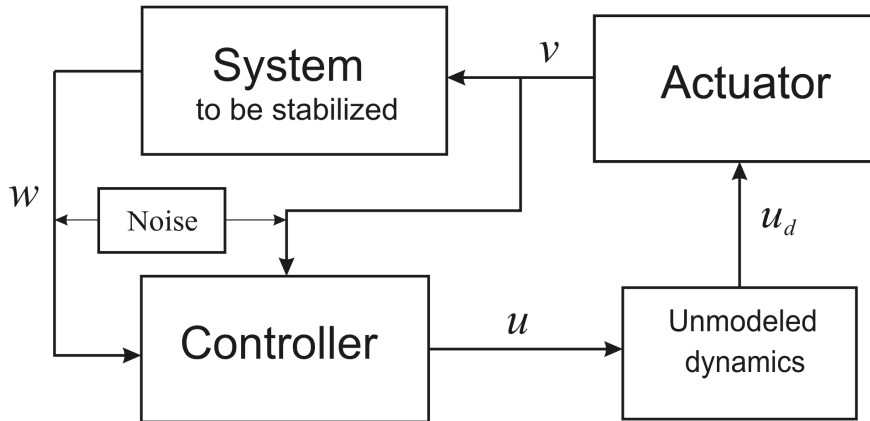


General Problem Statement (on the example of Problem 20)

Consider a system composed of a linear system to be stabilized, a non-linear actuator and some fast stable unmodeled dynamics of the actuator. All variables are real numbers.



1. The system to be stabilized: $\dot{w} = Aw + Bv$, $w \in \mathbb{R}^3, v \in \mathbb{R}$ is the input,

$$A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$$

2. The actuator output is defined as $v = x_1$, where the actuator equations are

$$\dot{x}_1 = 1 + \cos t - x_2,$$

$$\dot{x}_2 = a(x_1, x_2, x_3, t) - (3 + 2 \cos x_3) u_d,$$

$$\dot{x}_3 = \sin(x_1 - x_2 + x_3 - 5t) - x_3 + 0.5 u_d.$$

3. The “unmodeled” (singular) dynamics (actually a part of the actuator) is for the simulation presented by the equation

$$\ddot{u}_d = -2\varepsilon^{-1}\dot{u}_d - 3\varepsilon^{-2}u_d - 2\varepsilon^{-3}(u_d - u).$$

Here $\varepsilon > 0$ is a small parameter. The smaller $\varepsilon > 0$ the faster the dynamics is, and the closer its output u_d is to the input u .

The general task is to provide for the asymptotic stabilization of the system state w by means of a proper control u using the real-time-sampled values of $w \in \mathbb{R}^3$ and $v \in \mathbb{R}$. The problems for one and two students are subproblems of this one.

The solution is a combination of the controller u and simulation results represented in a pdf or word file by all formulas, parametric and initial values and essential graphs. Provide the accuracies of deviations w , the tracking error $v - v_c$ and its respective derivatives, as maximal absolute values over a steady-state time interval. **The integration/sampling step should not exceed 0.0001.**

Essential information appears in the document

https://www.tau.ac.il/~levant/Levant-SMC2019_for_students.pdf

I. Problem for one student: CONTROL OF ACTUATOR ONLY

The unmodeled dynamics is absent ($\varepsilon = 0$, $u_d \equiv u$).

A. Provide for $v = v_c(t)$ in finite time, by means of a discontinuous bounded control and for arbitrary initial conditions of x .

a. using full measurements of everything, including needed derivatives of $v_c(t)$.

b. using only real-time measurements of the difference $v(t) - v_c(t)$, actuator variables are not available.

The same controller is to work for both $a = \cos(3 + x_3)$ and $a = \cos(x_2 - 2x_3)$, as well as for the two command signals

$$\begin{aligned}v_c(t) &= 2 \sin(2t + 1) - 1.5 \cos(1.3t), \\v_c(t) &= 3 \cos(2t) - \cos(1.2t - 1) + 0.01 \cos 10t.\end{aligned}\tag{1}$$

A*. To solve the same problems by means of a continuous control with zero initial conditions of x and command signal (1) only.

B. To check the robustness of the designed output-feedback controllers with respect to small sampling noises of the measurements $v(t) - v_c(t)$. The noises of the magnitudes 0.01, 0.001 are to be considered.

Any noises can be taken, random are preferred, but also simple high-frequency harmonic noises will do.

Bonus (6 points): *Try larger noises if the differentiator filtering order is 1 and higher.*

The solution is a combination of chosen controllers and simulation results represented in a pdf or word file by all formulas, parameter values and essential graphs, including the graphs of the error $v - v_c$, and its derivatives (up to the order relative degree - 1), and of the control u . List the corresponding accuracies. Problem A alone costs maximum 80 points, A* - 90 points, B adds 6 points to A and A*. Both problems A+B, A*+ B are to be solved in order to get 100. No proofs are needed, if the controllers have one of the known forms. Only simulation is to be presented.

Hints: Take any standard controller and find the coefficients by simulation using full measurements. Only after step a. is done, include a differentiator. Find its parameter L by simulation. Problem A* is solved by artificially increasing the relative degree.

Any questions are welcome.

בהצלחה!

