# Instructions and clarification

The main hint for the solution is that the problem is divided into subproblems, which are solved separately one after another.

Integration is performed by the Euler method. **The integration/sampling step should not exceed 0.0001.**

**Useful option** (though probably not needed): One can postpone the application of the control until the differentiator converges. For example, one keeps $u = 0$ until $t = t_0 + 1$ (everything else is working, just $du = 0$ is kept with the continuous control or $u = 0$ with the discontinuous one), and only then starts to apply the control. In the continuous control case $u$ then starts to change from 0.

Essential information is provided in

https://www.tau.ac.il/~levant/Levant-SMC2019_for_students.pdf

For each solved problem I would like to see all the formulas of the controller (including integrator, and differentiator when applicable), all parameter values and characteristic graphs. This includes tracking results, graphs of $\sigma, \dot{\sigma}, \ddot{\sigma}$ for the 3-sliding mode and $\sigma, \dot{\sigma}$ for the 2-sliding one, stabilization graphs of $w$. Recall that $\sigma = v - v_c$. I do not need 40+ graphs, only the representative ones are required.

Stabilization accuracies of $w$ are requested to be provided at the end part of the simulation interval, since the convergence is exponential. The stabilization/tracking accuracies of $\sigma, \dot{\sigma}, \ddot{\sigma}$ for the 3-sliding mode and $\sigma, \dot{\sigma}$ for the 2-sliding mode are to be provided for the steady state interval.

# Project for one or two students.

The task is to make the output of a nonlinear system, called "actuator" in the general task, to track given signals, and to check the robustness of the obtained control. One student does not consider the unmodeled dynamics.

Only 2 controllers are to be designed: one discontinuous and one continuous.

Each of these two controllers is designed in two steps: 1. with all measurements available and 2. only the actuator output $v = x_1$ is available in "real time". In the latter case an output feedback controller is produced. Only simplest calculations are needed. Controllers and differentiators have the standard form, which appeared in the lectures. Please use the document

https://www.tau.ac.il/~levant/Levant-SMC2019_for_students.pdf

Let the "actuator" model be $\dot{x} = f(t,x,u), x \in \mathbf{R}^3, u \in \mathbf{R}, \sigma = v(t,x) - v_c(t)$. The task is to make σ vanish.

## A. Simulation of a **discontinuous controller**

a. **Full measurements are available**. Here the rel. degree is 2, $\sigma(t), \dot{\sigma}(t)$ are "measured in real time", i.e. they are calculated using the given formulas, $u(t) = \alpha U_2(\sigma(t), \dot{\sigma}(t))$ is one of the standard controllers, α is the magnitude of the control.

**Important**: the controller $U_2(\sigma, \dot{\sigma})$ is ± the standard form, depending on the sign of the obtained control coefficient in $\ddot{\sigma}$.

Choose the initial time $t_0$ and the final time $t_1$ for simulation. The convergence time should constitute about 1/3 of the whole simulation time. Thus usually $t_0 = 0$, $t_1 = 10$ is enough. Choose some initial conditions $x_0$. The simulation scheme:

$t = t_0$; $x = x_0$;

while($t \le t_1$)

{

$dx = f(t,x,u);$

$\sigma = v(t,x) - v_c(t); \dot{\sigma} = \dot{v} - \dot{v}_c;$

$u = \alpha U_2(\sigma, \dot{\sigma});$

$x := x + dx \cdot \tau;$

$t := t + \tau;$

}

b. **Output-feedback control.** Only $\sigma(t)$ is available in real time. Output-feedback controller includes a differentiator as a part of the controller. Here $\sigma(t), \dot\sigma(t)$ are evaluated in real time by the variables $z_0, z_1$ of a standard or filtering differentiator $\dot z = D_2(L, z_f, z, \sigma)$, where $L$ is its parameter, $z_f$ contains the corresponding number of filtering variables, $u(t) = \alpha U_2(z_0, z_1)$ is the controller previously chosen.

Note that $L$ depends on the control magnitude, i.e. on $\alpha$. Larger $\alpha$ requires larger $L$.

The simulation scheme:

$t = t_0$; $x = x_0$; $\sigma = v(t,x) - v_c(t)$;

$z_0 = \sigma; z_1 = 0;$

while( $t \le t_1$ )

{

$dx = f(t,x,u);$
$\sigma = v(t,x) - v_c(t);$
$d(z_f, z) = D_2(L, z_f, z, \sigma);$
$u = \alpha U_2(z_0, z_1);$
$x := x + dx \cdot \tau;$
$(z_f, z) := (z_f, z) + d(z_f, z) \cdot \tau;$
$t := t + \tau;$

}

## A*. Simulation of a **continuous controller**.

An integrator is included into the controller increasing the relative degree by one.

a. **Full measurements are available**. Here the rel. degree is 3, $\sigma(t), \dot\sigma(t), \ddot\sigma(t)$ are "measured in real time", i.e. they are calculated analytically using the given formulas, $\dot u(t) = \alpha U_3(\sigma(t), \dot\sigma(t), \ddot\sigma(t))$ is one of the standard controllers, $\alpha$ is the magnitude of the control.

