TWO TIER SEMANTICS FOR RELATIVE CLAUSES

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PART ONE: CLASSICAL SEMANTICS AND TWO TIER SEMANTICS

CLASSICAL SEMANTICS:
0. Compositionality: The meanings of complex expressions are built from the meanings of the parts.

1. Restricted set of available operations
-functional application
-function composition
-type shifting operations (algebraic lifts, domain shifts, general grammatically significant operations like intersection, closure under sum, supremum, existential closure, projection…)
2. Restricted notion of meaning
Meaning are truthfunctional contents.
3. Restricted domain of application:
locality: An operation used at a stage of the derivation can only use the information that is locally available (No access to earlier or later stages of the derivation).

CAVEATS:
The classical theory is aware of the following:
-Context: contextual restriction, contextual comparison class, etc.
-Pragmatic aspects of meanings, implicatures, scalarity effects, etc.
-Meanings that are not used in situ: wide scope effects.
-Information that is transferred through anaphoric connections, including discourse anaphora.

THE TOPIC OF THIS TALK:
-I will discuss some cases where a semantic meaning is used by the semantics at more than one stage of the derivation.
These cases concern generalized quantifiers (DPs), where the information concerning *what* was the interpretation of the head noun has to stay available in the derivation.
-I argue that these are violations of the classical theory.
-I propose a modification of the classical theory, two tier semantics.

CLAIM: IN THE CLASSICAL THEORY, WE USE THE MEANINGS OF THE DETERMINER *DET* AND THE NOUN *NOUN* TO FORM THE MEANING OF THE DP [$_{DP}$ *DET NOUN*].
ONCE THE DP MEANING IS FORMED, THE DET-MEANING IS NO LONGER GENERALLY SEMANTICALLY ACCESSIBLE.

DET	+ NOUN	$\rightarrow$	(DET(NOUN))
$<<e,t>,<<e,t>,t>>$	$<e,t>$		$<<e,t>,t>$
relation between sets	set		set of sets (= generalized quantifier)

Can we retrieve NOUN from (DET(NOUN))?
Answer: **There is no unified procedure for doing this and in some cases no procedure at all.**

We have a procedure that works well for quite a few cases:

-Input: a generalized quantifier denotation (a set of sets)
-Take the set of **minimal elements** in the gq-denotation
-Take the **union** of that set.
This gives you the noun-interpretation (if you are lucky).

Example:
-(SOME(BOY)) = $\{X: BOY \cap X \neq \emptyset\}$
-The set of minimal elements: $\{\{x\}: x \in BOY\})$
-The union: BOY

-(EVERY(BOY)) = $\{X: BOY \subseteq X\}$
-The set of minimal elements: $\{BOY\}$
-The union: BOY

But this doesn't work for downward entailing noun phrases like *no boy* or *at most three boys*. (UNION o MIN gives the empty set). You would need a different operation for those.

Hence: the noun meaning is not generally retrievable, since you would need to know WHAT operation to use to retrieve it. But for that you would need to know WHAT determiner was used to form the noun phrase, and that is, by assumption, no longer available information either.
The classical theory cannot retrieve the noun meaning from the generalized quantifier meaning, since it doesn't know by which operation to do so.

Worse, in some cases there is no operation to retrieve the noun interpretation.
Take a model where there are, say, 5 boys and 6 girls. Look at:
AT MOST 20(BOYS) = $\{X: |X \cap BOY| \leq 20\}$ = **pow(D)**
AT MOST 20(GIRLS) = $\{X: |X \cap GIRL| \leq 20\}$ = **pow(D)**
Thus:

AT MOST 20(BOYS) = AT MOST 20(GIRLS), even though BOY $\neq$ GIRL.
Obviously you cannot retrieve the noun interpretation in this case: the generalized quantifier interpretation is trivial, even though the noun interpretation is not.

Note that the generalized quantifier interpretation **SHOULD be trivial** in this case, since it is, by assumption, the truth conditional content, and if there **are** only 5 boys, then for every predicate P: *at most 20 boys have P* is trivially true) since, by the meaning of *at most*, it is **only** false if **more** than 20 boys have P, and there aren't more than 20 boys.

Hence: in some cases there is no operation retrieving the noun interpretation from the generalized quantifier interpretation.

