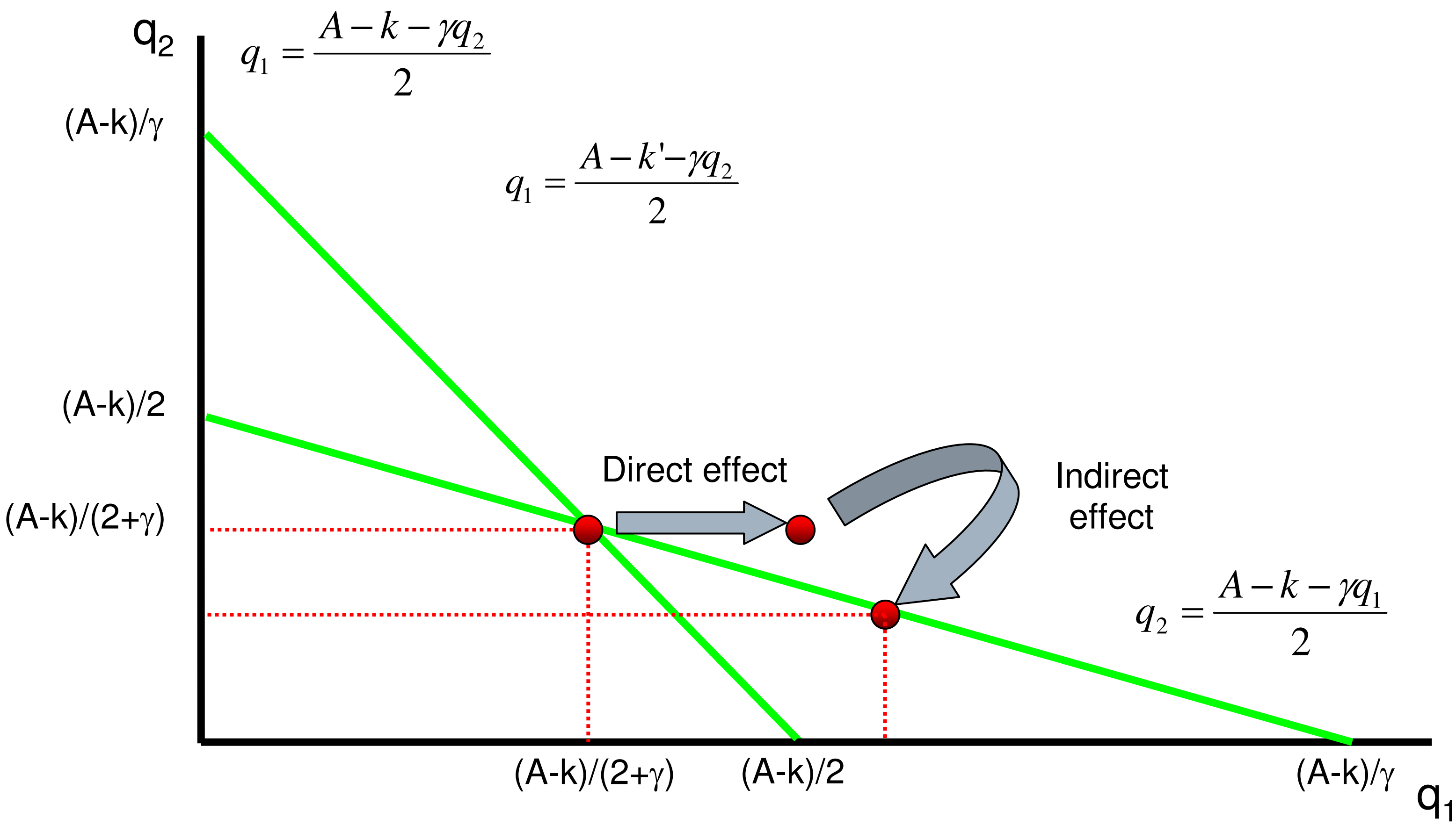
$$\frac{\partial \pi_1}{\partial q_1} = \underbrace{A - q_1 - \gamma q_2 - q_1}_{\text{Marginal revenue}} - k = A - 2q_1 - \gamma q_2 - k = 0$$

