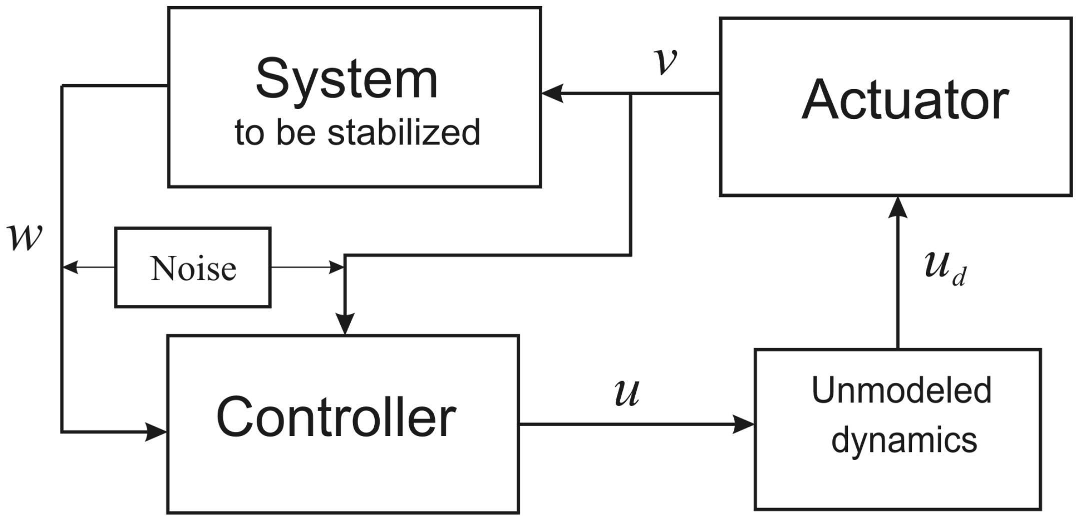
Problem 6

The system is composed of a linear system to be stabilized, a non-linear actuator and some fast stable unmodeled dynamics of the actuator. The general detailed problem description is singled out.



1. The system to be stabilized: $\dot{w} = Aw + Bv, \quad w \in \mathbb{R}^3, v \in \mathbb{R}$ is the input,

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0.5 & -1 \\ 1 & 0 & 0.5 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}.$$

2. The actuator output is defined as $v = x_1$, where the actuator equations are

$$\dot{x}_1 = -\sin t + 0.5 \cos t - x_2 ,$$

$$\dot{x}_2 = a(x_1, x_2, x_3, t) + (3 + \cos(x_1 + x_3))\, u_d ,$$

$$\dot{x}_3 = \cos(0.5\, x_1 + x_3 - 2t) - x_3 + 0.2 u_d .$$

3. The "unmodeled" (singular) dynamics (actually a part of the actuator) is for the simulation presented by the equation

$$\dddot{u}_d = -2\varepsilon^{-1}\ddot{u}_d - 3\varepsilon^{-2}\dot{u}_d - 4\varepsilon^{-3}(u_d - u).$$

4. $a(x,t)$ represents the system uncertainty. **For a single student (I) and for a pair of students (II):** for both $a = \cos(x_1 + 0.5\, x_3)$ and $a = \cos(x_1 - 0.5\, x_3)$ the same controller is to provide for the exact actuator tracking of the two command signals

$$v_c(t) = \cos(t - 2) + \sin(0.5\, t - 1), \qquad (1)$$

$$v_c(t) = \sin(t - 2) - 0.3 \cos(2t - 2) + 0.01 \cos 10t.$$

בהצלחה!