The case discussed in Landman 2004:

(1) a. The guests were two girls and at most two boys.
b. The guests were at most two boys.

(1b) entails (1c):
` c. The guests were boys.

Predicate interpretation: *at most two boys* $\rightarrow \lambda x.{}^*\text{BOY}(x) \wedge |x|=2$
This is (in the classical theory) not derivable from the downward entailing DP interpretation of *at most two boys*.

(In this case there is an alternative: derive the DP interpretation from the predicate interpretation, but, I argue in Landman 2004, that only postpones the violation of the classical theory, see below.)

Before discussing the cases I am interested in in this paper, I will introduce the idea of two-tier semantics.

TWO TIER SEMANTICS

0. Compositionality	**the same as in the classical theory**
1. Restricted set of available operations	**the same as in the classical theory**
2. Two tier notion of meaning	
3. Restricted domain of application: locality	**the same as in the classical theory**

The semantics builds up compositionally **simultaneously**:
Tier 1: -a **propositional meaning** (truth conditional content)
Tier 2: -a **relational meaning** (the domain on which the thruth conditional content is evaluated)
The main idea in an example

Every boy kissed some girl

every		*boy*	
Tier 1: $\lambda Q\lambda P.\forall x[Q(x) \rightarrow P(x)]$	The subset relation between sets	**Tier 1**: BOY	
Tier 2: $\lambda Q.Q$	The identity function on sets	**Tier 2**: BOY	

hence:

every boy

Tier 1: $\lambda P.\forall x[BOY(x) \rightarrow P(x)]$ The set of properties that every boy has (GQ)

Tier 2: BOY The predicate BOY

Similarly:

some girl

Tier 1: $\lambda P.\exists y[GIRL(y) \wedge P(y)]$ The set of properties that some girl has (GQ)

Tier 2: GIRL The predicate GIRL

kissed

Tier 1: KISS

Tier 2: <KISS, <<2,Th>,<1,Ag>>

Main idea:

-on the first tier, the verb combines with the two generalized quantifiers in the usual way with the usual operations

-on the second tier, you **conjoin the relevant properties and relations, IDENTIFYING coarguments.** (Thus, when *kiss* combines with *some girl* on the second tier, you use the role <2,Th> to co-identiy argument of the predicate *girl* and the second argument of the relation *kiss*)

Hence:

Every boy kissed some girl

Tier 1: $\forall x[BOY(x) \rightarrow \exists y[GIRL(y) \wedge KISS(x,y)]$

Tier 2: $\lambda y\lambda x.\ BOY(x) \wedge GIRL(y) \wedge KISS(x,y))$

the set of pairs of boys and girls where the first kissed the second.

Computational semantics: the relation is the domain on which the propositional content is evaluated (database relation).

Compositional dynamic semantics: the variables quantified over in the propositional meanining are still abstracted over in the relational meaning.

Cumulative readings: Scha 1982, Landman 2000/2004 and others:

(2) At most three boys kissed at most four girls

Landman 2000: Davidsonian existential closure is existential closure ***cum* maximalization:** $\boldsymbol{\exists e \in \alpha \wedge \sqcup(\alpha^{+}) \in \alpha}$

In here α^{+} is (a davidsonian version of) Scha's relation:

$\lambda x\lambda y.BOY(x) \wedge GIRL(y) \wedge KISS(x,y)$.

This relation was derived in a complex way in Landman 2000. In two tier it is derived directly. So: reformulation of Landman 2000:

Existential closure *cum* maximalization is an operation that takes both tiers as input.

PART TWO: DOMAIN RESTRICTION IN FUNCTIONAL RELATIVE CLAUSES

Functional readings of questions: Groenendijk and Stokhof 1980/84, Engdahl 1980/86
of relative clauses: Sharvit 1997/99, etc. etc.

Individual reading:

(1) a. Which woman does every Englishman admire? *The queen*.
b. The woman that every Englishman admires is *the queen*.

Functional reading:

(2) a. Which woman does every Englishman admire? *His mother*.
b. The woman that every Englishman admires is *his mother*.

Functional readings are possible with any kind of DP

(3) a. Which woman does *no* Englishman admire? *His mother in law*.
b. The woman that *no* Englishman admires is *his mother in law*.