- The best-response function of firm 1

$$q_1 = BR(q_2) = \frac{A - k - \gamma q_2}{2}$$

- The best response function of firm 2 is analogous

Gaining strategic advantage with cost reduction from k to k'

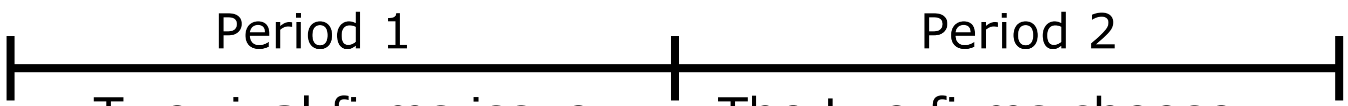


The limited liability effect

- Marginal cost is c_H with prob. α or c_L with prob. $1-\alpha$ (oil prices fluctuate)
- The firm chooses quantity <u>before</u> knowing if cost is high or low (an airline determines its flight schedule in advance and buys fuel on the spot market)
- Expected profit: $E\pi = \alpha(p - c_H)Q + (1-\alpha)(p - c_L)Q$
- The firm issues debt with face value D that can be repaid in full only if cost is c_L: $D > (p-c_H)Q$
- If cost is c_H, the firm goes bankrupt; due to limited liability equityholders get a payoff of 0
- The firm acts on behalf of equityholders and therefore maximizes:
$$(1-\alpha)(p - c_L)Q$$
- The firm behaves as if its cost is c_L rather than $\alpha c_H + (1-\alpha)c_L$
  - The firm gains a strategic advantage over rivals if strategies are strategic substitutes
  - The strategy is costly: the firm goes bankrupt with probability α

The general model

- The timing:



Period 1: Two rival firms issue debt with faces values D^1 and D^2

Period 2: The two firms choose output levels q_1 and q_2

Profits are $R^1(q_1,q_2,z)$ and $R^2(q_1,q_2,z)$

- $R^i(q_i,q_j,z)$ has a unique maximizer q_i
- $R^i_j(q_i,q_j,z) < 0$ – firm j has a negative externality on firm i
- $R^i_z(q_i,q_j,z) > 0$: z is a positive shock (either supply of demand shock)
- $z \sim [0, \infty)$ according to $f(z)$ with CDF $F(z)$

Examples

- Cost shocks:
  - $C_i = C(q_i)/z$
  - $C_i = (1-z)C(q_i)$, $z \sim [0, 1]$

- Demand shocks
  - $p_i = zp_i(q_i)$, where $p_i = A-q_i-\gamma q_j$
  - $p_i = Az-q_i-\gamma q_j$

All-equity firm

- Firm i chooses q_i to maximize its expected profit taking as given q_j

$$\underset{q_i}{Max}\, V^i\left(q_i, q_j, z\right) = \int_0^{\infty} R^i\left(q_i, q_j, z\right) dF(z)$$

- The best response of firm i against q_j is defined by

$$V_i^i = \int_0^{\infty} R_i^i\left(q_i, q_j, z\right) dF(z) = 0$$

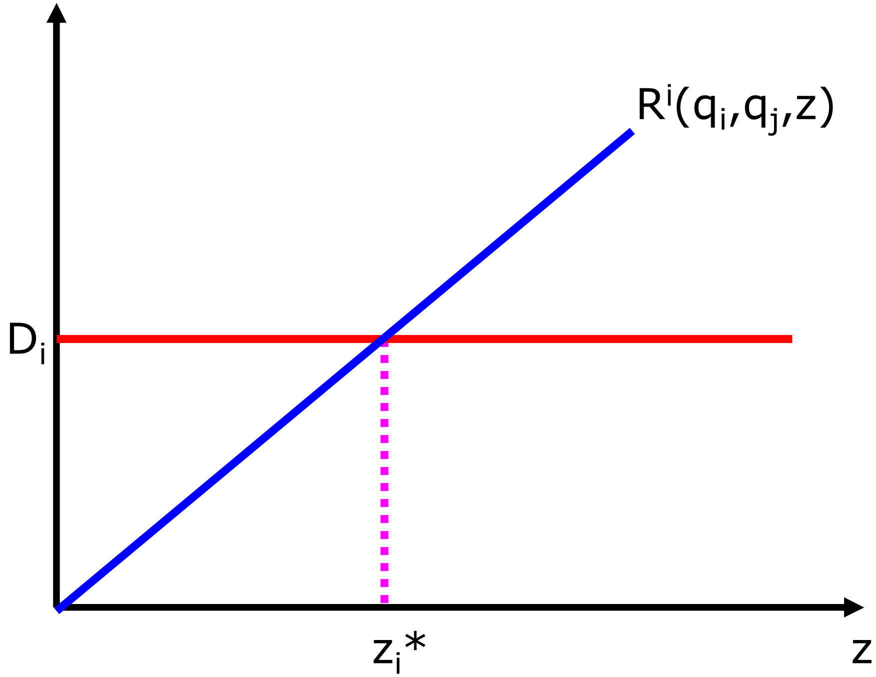
- The NE is defined implicitly by the system:

$$V_i^i\left(q_i, q_j, z\right) = 0 \qquad V_j^j\left(q_i, q_j, z\right) = 0$$

Leveraged firm

- The firm is solvent iff:

$$z \geq z_i^* \qquad R^i\left(q_i, q_j, z_i^*\right) = D_i$$



Comparative statics:

- $D_i \uparrow \;\Rightarrow\; z_i^* \uparrow$

The problem of a leveraged firm

- Firm i's problem:

$$\underset{q_i}{Max} V^i(q_i, q_j, z) = \int_{z_i*}^{\infty} \left(R^i(q_i, q_j, z) - D^i\right) dF(z)$$

- The best response of firm i against q_j is defined by

$$V_i^i(q_i, q_j, z) = -\underbrace{\left(R^i(q_i, q_j, z_i*) - D^i\right)}_{=0} f(z_i*) \frac{\partial z_i*}{\partial q_i} + \int_{z_i*}^{\infty} R_i^i(q_i, q_j, z) dF(z) = 0$$

- The NE is defined implicitly by the system:

$$V_i^i(q_i, q_j, z) = 0 \qquad V_j^j(q_i, q_j, z) = 0$$

The effect of leverage on the firm's strategy

- To find the effect of D_i on q_i, fully differentiate f.o.c.:

$$\underbrace{V_{ii}^{\,i}\left(q_i, q_j, z\right)}_{\text{(-) by s.o.c}}\partial q_i + V_{iD}^{\,i}\left(q_i, q_j, z_i^*\right)\partial D_i = 0 \quad \Rightarrow \quad \frac{\partial q_i}{\partial D_i} = \frac{V_{iD}^{\,i}\left(q_i, q_j, z\right)}{-V_{ii}^{\,i}\left(q_i, q_j, z\right)}$$

- The change in firm i's best response function depends on the sign of $V^i_{iDi}(q_i,q_j,z)$

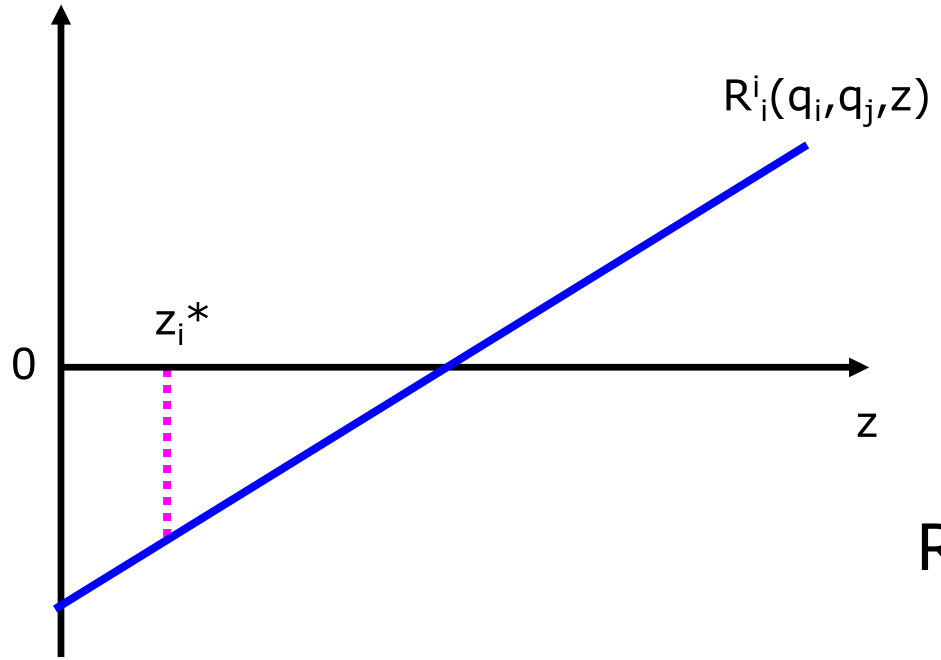
$$V_{iD}^{\,i}\left(q_i, q_j, z\right) = \frac{\partial}{\partial D_i}\int_{z_i^*}^{\infty} R_i^{\,i}\left(q_i, q_j, z\right) dF(z)$$

