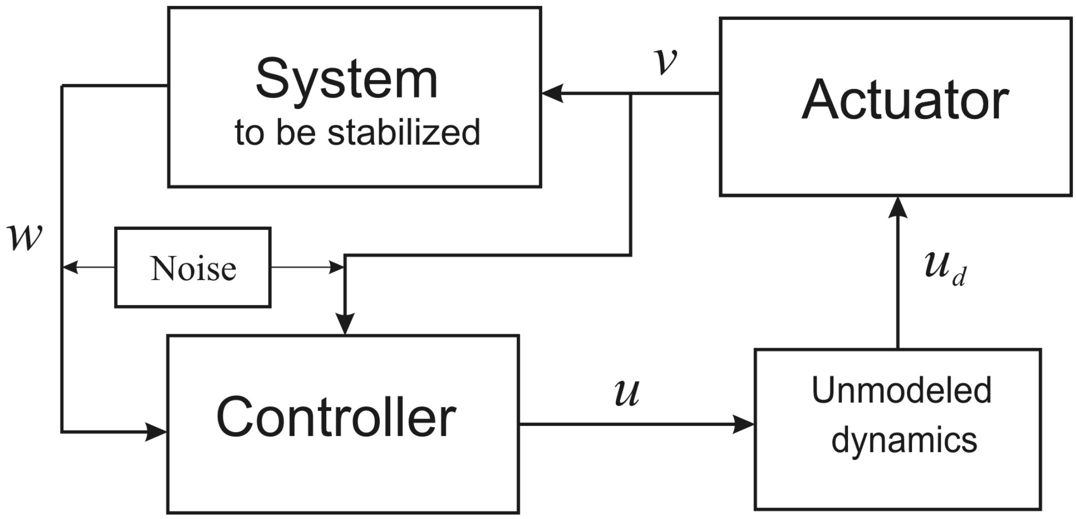
Problem 17

The system is composed of a linear system to be stabilized, a non-linear actuator and some fast stable unmodeled dynamics of the actuator. The general detailed problem description is singled out.



1. The system to be stabilized: $\dot{w} = Aw + Bv, \quad w \in \mathbb{R}^3, v \in \mathbb{R}$ is the input,

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$$

2. The actuator output is defined as $v = x_1$, where the actuator equations are

$$\dot{x}_1 = -1 + \cos 2t + 2\,x_2\,,$$

$$\dot{x}_2 = a(x_1, x_2, x_3, t) + (2 + 0.5\cos x_1)\,u_d\,,$$

$$\dot{x}_3 = \cos(x_1 - 2\,x_2 + x_3 - 10\,t) - 2\,x_3 + 0.5\,u_d\,.$$

3. The “unmodeled” (singular) dynamics (actually a part of the actuator) is for the simulation presented by the equation

$$\dddot{u}_d = -4\varepsilon^{-1}\ddot{u}_d - 1.5\varepsilon^{-2}\dot{u}_d - 5\varepsilon^{-3}(u_d - u)\,.$$

4. $a(x,t)$ represents the system uncertainty. **For a single student (I) and for a pair of students (II):** for both $a = \cos(3\cos t - x_3)$ and $a = \cos(1 + x_3)$ the same controller is to provide for the exact actuator tracking of the two command signals

$$v_c(t) = \sin(2\,t + 1) - 0.8\cos(1.3\,t), \qquad (1)$$

$$v_c(t) = -2\sin(3\,t) - \cos(t - 1) + 0.01\cos 10t.$$

בהצלחה!