II. Problem for two students: ACTUATOR WITH UNMODELED DYNAMICS

A. The unmodeled dynamics is absent: $u_d \equiv u$, i.e. $\varepsilon = 0$. Provide for $v = v_c(t)$ in finite time, by means of a discontinuous bounded control and for arbitrary initial conditions of x ,

a. using full measurements of everything, including needed derivatives of $v_c(t)$.

b. using only real-time measurements of the difference $v(t) - v_c(t)$, actuator variables are not available.

The same controller is to work for both $a = \cos(3 + x_3)$ and $a = \cos(x_2 - 2x_3)$, as well as for the two command signals

$$v_c(t) = 2 \sin(2t + 1) - 1.5 \cos(1.3t), \quad (1)$$

$$v_c(t) = 3 \cos(2t) - \cos(1.2t - 1) + 0.01 \cos 10t.$$

A*. To solve the same problems by means of a continuous control with zero initial conditions of x and command signal (1) only.

B. To check the robustness of the designed two output-feedback controllers (A, A*) to the presence of the unmodeled dynamics and noisy measurements of v . The initial values of the dynamics satisfy $|\ddot{u}_d|, |\dot{u}_d|, |u_d| \leq 5$. The values $\varepsilon = 0.02, 0.01, 0.005$ are to be considered. The noise magnitudes are to be 0.01 and 0.001. Any noises can be taken, random are preferred, but also simple high-frequency harmonic noises will do. **Bonus (6 points):** Try larger noises if the differentiator filtering order is 1 and higher.

The solution is a combination of chosen controllers and simulation results represented in a pdf or word file by all formulas, parameter values and essential graphs, including the graphs of the error $v - v_c$, and its derivatives (up to the order the relative degree - 1), and of the control u . List the corresponding accuracies. Problem A alone costs maximum 50 points, A* - 60 points, B adds 30 points to A and A*. Both problems A+B, A*+ B are to be solved in order to get 100. No proofs are needed, if the controllers have one of the known forms. Only simulation is to be presented.

Hints: Take any standard controller and find the coefficients by simulation using full measurements without unmodeled dynamics. Only after step a. is done, include a differentiator (step b.). Find its parameter L by simulation. Problem A* is solved by artificially increasing the actuator relative degree. Now include the unmodeled dynamics and noises.

Any questions are welcome.

בהצלחה!

III. Problem for three students: FULL SYSTEM STABILIZATION

A. The unmodeled dynamics is absent: $u_d \equiv u$, i.e. $\varepsilon = 0$. Develop a linear feedback $v = v_c = kw$, providing for the asymptotic stabilization of w at zero. Provide for $v \equiv v_c$ in finite time by means of a discontinuous bounded control and for arbitrary initial conditions of x, w , $|x_i|, |w_i| \leq 5$:

a. using full measurements of everything, including needed derivatives of $v_c(t)$.

b. only using real-time measurements of the difference $v(t) - v_c(t)$ and of the coordinates $w \in \mathbb{R}^3$. Internal actuator variables are not available.

Respectively the system is asymptotically stabilized. The same controller is to work for both $a = \cos(3 + x_3)$ and $a = \cos(x_2 - 2x_3)$.

A*. To solve the same problems by means of continuous control u with zero initial conditions of x .

B. To check the robustness of the designed two output-feedback controllers (A, A*) to the presence of the unmodeled dynamics and noisy measurements of w and v . The initial values of the dynamics satisfy $|\ddot{u}_d|, |\dot{u}_d|, |u_d| \leq 5$. The values $\varepsilon = 0.02, 0.01, 0.005$ are to be considered. The noise magnitudes are to be 0.01 and 0.001. Any noises can be taken, random are preferred, but also simple high-frequency harmonic noises will do. The magnitude of a Gaussian signal is defined as two standard deviations.

Bonus (6 points): *Try larger noises if the differentiator filtering order is 1 and higher.*

The solution is a combination of chosen controllers and simulation results represented in a pdf or word file by all formulas, parameter values and essential graphs, including the graphs of w_1, w_2, w_3 , of the error $v - v_c$, and its derivatives (up to the order relative degree - 1), and of the control u . List the corresponding accuracies. Problem A alone costs maximum 60 points, A* - 70 points, B adds 20 points to A and A*. Both problems A+B, A* + B are to be solved in order to get 100. No proofs are needed, if the controllers have one of the known forms. Only simulation is to be presented.

Hints: Take any standard controller and find the coefficients by simulation, using full measurements without unmodeled dynamics and with a fixed function $v_c(t)$. Only after step a. is done, include a differentiator (step b.). Find its parameter L by simulation and take it large enough. Separately stabilize the w -system. Then include it into the composite one. Possibly increase L . Problem A* is solved by artificially increasing the actuator relative degree. Now include the unmodeled dynamics and noises.

Any questions are welcome.

בהצלחה!

IV. Problem for four students: FULL SYSTEM STABILIZATION

The tasks are the same as for three students with the following difference and addition:

1. The bonus is removed, but the students are still asked to take the differentiator filtering order 1 or higher and to try larger noises.
2. In addition to other tasks, in the task B the students are requested to check the performance of the output-feedback controller for **different** filtering orders of the differentiator varying from 1 to 7. If the subtask A* is done, then it is enough to perform the checking only for A* + B (not for A + B).

Any questions are welcome.

בהצלחה!