$$= -R_i^{\,i}\left(q_i, q_j, z_i^*\right) f\left(z_i^*\right)\frac{\overbrace{\partial z_i^*}^{(+)}}{\partial D_i}$$

The sign of $R^i_i(q_i,q_j,z_i^*)$

- Suppose that $R^i_{iz} > 0$ – the shock z boosts the marginal profit

- Recall that total area between the horizontal axis and the graph of $R^i_i(q_i,q_j,z_i)$ sums up to 0



$R^i_i(q_i,q_j,z_i^*) < 0$

Effect of D^i

- If $R^i_{iz} > 0$, then $q^i \uparrow$ with D_i – firm i becomes more aggressive when it issues debt
- Why?
  - Due to limited liability, firm i ignores low states of z in which it goes bankrupt and hence behaves as if z was on average high
  - When $R^i_{iz} > 0$, the firm produces more when z is high
- In the Cournot model, when firm i is more aggressive, firm j is softer and firm i enjoys a strategic advantage vis-à-vis firm j

The choice of D^i

- The value of debt:

$$B_i(D_i) = \int_0^{z_i^*} R^i\left(q_i^*, q_j^*, z\right)dF(z) + \int_{z^*}^{\infty} D_i dF(z)$$

- The value of equity:

$$E_i(D_i) = \int_{z_i^*}^{\infty} \left(R^i(q_i^*, q_j^*, z) - D_i\right)dF(z)$$

- The total value of the firm:

$$Y_i(D_i) = \int_0^{\infty} R^i(q_i^*, q_j^*, z)dF(z)$$

The optimal choice of D_i

$$Y_i'(D_i) = \int_0^\infty \left[R_i^i(q_i^*, q_j^*, z) \frac{\partial q_i^*}{\partial D_i^*} + R_j^i(q_i^*, q_j^*, z) \frac{\partial q_j^*}{\partial D_i^*} \right] dF(z)$$

$$= \int_0^{z_i^*} R_i^i(q_i^*, q_j^*, z) dF(z) \frac{\partial q_i^*}{\partial D_i^*} + \underbrace{\int_{z_i^*}^\infty R_i^i(q_i^*, q_j^*, z) dF(z)}_{=0 \text{ by F.O.C for } q_i} \frac{\partial q_i^*}{\partial D_i^*}$$

$$+ \int_0^\infty \underbrace{R_j^i(q_i^*, q_j^*, z)}_{(-)} dF(z) \underbrace{\frac{\partial q_j^*}{\partial D_i^*}}_{(-)}$$

$$= \underbrace{\int_0^{z_i^*} R_i^i(q_i^*, q_j^*, z) dF(z)}_{(-)} \underbrace{\frac{\partial q_i^*}{\partial D_i^*}}_{(+)} + \underbrace{\int_0^\infty R_j^i(q_i^*, q_j^*, z) dF(z) \frac{\partial q_j^*}{\partial D_i^*}}_{(+)}$$