Important for our purposes.
Sharvit 1999 claims that functional readings in relative clauses are only possible if the DP with the relative clause in question is part of an **equational construction** (as it is in the above examples).
But this is not correct, examples with predicative constructions which are ok (according to my informants and my dutch intuitions) are often fine:

(4) a. [Radio host on the day before Christmas] The woman that most American men spend Chistmas with is right now standing in the kitchen making the traditional Christmas pudding. [*their mother*]
b. The woman that no haredi man sees on the morning of his wedding day is waiting for him later that day in front of the canopee [*his bride to be*]
c. The thing that no superstitious person travels without hangs on a chain around his neck [*his rabbit foot*]
d. The woman that most pianists have nightmares about made them practice Hanon for hours daily [*their first piano teacher*]

Groenendijk and Stokhof's analysis of functional readings. Step one:
The **gap** is interpreted as a **variable complex**: $f_i(x_n)$
f_i is a variable over functions of type<e,e>, x_n an individual variable.
-f_i is abstracted over by question formation/relativization
-x_n is bound locally by the scope mechanism (to *every Englishman*)

Groenendijk and Stokhof's analysis of functional readings. Step two:
Form a question abstract/relative clause:

wh/that DET englishman admire -

λf.DET(E-MAN,λx.ADMIRE(x,f(x)))

the set of functions f such that *det* englishmen admire their f-value

Groenendijk and Stokhof's analysis of functional readings. Step three:

a. **head noun** *woman* shifts to a predicate of functions of type <e,e>:

woman

WOMAN $\rightarrow$ λf.$\forall$x $\in$ dom(f): WOMAN(f(x))

The set of woman valued functions

b. The two combine by intersection:

woman that det englishmen admire

λf. $\forall$x $\in$ dom(f): WOMAN(f(x)) $\wedge$ DET(E-MAN,λx.ADMIRE(x,f(x)))

The set of woman-valued functions f such that det englishmen admire their f-value.

Sharvit's analysis of DPs with functional relative clauses. Step one:

Restriction to 'NATURAL, contextually relevant functions'

woman that det englishmen admire

λf. **NAT**(f) $\wedge$ $\forall$x $\in$ dom(f): WOMAN(f(x)) $\wedge$ DET(E-MAN,λx.ADMIRE(x,f(x)))

Sharvit's analysis of DPs with functional relative clauses. Step two:

Apply the determiner to the predicate of functions:

the woman that det englishmen admire

σ(λf. **NAT**(f) $\wedge$ $\forall$x $\in$ dom(f): WOMAN(f(x)) $\wedge$ DET(E-MAN,λx.ADMIRE(x,f(x))))

Reason for restriction to 'natural' functions. There are **by far too many** functions satisfying the description for σ to be applicable (for instance, take two functions f and g that map every Englishmen onto their mother, but f maps me onto my mother, while g maps me onto my wife; since I am not English, both satisfy the bill.)

However, the notion of 'natural' function is dubious (it can be argued that, as far as the linguistic data go, every function is natural). I will not argue this point, because the analysis has more general problems:

(5) (The) two women that every boy adores are his mother and his teacher.
Sharvit: There are **two natural functions** f_1 and f_2 such that every boy admires his f_1 value and every boy admires his f_2 value, and these functions are: f_1 is the mother function, and f_2 is the teacher function.

Look at the following situation.
In the coal-mining village there are 100 boys. 50 of them go to school and study with the adorable teacher Miss Prism, they also adore their mother. The other 50 belong to a different religion and receive home education. Since all the fathers work in the mines, they are educated by their mothers, whom they adore. None of these kids adore any other women.

Intuition: (5) is **false** in the situation sketched.

But: there **are** two natural, relevant, contextually given functions, f_1 and f_2 that satisfy the requirement in (5). Though the functions map half of the boys onto the same value, the functions are **extensionally** distinct, and –more importantly – they are **intensionally** distinct (as 'natural' functions). So Sharvit **predicts** that (5) is true.

Crucial observation: *two* in (5) does not count woman-functions, but women: (5) requires there to be a function which maps every boy onto two women.
In the context given, there is no such function, that's why (5) is false.

