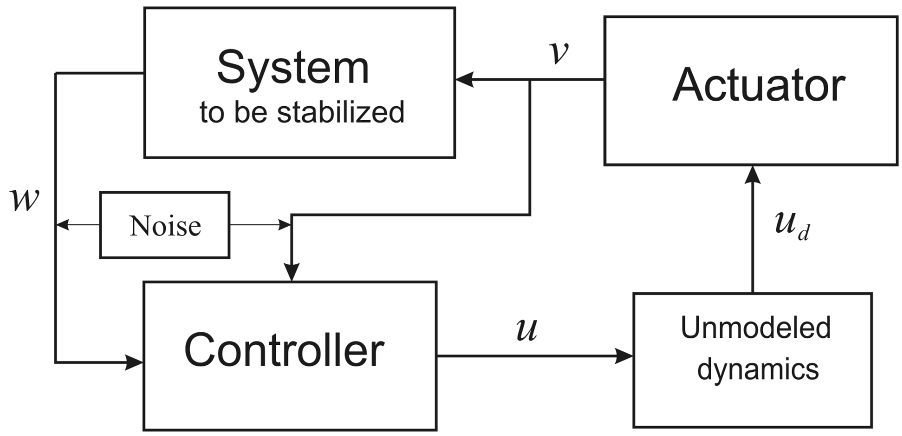
General Problem Statement (on the example of Problem 20)

Consider a system composed of a linear system to be stabilized, a non-linear actuator and some fast stable unmodeled dynamics of the actuator. All variables are real numbers.



1. The system to be stabilized: $\dot{w} = Aw + Bv, \quad w \in \mathbb{R}^3, v \in \mathbb{R}$ is the input,

$$A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}.$$

2. The actuator output is defined as $v = x_1$, where the actuator equations are

$$\dot{x}_1 = 1 + \cos t - x_2 \,,$$

$$\dot{x}_2 = a(x_1, x_2, x_3, t) - (3 + 2\cos x_3)\, u_d \,,$$

$$\dot{x}_3 = \sin(x_1 - x_2 + x_3 - 5t) - x_3 + u_d \,.$$

3. The “unmodeled” (singular) dynamics (actually a part of the actuator) is for the simulation presented by the equation

$$\dddot{u}_d = -2\varepsilon^{-1}\ddot{u}_d - 3\varepsilon^{-2}\dot{u}_d - 2\varepsilon^{-3}(u_d - u).$$

Here $\varepsilon > 0$ is a small parameter. The smaller $\varepsilon > 0$ the faster the dynamics is, and the closer its output u_d is to the input u.

The general task is to provide for the asymptotic stabilization of the system state w by means of a proper control u using the real-time-sampled values of w and v. The problems for one and two students are subproblems of this one.

The solution is a combination of a controller and simulation results represented in a pdf or word file by all formulas, parametric and initial values and essential graphs. Provide the accuracies of deviations w, the tracking error $v - v_c$ and its respective derivatives, as maximal absolute values over a steady-state time interval. **The integration/sampling step should not exceed 0.0001.**

Essential information appears in the document

https://www.tau.ac.il/~levant/Levant-SMC2019_for_students.pdf

General Problem Statement (on the example of Problem 20)

I. Problem for one student: CONTROL OF ACTUATOR ONLY

The unmodeled dynamics is absent ($\varepsilon = 0$, $u_d \equiv u$).

A. Provide for $v = v_c(t)$ in finite time, by means of a discontinuous bounded control and for arbitrary initial conditions of x.

a. using full measurements of everything, including needed derivatives of $v_c(t)$.

b. using only real-time measurements of the difference $v(t) - v_c(t)$, actuator variables are not available.

The same controller is to work for both $a = \cos(3 + x_3)$ and $a = \cos(x_2 - 2x_3)$, as well as for the two command signals

$$v_c(t) = 2\sin(2t + 1) - 1.5\cos(1.3t), \qquad (1)$$

$$v_c(t) = 3\cos(2t) - \cos(1.2t - 1) + 0.01\cos 20t.$$

A*. To solve the same problems by means of a continuous control with zero initial conditions of x and command signal (1) only.

B. To check the robustness of the designed output-feedback controllers with respect to small sampling noises of the measurements $v(t) - v_c(t)$. The noises of the magnitudes 0.01, 0.001 are to be considered. Any noises can be taken, random are preferred, but also simple high-frequency harmonic noises will do.

Bonus (6 points): *Try larger noises if the differentiator filtering order is 1 and higher.*

The solution is a combination of chosen controllers and simulation results represented in a pdf or word file by all formulas, parameter values and essential graphs, including the graphs of the error $v - v_c$, and its derivatives (up to the order relative degree - 1), and of the control u. List the corresponding accuracies. Problem A alone costs maximum 80 points, A* - 90 points, B adds 6 points to A and A*. Both problems A+B, A*+ B are to be solved in order to get 100. No proofs are needed, if the controllers have one of the known forms. Only simulation is to be presented.

Hints: Take any standard controller and find the coefficients by simulation using full measurements. Only after step a. is done, include a differentiator. Find its parameter L by simulation. Problem A* is solved by artificially increasing the relative degree.

Any questions are welcome.

בהצלחה!