The optimal choice of D_i

- The f.o.c for D_i:

$$Y_i'(D_i) = \underbrace{\int_0^{z_i^*} R_i^{\,i}(q_i^*, q_j^*, z)\,dF(z)}_{(-)}\underbrace{\frac{\partial q_i^*}{\partial D_i^*}}_{(+)} + \underbrace{\int_0^{\infty} R_j^{\,i}(q_i^*, q_j^*, z)\,dF(z)\frac{\partial q_j^*}{\partial D_i^*}}_{(+)} = 0$$

- The second term is the positive strat. effect
- The first term is the negative direct effect: debt distorts the firm's choice of quantity
- As $D_i \rightarrow 0 \Rightarrow z_i^* \rightarrow 0 \Rightarrow$ the negative direct effect vanishes $\Rightarrow Y_i'(D_i) > 0 \Rightarrow D_i^* > 0$

Spiegel, JEMS 1996

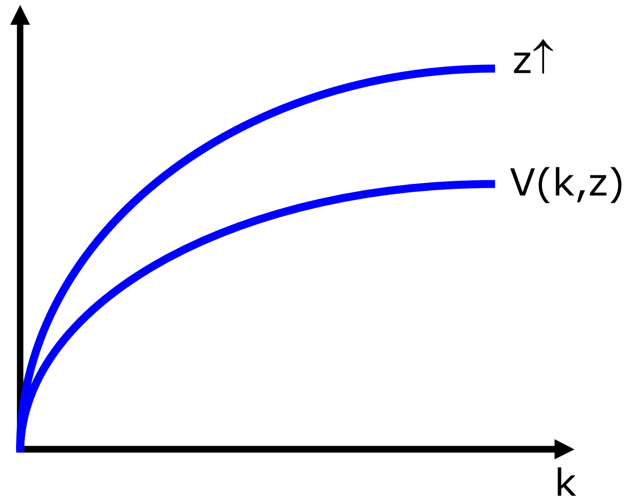
“The Role of Debt in Procurement Contracts”

The model

- The timing:

Period 1	Period 2	Period 3
A firm invests k which determines the net surplus from trading with a buyer, V(k,z)	The firm issues debt with face value D	z is realized, the firm and the buyer bargain over a price, D is due

- $V_k(k,z) > 0 > V_{kk}(k,z)$
- $V_z(k,z) > 0$ and $V_{kz}(k,z) \geq 0$
- $z \sim U[0, 1]$



The bargaining stage

- The buyer and the firm trade iff V(k,z) ≥ D, o/w the price the buyer pays is not enough to ensure solvency
- Let z* be such that V(k,z*) = D
- If V(k,z) ≥ D, the buyer makes a TIOLI with prob. γ and the firm makes an offer with prob. 1-γ
- The firm's expected payoff when it issues debt is

$$Y(k,D) = (1-\gamma)\underbrace{\int_{z^*}^{1} \left(V(k,z) - D\right) dz}_{\text{Equity}} + \underbrace{\int_{z^*}^{1} D dz}_{\text{Debt}} - k$$