[Observation: For nouns like *woman*, **quantification over functions** doesn't seem to be felicitous. In this, they differ from nouns like *present* (which are more inherently functional):

(6) a. **Every** present/**most** presents that EVERY child got was/were cheap.
Besides individual presents, they all got their initial in chocolate, a bag of candies, a note pad, and a subscription to the museum (the latter, in the case of *most* was more expensive)

b.#**Every** woman/**most** women that EVERY child adores is a/are family members.
Besides individual adorations, they all adore both of their grandmothers, their mother, their sister (and in the case of *most*, their school teacher)]

THE ANALYSIS

Analysis: step 1

Relative clause: Like G&S

that DET boys admire –

$\lambda f.\mathrm{DET}(\mathrm{BOY},\lambda x.\mathrm{ADMIRE}(x,f(x))$

Analysis: step 2 Small change from G&S

Type shifting: **From a relation to a set of functions (not from a property like G&S)**

$\mathbf{R} \rightarrow \lambda f.\forall x \in \mathrm{dom}(f): \mathbf{R}(x,f(x))$

Analysis: step 3

WOMAN is of type <e,t>. Given step 2, this is the wrong type (should be a relation).

Solution: find a contextually relevant relation C and form:

$\boldsymbol{\lambda x \lambda y.\mathrm{WOMAN}(y) \wedge C(x,y)}$

Analysis: step 4

Definite and indefinite noun phrases can be predicates

Definite and indefinite relational noun phrases can be relations.

I assume that *two women* and *the two women*, with *women* interpreted relationally, have themselves **relational** interpretations:

two women

$\lambda \underline{x}\lambda y.{}^*\mathrm{WOMAN}(y) \wedge C(x,y) \wedge |y|=2$ the relation that holds between x and y iff x stands in relation C to a sum y of two women

the two women

$\lambda \underline{x}\lambda y.y=\sigma(\lambda y \mathrm{WOMAN}(y) \wedge C(x,y))$ the relation that holds between x and y iff y is the unique woman that x stands in relation C to

Analysis: step 5
We combine step 2 and step 5: Function shift

two women $\rightarrow \lambda f.\forall x \in dom(f): {}^{*}WOMAN(f(x)) \wedge C(x,f(x)) \wedge |f(x)|=2$

The set of functions that map every element x of their domain onto two women that x stands in contextual relation C to.

the woman $\rightarrow \lambda f.\forall x \in dom(f): f(x) = \sigma(\lambda y.WOMAN(y) \wedge C(x,y))$

The set of functions that map every element x of their domain onto the woman that x uniquely stands in contextual relation C to.

Analysis: step 6 (G&S): intersection

two women that DET boy(s) admire

$\lambda f.\forall x \in dom(f): {}^{*}WOMAN(f(x)) \wedge C(x,f(x)) \wedge |f(x)|=2 \wedge DET(BOY,\lambda x.{}^{*}ADMIRE(x,f(x)))$

The set of functions f that map every element x of their domain onto two women that x stands in contextual relation C to, such that DET boys admire each of their f-values.

the woman that DET boy(s) admire

$\lambda f.\forall x \in dom(f): f(x) = \sigma(\lambda y.WOMAN(y) \wedge C(x,y)) \wedge DET(BOY,\lambda x.ADMIRE(x,f(x)))$

The set of functions that map every element x of their domain onto the woman that x uniquely stands in contextual relation C to, such that DET boys admire their f value.

Predictions so far:
1. Sentence (5) is not true in the scenario given, because, on my analysis, (5) is only true if each boy adores **two** women, which is not the case in the scenario given.
2. The lifting analysis only works for definites and indefinites, no analysis is provided for 'real' quantifiers like *every* and *most* (which is good if those cases are infelicitous)
3. The contextual restriction with restriction with relation C assigns **very weak** uniqueness requirements to functional readings.
Weak enough to account for Groenendijk and Stokhof's observation that the functional reading of (7) is compatible with individual Englishmen admiring more than one woman:

(7) The woman that every Englishman admires is *his mother*.

But, in fact, it is much weaker: the uniqueness requirement is not that every Englishman admires one and only one woman., but that every Englishman stands in contextual relation C to one and only one woman. (Of course, in context C **can** be the admire-relation, and then we would have the first presupposition)
Thus, with Kadmon, the uniqueness requiement imposed by the definite article is that, for each argument x, x is related to a woman which is unique to x in some contextual way.