**Important**: the controller $U_3(\sigma, \dot\sigma, \ddot\sigma)$ is $\pm$ the standard form, depending on the sign of the obtained control coefficient in $\dddot\sigma$.

Choose the initial value $u_0$ of the control, usually $u_0 = 0$. The simulation scheme takes the form

$t = t_0$; $x = x_0$; $u = u_0$;

while($t \le t_1$)

{

$dx = f(t,x,u);$

$\sigma = v(t,x) - v_c(t); \dot{\sigma} = \dot{v} - \dot{v}_c; \ddot{\sigma} = \ddot{v} - \ddot{v}_c;$

$du = \alpha U_3(\sigma,\dot{\sigma},\ddot{\sigma});$

$x := x + dx \cdot \tau;$

$u := u + du \cdot \tau;$

$t := t + \tau;$

}

b. **Output-feedback control.** Only $\sigma(t)$ is available in real time. Output-feedback controller includes a differentiator as a part of the controller. Here $\sigma(t), \dot{\sigma}(t), \ddot{\sigma}(t)$ are evaluated in real time by the variables $z_0, z_1, z_2$ of a standard or filtering differentiator $(\dot{z}_f, \dot{z}) = D_3(L, z_f, z, \sigma)$, where $L$ is its parameter, $z_f$ are the filtering variables, $\dot{u}(t) = \alpha U_3(z_0, z_1, z_2)$ is the controller previously chosen.

The simulation scheme:

$t = t_0$; $x = x_0$; $\sigma = v(t,x) - v_c(t);$

$z_0 = \sigma; z_1 = 0; z_2 = 0;$ $z_f = 0;$

while($t \le t_1$)

{

$dx = f(t,x,u);$

$\sigma = v(t,x) - v_c(t);$

$d(z_f, z) = D_3(L, z_f, z, \sigma);$

$du = \alpha U_3(z_0, z_1, z_2);$

$x := x + dx \cdot \tau;$

$u := u + du \cdot \tau;$

$(z_f, z) := (z_f, z) + d(z_f, z) \cdot \tau;$

$t := t + \tau;$

}

Inclusion of the differentiator does not change the controller using full measurements, it only supplies these measurements.

Thus, at the end, all over the project you have only two values of α: one for the discontinuous controller and one for the continuous one. The same is true for the differentiator parameter *L*.

## B. Adding unmodeled dynamics and noises to check robustness.

The sense of the unmodeled dynamics is that the designer does not know on its existence. The designer also assumes that there is a measurement noise, but he knows nothing about its features or intensity.

Only the homogeneous output-feedback controller is robust with respect to unmodeled dynamics. **Without a differentiator the robustness may be lost.** The noises and the unmodeled dynamics are added to the output feedback controllers developed at the substages b and b* of the A stage. Let the unmodeled dynamics be $\varepsilon\dot{\zeta} = \Phi(\zeta,u), u_d = U_d(\zeta)$, $\zeta \in \mathbf{R}^3$. So that with $\varepsilon = 0$ one gets $u_d = u$.

Choose the initial value $\zeta_0$ of the state of the unmodeled dynamics.

The noise is defined by some random or quasi-random function $\eta(t)$ of a given magnitude.

Now the simulation of the **continuous output-feedback control** looks as follows**.**

$t = t_0$; $x = x_0$; $\zeta = \zeta_0$; $\sigma = v(t,x) - v_c(t)$;

$z_0 = \sigma(t,x) + \eta(t); z_1 = 0; z_2 = 0;$ $z_f = 0$;

while( $t \le t_1$ )

{

$d\zeta = \varepsilon^{-1}\Phi(\zeta,u);$
$u_d = U_d(\zeta);$
$dx = f(t,x,u_d);$
$\sigma = v(t,x) - v_c(t);$
$d(z_f,z) = D_3(L, z_f, z, \sigma + \eta(t));$
$du = \alpha U_3(z_0, z_1, z_2);$
$\zeta := \zeta + d\zeta \cdot \tau;$
$x := x + dx \cdot \tau;$
$u := u + du \cdot \tau;$
$(z_f, z) := (z_f, z) + d(z_f, z) \cdot \tau;$
$t := t + \tau;$

}

Thus the solution of the problem consists of a discontinuous controller + its output feedback version and a continuous controller + its output feedback version, which provide for the solution of the stated tracking problems. Respectively one has one pair $\alpha, L$ for the discontinuous controller and another pair $\alpha, L$ for the continuous one. Usually, $\alpha$ =10, L = 50 are sufficient for the discontinuous controller and $\alpha$ =20, L = 100 are sufficient for the continuous controller. These numbers are only listed for an example, without any guaranty for each concrete problem.

For each solved problem I would like to see all the formulas of the controller (including integrator, and differentiator when applicable), all parameter values and characteristic graphs. This includes tracking results, graphs of $u, \dot{u}$, $\sigma, \dot{\sigma}, \ddot{\sigma}$ for the 3-sliding mode and $u$, $\sigma, \dot{\sigma}$ for the 2-sliding one. I do not require 40+ graphs, but only the most representative ones.