General Problem Statement (on the example of Problem 20)

II. Problem for two students: ACTUATOR WITH UNMODELED DYNAMICS

A. The unmodeled dynamics is absent: $u_d \equiv u$, i.e. ε = 0. Provide for $v = v_c(t)$ in finite time, by means of a discontinuous bounded control and for arbitrary initial conditions of x,

a. using full measurements of everything, including needed derivatives of $v_c(t)$.

b. using only real-time measurements of the difference $v(t) - v_c(t)$, actuator variables are not available.

The same controller is to work for both $a = \cos(3 + x_3)$ and $a = \cos(x_2 - 2x_3)$, as well as for the two command signals

$$v_c(t) = 2\sin(2t + 1) - 1.5\cos(1.3t), \qquad (1)$$

$$v_c(t) = 3\cos(2t) - \cos(1.2t - 1) + 0.01\cos 20t.$$

A*. To solve the same problems by means of a continuous control with zero initial conditions of x and command signal (1) only.

B. To check the robustness of the designed two output-feedback controllers (A, A*) to the presence of the unmodeled dynamics and noisy measurements of v. The initial values of the dynamics satisfy $|\ddot{u}_d|, |\dot{u}_d|, |u_d| \leq 5$. The values $\varepsilon = 0.02, 0.01, 0.005$ are to be considered. The noise magnitudes are to be 0.01 and 0.001. Any noises can be taken, random are preferred, but also simple high-frequency harmonic noises will do. **Bonus (6 points)**: *Try larger noises if the differentiator filtering order is 1 and higher.*

The solution is a combination of chosen controllers and simulation results represented in a pdf or word file by all formulas, parameter values and essential graphs, including the graphs of the error $v - v_c$, and its derivatives (up to the order the relative degree - 1) , and of the control u. List the corresponding accuracies. Problem A alone costs maximum 50 points, A* - 60 points, B adds 30 points to A and A*. Both problems A+B, A*+ B are to be solved in order to get 100. No proofs are needed, if the controllers have one of the known forms. Only simulation is to be presented.

Hints: Take any standard controller and find the coefficients by simulation using full measurements without unmodeled dynamics. Only after step a. is done, include a differentiator (step b.). Find its parameter L by simulation. Problem A* is solved by artificially increasing the actuator relative degree. Now include the unmodeled dynamics and noises.

Any questions are welcome.

בהצלחה!

General Problem Statement (on the example of Problem 20)

III. Problem for three students: FULL SYSTEM STABILIZATION

A. The unmodeled dynamics is absent: $u_d \equiv u$, i.e. ε = 0. Develop a linear feedback $v = v_c = cw$, providing for the asymptotic stabilization of *w* at zero. Provide for $v \equiv v_c$ in finite time by means of a discontinuous bounded control and for arbitrary initial conditions of *x*, *w*, $|x_i|, |w_i| \leq 5$:

a. using full measurements of everything, including needed derivatives of $v_c(t)$.

b. only using real-time measurements of the difference $v(t)$ - $v_c(t)$ and of the coordinates $w \in \mathbb{R}^3$. Internal actuator variables are not available.

Respectively the system is asymptotically stabilized. The same controller is to work for both $a = \cos(3 + x_3)$ and $a = \cos(x_2 - 2x_3)$.

A*. To solve the same problems by means of continuous control *u* with zero initial conditions of *x*.

B. To check the robustness of the designed two output-feedback controllers (A, A*) to the presence of the unmodeled dynamics and noisy measurements of *w* and *v*. The initial values of the dynamics satisfy $|\ddot{u}_d|, |\dot{u}_d|, |u_d| \leq 5$. The values $\varepsilon = 0.02, 0.01, 0.005$ are to be considered. The noise magnitudes are to be 0.01 and 0.001. Any noises can be taken, random are preferred, but also simple high-frequency harmonic noises will do. **Bonus (6 points)**: *Try larger noises if the differentiator filtering order is 1 and higher.*

The solution is a combination of chosen controllers and simulation results represented in a pdf or word file by all formulas, parameter values and essential graphs, including the graphs of $w_{1,2,3}$, of the error v - v_c, and its derivatives (up to the order relative degree - 1), and of the control *u*. List the corresponding accuracies. Problem A alone costs maximum 60 points, A* - 70 points, B adds 20 points to A and A*. Both problems A+B, A* + B are to be solved in order to get 100. No proofs are needed, if the controllers have one of the known forms. Only simulation is to be presented.

Hints: Take any standard controller and find the coefficients by simulation, using full measurements without unmodeled dynamics and with a fixed function $v_c(t)$. Only after step a. is done, include a differentiator (step b.). Find its parameter *L* by simulation and take it large enough. Separately stabilize the *w*-system. Then include it into the composite one. Possibly increase *L*. Problem A* is solved by artificially increasing the actuator relative degree. Now include the unmodeled dynamics and noises.

Any questions are welcome.

בהצלחה!

IV. Problem for four students: FULL SYSTEM STABILIZATION

The tasks are the same as for three students with the following difference and addition:

1. The bonus is removed, but the students are still asked to take the differentiator filtering order 1 or higher and to try larger noises.

2. In addition to other tasks, in the task B the students are requested to check the performance of the output-feedback controller for **different** filtering orders of the differentiator varying from 1 to 4. If the subtask A* is done, then it is enough to perform the checking only for A* + B (not for A + B). *That is a research problem: I have some conjecture here.*

Any questions are welcome.

בהצלחה!