THE PROBLEM OF DOMAIN RESTRICTION

The analysis is unfinished. We have derived an NP-meaning, but not yet a DP meaning, of type $<e,e>$, denoting a function, or $(<<e,e>,<<e,e>,t>$, denoting a generalized quantifier of functions.
The determiner was interpreted predicate internally, so: **default interpretation rule**.
But which?

α is the complex noun interpretation

Suggestion 1: DP = $\sigma(\alpha)$ (cf Jacobson 1994 on free relatives)
Problem:

(8) Two women that every boy admires are his mother and his teacher.

Intuitively (8) can be true, even if Fred admires three women.
With suggestion 1, however (8) would be undefined in that case (because $\sqcup\alpha \notin \alpha$).

Suggestion 2: DP = $\sqcup(\alpha)$
Problem:

(9) Two women that every boy admires play an important role in his life.

The main predicate will apply distributively per argument:

$\forall x \forall a \in \text{ATOM}(f(x))$: a plays an important role in x's life.

With suggestion 2, in the case where one boy happens to admire a thousand women, the prediction is that (9) **expresses** that all one thousand of them play an important role in that boys life, because all one thousand of them are in $\text{ATOM}([\sqcup(\alpha)](x))$. But, of course, (9) expresses no such thing.

Suggestion 3: Existential closure: DP = $\lambda P.\exists f \in \alpha: P(f)$
Problem:

(10) The woman that no boy adores is wise.

Let us choose for C the mother relation. Assume that each girl has a wise mother, but each boy has a foolish mother. Assume that each girl adores her mother and no boy adores his. Under the conditions given, (10) should be false.
Unfortunately, on the existential closure analysis, (10) does not come out as false, but as **true**!

Why is this?

$$\alpha = \lambda f.\forall x \in \text{dom}(f): f(x) = \sigma(\lambda y \text{WOMAN}(y) \wedge \text{MOTHER}(y,x)) \wedge \text{NO}[\text{BOY},\lambda x.\text{ADORE}(x,f(x))]$$

Unfortunately, the following function is in α, making (10) true:
f: **GIRL** $\rightarrow$ WOMEN such that: $\forall x \in \text{GIRL}: f(x) = \sigma(\lambda y.\text{MOTHER}(y,x))$.

The crucial point is: since f is **undefined** for boys, BOY $\cap$ λxADORE(x,f(x))=Ø, hence NO[BOY, λxADORE(x,f(x))] is true, and hence (10) is predicted to be true.

What goes wrong? The intuition is: f **should be irrelevant** because the domain of f does not include all semantically relevant objects (the boys). We need to achieve that the functions in α are restricted to **functions that are defined for the semantically relevant objects.**
But how do we do that?

Suggestion 4: existential quantification over **maximal** functions.

f is **maximal** in β iff $f \in \beta$ and $\neg\exists g \in \beta$: $f \sqsubseteq g$ and $f \neq g$

Now we need to be careful. We need to build in this maximality into the shift operation from R to a set of functions:

$R \rightarrow \lambda g.g$ is maximal in $\lambda f.\forall x \in dom(f)$: $R(f(x),x)$.

This will give us:
$\alpha = \lambda g.g$ is maximal in $\lambda f.\forall x \in dom(f)$: $f(x) = \sigma(\lambda y$WOMAN(y) $\wedge$MOTHER(y,x)) $\wedge$
EVERY[GIRL,λx.ADORE(x,g(x))] $\wedge$ NO[BOY,λx.ADORE(x,g(x))]
And existential closure will apply to this.

With this move, the function f, discussed before, will not be in α, since it is not the maximal function from individuals to their mother (since it assigns no value to the boys). A function that is maximal will map boys and girls onto their mother. But then, it is not going to be the case that the value for every argument is wise, since the value for the boys is foolish. Thus such a function is not going to make (10) true.

Since G&S don't put in any domain specification, it is quite possible that this is what they had in mind.

Problem: the same problem as suggestion 2:

(4a) The woman that most American men spend Christmas with is right now preparing the Christmas pudding. *his mother.*

Which mothers are required to be in the kitchen now, according to (4a)?