The choice of debt

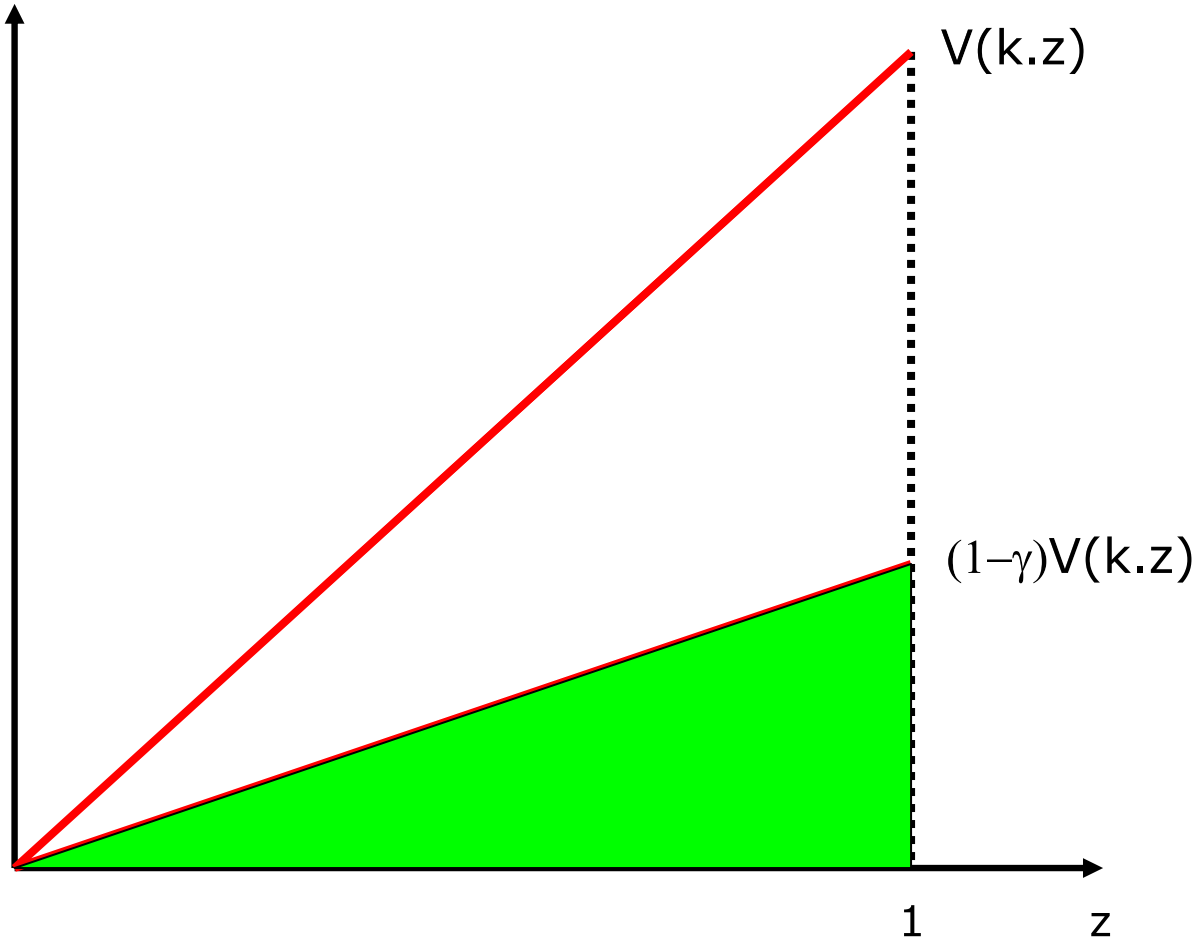
- The f.o.c for the firm's value:

$$Y_D(k,D) = -(1-\gamma)(1-z^*) + (1-z^*) - D\frac{\partial z^*}{\partial D}$$

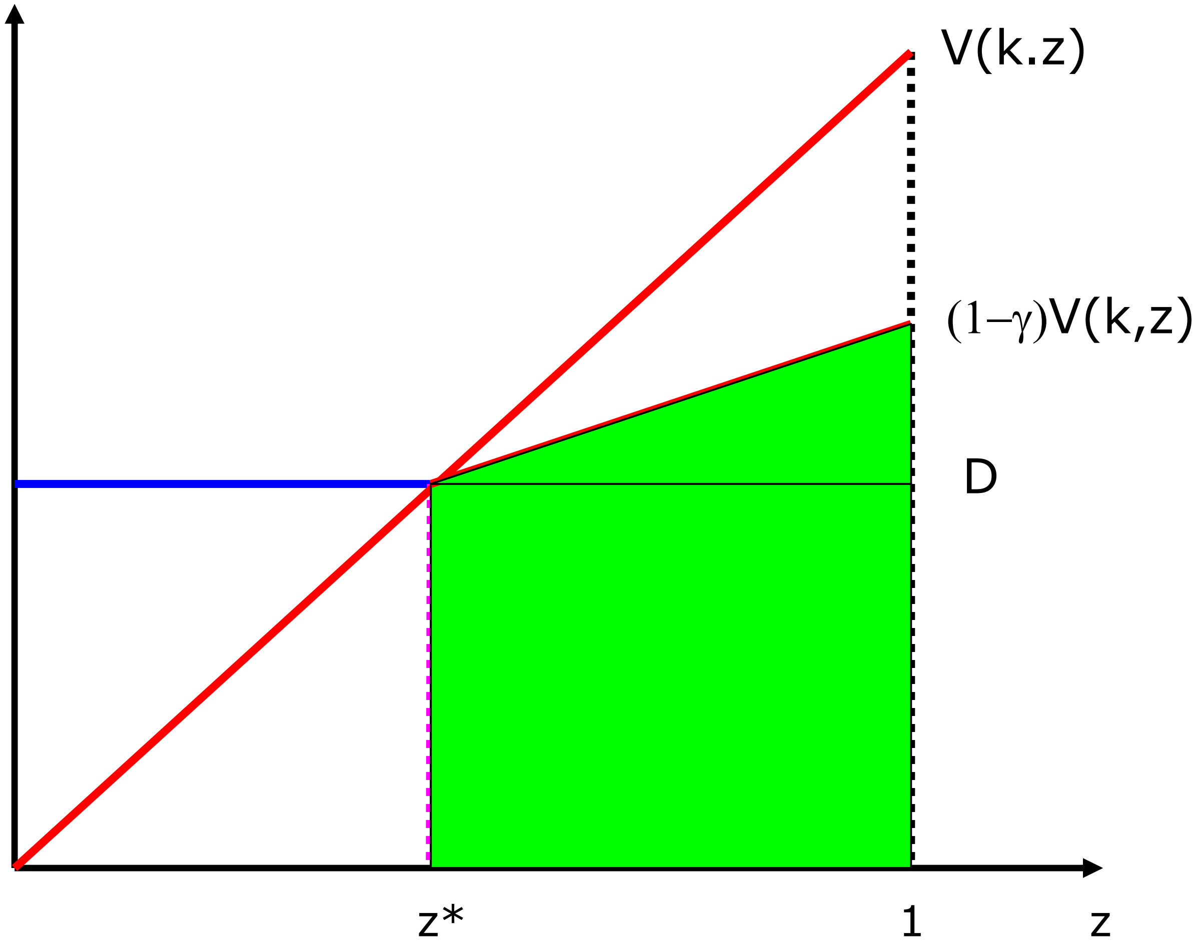
$$= \gamma(1-z^*) - D\underbrace{\frac{\partial z^*}{\partial D}}_{(+)} = 0$$

- At D = 0, $Y_D(k,z) > 0 \Rightarrow D^* > 0$

Illustrating the model – all equity



Illustrating the model



Investment

- The firm's value as a function of k:

$$Y(k,D) = (1-\gamma)\underbrace{\int_{z^*}^{1}\left(V(k,z) - D^*\right)dz}_{\text{Equity}} + \underbrace{\int_{z^*}^{1} D^* dz}_{\text{Debt}} - k$$

- The f.o.c for k:

$$Y_k(k,D) = \underbrace{Y_D(k,D^*)}_{=0}\frac{\partial D^*}{\partial k} + (1-\gamma)\int_{z^*}^{1} V_k(k,z)dz - D^*\underbrace{\frac{\partial z^*}{\partial k}}_{(-)} - 1$$

$$= (1-\gamma)\int_{z^*}^{1} V_k(k,z)dz - D^*\frac{\partial z^*}{\partial k} - 1 = 0$$

The effect of debt on investment

- The marginal benefit of k:

$$(1-\gamma)\int_{z^*}^{1} V_k(k,z)dz - D^* \frac{\partial z^*}{\partial k}$$

- Since V(k,z*) = D: $\frac{\partial z^*}{\partial k} = -\frac{V_k(k,z^*)}{V_z(k,z^*)} < 0$

- The effect of D:

$$-(1-\gamma)V_k(k,z^*)\frac{\partial z^*}{\partial k} - \frac{\partial z^*}{\partial k} - D^* \frac{\partial^2 z^*}{\partial k \partial D}$$

The effect of debt on investment

- Suppose that $\frac{\partial^2 z^*}{\partial k \partial D} < 0$
- The marginal benefit of k increases with D
- The firm invests more when it issues debt