Problem 3

The system is composed of a linear system to be stabilized, a non-linear actuator and some fast stable unmodeled dynamics of the actuator. The general detailed problem description is singled out.

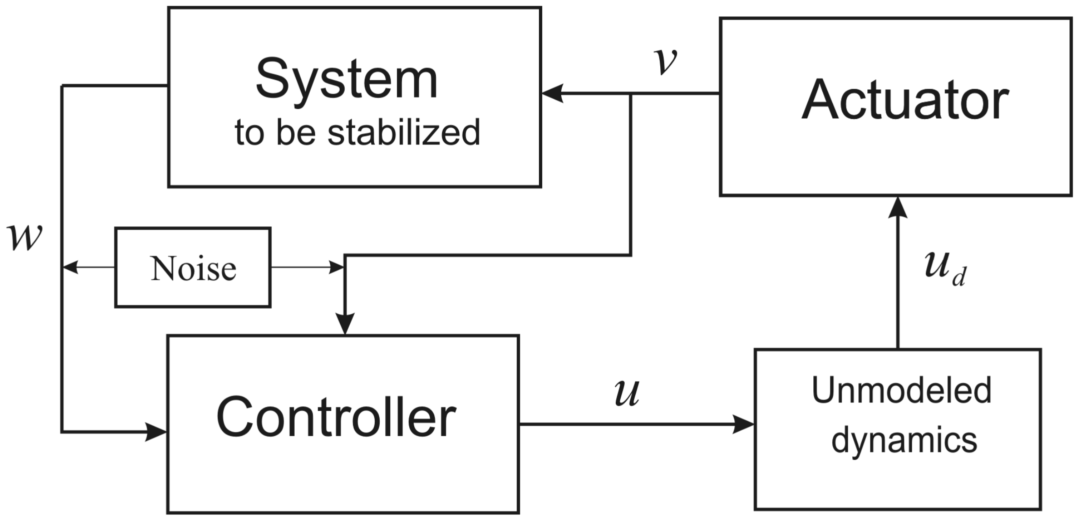

Fig. 1

1. The system to be stabilized: $\dot{w} = Aw + Bv, \quad w \in \mathbb{R}^3, v \in \mathbb{R}$ is the input,

$$A = \begin{pmatrix} 0.5 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & 0 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}.$$

2. The actuator output is defined as $v = x_1$, where the actuator equations are

$$\dot{x}_1 = \sin t + \cos 2t + 1 + x_2 ,$$

$$\dot{x}_2 = a(x_1, x_2, x_3, t) - (2 + \cos(x_1 + x_3))\, u_d ,$$

$$\dot{x}_3 = \cos (x_1 - x_3 - 2t) - x_3 - 0.4\, u_d .$$

3. The “unmodeled” (singular) dynamics (actually a part of the actuator) is for the simulation presented by the equation

$$\dddot{u}_d = -\varepsilon^{-1}\ddot{u}_d - 3\varepsilon^{-2}\dot{u}_d - \varepsilon^{-3}(u_d - u).$$

4. $a(x,t)$ represents the system uncertainty. **For a single student (I) and for a pair of students (II):** for both $a = \cos (x_1 + 2x_3)$ and $a = \cos (x_1 - 2x_3)$ the same controller is to provide for the exact actuator tracking of the two command signals

$v_c(t) = \cos 2t - \sin t + 1.5,$ (1)

$v_c(t) = 2 \sin t + 0.3 \cos 2t - 0.01 \cos 10t.$

בהצלחה!