The problem with the maximal function approach is that – with the distributive interpretation of the main predicate – it predicts that **everybody's mother** (yours, mine, and the cat's) is in the kitchen preparing the Christmas pudding. (I.e. it predicts that (4a) is false if **anybody's** mother is not standing in the kitchen).

But intuitively, the mothers that are required to be in the kitchen according to what (4a) says should be the mothers of American men that spend Christmas with their mother.

So: **the maximal function approach makes the domain of the functions too big.** (cf. Grosu and Krifka).

Similarly, look at the following inference:

(11) The book that every religious jew carries with him is his prayerbook.
(12) The book that every religious muslim carrier with him is his prayerbook.
Hence:
(13) The book that every religious jew carries with him is the book that every religious muslim carries with him.

The inference from (11)+(12) to (13) is **not necessarily valid.**

Maximalist strategy: The natural interpretation for relation C in (11) and (12) is: 'x uses y for saying x's daily prayers.' Hence: the existential quantifier in (11) and (12) range over functions that map **anybody** for whom there is a unique book that he/she uses for daily prayers onto that book. In particular, (11) and (12) range over functions that map jews and muslims **alike** onto their prayerbook. Assuming they all carry their prayerbook with them, the conclusion (13) becomes practically **unavoidable**: (13) expresses the selfidentity of the maximal function that maps jews and muslims onto their prayerbook.

A DIFFERENT APPROACH TO FUNCTION IDENTITY

Intuition:

(14) The present Harry got from Mrs. Weasley for Christmas was the same as the present all the Weasley kids got: *A sweater with his initial on it*.

It seems unwise to me to assume that this involves the 'natural' function which maps anybody in the whole wide world who has a sweater with his initial on it onto that sweater. Rather, it involves two specified **partial** functions:

> f_1: $\{<h,S_h>\}$ the function that maps Harry onto a sweater.
> f_2: $W \rightarrow S$, the function that maps precisely the Weasley kids onto their sweater.

(14) expresses that f_1 is ' the same' function as f_2.

What does 'the same function' mean?
Contextual identity:

> f_1 is **contextually identical to** f_2 if in the context there is a contextually relevant relation C such that f_1 is **C-identical to** f_2, $f_1 =_C f_2$

> $f_1 =_C f_2$ iff: 1. $f_1 \cup f_2$ is a partial function
> 2. $\forall x \in \text{dom}(f_1 \cup f_2)$: $C(x, f_1 \cup f_2(x))$
> 3. homogeneity

-1. **integrity constraint**: for two partial functions to be contextually identical, they should not map the same object onto different values.
-2. **contextual identity is a rather weak relation:** find a contextual relation C (like 'y is a sweater with x's initial on it'). The requirement is that all the arguments of the contextually identical functions should stand **in the same relation** to their values.
-3, **Homogeneity** is an intuition that is hard to formalize, but it is basically the following:

> **Homogeneity: Don't choose for C a relation that divides $f_1 \cup f_2$ as much as it unities it.**

The prayerbook relation is a case in point. If we assume that (11) involves the **partial** function f_1 that maps jews onto their prayerbook, while (12) involves the **partial** function f_2 that maps muslims onto their prayerbook, then normally $f_1 \cup f_2$ will indeed be a function, and the prayerbook relation indeed holds between any element of $dom(f_1 \cup f_2)$ and its value, but **still** we may deny that f_1 contextually identical to f_2, because the prayerbook relation, divides jews and muslims as much as it unites them: it naturally partitions the $domain(f_1 \cup f_2)$ precisely along $dom(f_1)$ and $dom(f_2)$.

