●●● Pursuing the Unknown ●●● בעקבות הלא נודע

Tel Aviv University
Faculty of Exact Sciences
Exam in ODE-1, 0366-2103-01, Fall semester, Term A
25.03.2024
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Exam is 3 hours long. Simple calculator is allowed, but not needed. There are 5 questions and a bonus for group 1+ (i.e. 28).
4 questions are to be chosen to answer and are to be explicitly listed in the exam notebook.
Each question costs 25 points, bonus costs 10 points.

1.

a. (13) The DE $\dddot{y}+a_1\ddot{y}+a_2\dot{y}+\alpha y=0$ with constant real coefficients has the particular solution $\cos t-2\sin t$. Find its general solution and the coefficients a_1, a_2 as the functions of α.

b. (12) Determine whether the following constant solutions of first-order DEs are stable or unstable: $\dot{y}=-y\sqrt{|y|},\ y(0)=0$; $\dot{y}=|\sin y|,\ y(0)=0$; $\dot{y}=\cos(y^2),\ y(0)=\sqrt{\frac{\pi}{2}}$

2.

a. (14) Find the general solution and the solution of the Cauchy problem for the DE $y'-\frac{1}{2}\tan(x)y=\cos(x)y^3$ with the initial condition $y(0)=1$ (maybe as an implicit function).

b. (11) Check the stability of the equilibrium at the origin for $\alpha=0,2$, $\begin{cases}\dot{x}=\sin(x-3y)+x\cos y\\ \dot{y}=\alpha e^{x-2y}+\ln(1+x)-\alpha\end{cases}$.

3.

a. (14) Find the general solution of the DE $(x^2-xy-y^2)dx+x^2dy=0$ (maybe as an implicit function).

b. (11) The function $f:\mathbb{R}^2\to\mathbb{R}$ is known to be locally Lipschitz. Can the function $\cos t-1$ satisfy the DE $\ddot{y}=f(y,\dot{y})y$ for $t\in[-1,1]$ or for $t\in[5,7]$? Base the answer.

4. Consider the matrix $A=\begin{pmatrix}0 & 1\\ -1 & 2\end{pmatrix}$. Solve the DE $\dot{y}=Ay$, $y\in\mathbb{R}^2$, for arbitrary general initial conditions $y(0)=C\in\mathbb{R}^2$, and calculate e^{At}.

5. Solve the DE $\dot{y}=Ay+\begin{pmatrix}0\\ 0\\ e^t+1\end{pmatrix}$, $y\in\mathbb{R}^3$, for the matrix $A=\begin{pmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -2 & 1 & 2\end{pmatrix}$. One eigenvalue of A is known to be 2.

Bonus (group 1+, 10 pt): The function $f:\mathbb{R}^2\to\mathbb{R}$ is locally Lipschitz. Can both functions $\sin(2t), 2\arctan t$ simultaneously be solutions of the DE $\ddot{y}=f(y,\dot{y})$ for all $t\in[-1,1]$? Base the answer.

Good Luck!