# Project for three students.

A. First the A, A* problems for one student are solved. Carefully read all the text above. Choose any simple signal like $\cos(5t)$ for the tracking by the actuator output and check that the tracking problem is solved.

Now construct a stabilizing input $v_c = cw$ for the linear dynamics $\dot{w} = Aw + Bv_c$. Choose **small negative eigenvalues** for the closed loop linear dynamics $\dot{w} = (A + Bc)w$. For example choose the eigenvalues -0.1, -0.2, -0.3 in order to get a non-demanding tracking problem for the actuator.

Do not take large initial values of *w*, initial values of the order of 1 will suffice. The simulation interval for the whole closed-loop system should be a bit longer ($t \in [0,20]$ or $t \in [0,30]$)

The simulation scheme for **continuous output-feedback control** looks as follows**.**

$t = t_0$; $x = x_0$; $w = w_0$; $\sigma = v(t,x) - cw$;

$z_0 = \sigma; z_1 = 0; z_2 = 0;$

while($t \le t_1$)

{

$v = v(t,x); \sigma = v(t,x) - cw;$

$dw = Aw + Bv;$

$dx = f(t,x,u);$

$d(z_f, z) = D_3(L, z_f, z, \sigma);$

$du = \alpha U_3(z_0, z_1, z_2);$

$x := x + dx \cdot \tau;$

$u := u + du \cdot \tau;$

$(z_f, z) := (z_f, z) + d(z_f, z) \cdot \tau;$

$w := w + dw \cdot \tau;$

$t := t + \tau;$

}

Choose sufficiently large α, L to provide for the convergence of the system. One will probably needs larger α and L to track the virtual control $v_c$ designed for the linear system. The values α = 100, *L* =500 are still feasible, increasing them more one will need to decrease the integration step to $10^{-5}$ or even less.

B. **Add unmodeled dynamics and noises to the system with the output-feedback controller**. In order to decrease the influence of the unmodeled dynamics and noises take small noises and small epsilon. Don't change α and L. You also have only one structure of the control $v_c$ for *w*.

Choose some 3-dimensional noise $\eta_w(t)$ of a given magnitude for the measurements of *w*. Denote $\hat{\sigma}$ the result of the noisy measurements. Now the simulation scheme for **continuous output-feedback control** looks as follows**.**

$$
\begin{aligned}
&t = t_0;\; x = x_0;\; \zeta = \zeta_0;\; w = w_0;\; \sigma = v(t,x) - cw;\\
&z_0 = \hat{\sigma} = v(t,x) - c(w + \eta_w(t)) + \eta(t); z_1 = 0; z_2 = 0;\\
&\text{while}(t \le t_1)\\
&\{\\
&\qquad d\zeta = \varepsilon^{-1}\Phi(\zeta, u);\\
&\qquad u_d = U_d(\zeta);\\
&\qquad v = v(t,x); \sigma = v(t,x) - cw;\\
&\qquad \hat{\sigma} = v(t,x) - c(w + \eta_w(t)) + \eta(t);\\
&\qquad dw = Aw + Bv;\\
&\qquad dx = f(t, x, u_d);\\
&\qquad d(z_f, z) = D_3(L, z_f, z, \hat{\sigma});\\
&\qquad du = \alpha U_3(z_0, z_1, z_2);\\
&\qquad \zeta := \zeta + d\zeta \cdot \tau;\\
&\qquad x := x + dx \cdot \tau;\\
&\qquad u := u + du \cdot \tau;\\
&\qquad (z_f, z) := (z_f, z) + d(z_f, z) \cdot \tau;\\
&\qquad w := w + dw \cdot \tau;\\
&\qquad t := t + \tau;\\
&\}
\end{aligned}
$$

Thus the solution of the problem consists of a discontinuous controller + its output feedback version and a continuous controller + its output feedback version, which provide for the solution of the stated tracking problems. Respectively one has the row vector *c* for the stabilization of *w*, one pair $\alpha, L$ for the discontinuous controller and another pair $\alpha, L$ for the continuous one. **The problem can only be solved locally in such a way.** It means that the larger are the initial values of *w* the larger $\alpha, L$ one needs.

If you don't succeed to stabilize the system, take smaller initial values of *w*, smaller eigenvalues, larger $\alpha, L$, and, possibly, decrease the integration step. That is not a practical problem, which means that one can choose very large $\alpha, L$. But $\alpha$ in thousands and *L* in tens of thousands will require the integration step $10^{-5}$ or probably even $10^{-6}$. If the problem appears with unmodeled dynamics and noises, decrease the parameter $\varepsilon$ and the noises. For example, take $\varepsilon = 0.001$. It also can require smaller integration step.

For each solved problem I would like to see all the formulas of the controller (including integrator, and differentiator when applicable), all parametric values and characteristic graphs. This includes graphs of the norm of *w*, tracking results, graphs of $\sigma, \dot{\sigma}, \ddot{\sigma}$ for the 3-sliding mode and $\sigma, \dot{\sigma}$ for the 2-sliding one. I do not require 40+ graphs, but only the most representative ones.