This seems to be a reasonable account of the failure of the inference, but it presupposes that f_1 and f_2 are **not** required to be maximal functions, but indeed, functions restricted to jews, vs. muslims.
From this we must conclude that there is a **domain restriction** on the functions involved: indeed **a restriction to include the objects that are relevant for the quantification expressed in the relative clause, but not necessarily more.**

-This restriction is derived from material in the relative clause, in particular, in (11) from the property **RELIGIOUS JEW** and it is a restriction on the **domain of function f.**
-In the relative clause *religious jew* is the **head noun** of the DP *every religious jew*.
-The earliest stage in the semantic derivation where the meanings involving f and involving that of *religious jew* are both present is at the level where the generalized quantifier combines with the main predicate:

$$(DET(RELIGIOUS\ JEW))\ (\lambda x.CARRY(x,f(x)))$$

-This means that if we want to state the restriction at **this stage or a later stage the property RELIGIOUS JEW** should be derivable from the locally available meanings.
-But, of course, I have argued that in the classical theory **this information is not available.**

Note that the the restriction to include all the objects that are 'relevant for the quantification' is **semantic** in the sense that you get **not just a different meaning but the wrong meaning** if you don't obey it.
In that sense it is **unlike** normal contextual restriction

TWO TIER SEMANTICS FOR DOMAIN RESTRICTION

the woman that DET boys adore

Tier 1: $\lambda f.\forall x \in dom(f){:}\ f(x) = \sigma(\lambda y.WOMAN(y) \wedge C(x,y)) \wedge$
$DET(BOY,\lambda x.ADORE(x,f(x)))$

What should the second tier interpretation be?
The **second** tier interpretation of the **relative clause** *that DET boys adore* is straightforward, given the two tier-idea: the predicate BOY is conjoined with the ADORE relation:

that DET boys admire
Tier 2: $\lambda f\lambda x.BOY(x) \wedge ADORE(x,f(x))$

But what should the second tier be of the head predicate *the woman*?
There are two reasonable choices, depending on how we think about the type shifting: the relation before type shifting or the set of functions after type shifting. I will assume that both options are possible, and this represents a **second tier ambiguity**.

Thus:

the woman
Tier 2-after: $\lambda f.\forall x \in dom(f){:}\ f(x) = \sigma(\lambda y.WOMAN(y) \wedge C(x,y)$
Tier 2-before: $\lambda y\lambda x.\ y = \sigma(\lambda y.WOMAN(y) \wedge C(x,y)$

Identifying corresponding arguments give us the following two second tier interpretations:

the woman that DET boys adore
Tier 2-after: $\lambda f\lambda x.\ BOY(x) \wedge$
$\forall z \in dom(f){:}\ f(z) = \sigma(\lambda y.WOMAN(y) \wedge C(z,y)) \wedge$
$ADORE(x,f(x))$
Tier 2-before: $\lambda f\lambda y\lambda x.\ BOY(x) \wedge$
$y = \sigma(\lambda yWOMAN(y) \wedge C(x,y))$
$\wedge\ ADORE(x,f(x))$

Tier 2-after is a two place relation, **tier 2-before** a three-place relation.
What will be relevant in both cases is the **first projection of that relation**

$[R^2]^1 = \lambda x.\exists y[R(x,y)]$
$[R^3]^1 = \lambda x.\exists y\exists x[R(x,y,z)]$

[**Tier 2-after**]^1 = $\lambda x.\exists f[\ BOY(x) \wedge \forall z \in dom(f): f(z) = \sigma(\lambda y.WOMAN(y) \wedge C(z,y))$
$\wedge\ ADORE(x,f(x))]$

$= \lambda x.BOY(x) \wedge ADORE(x,\sigma(\lambda y.WOMAN(y) \wedge C(x,y))$
The set of boys that adore the woman they stand uniquely in relation C to.

[**Tier 2-before**]^1 = $\lambda x\exists y\exists f[BOY(x) \wedge y = \sigma(\lambda y.WOMAN(y) \wedge C(x,y))$
$\wedge\ ADORE(x,f(x))$

$= \lambda x.BOY(x) \wedge \sigma(\lambda y.WOMAN(y) \wedge C(x,y)){\neq}\bot \wedge \exists f[ADORE(x,f(x))]$

The set of boys for whom there is a woman they stand uniquely in relation C to, such that for some function f they stand in the adore-relation to f(x).

What does the clause $\exists f[ADORE(x,f(x))]$ mean?
Since the f argument here is grammatically **unrestricted** by any information derived from the arguments of the relation (i.e. the subject, the gap) (**unlike** in the case of tier 2-after), I will ssume that:

> $\exists f[ADORE(x,f(x))]$ is **too unspecified to be semantically useful**, **and is *de facto* semantically neutralized.**

One way of achieving this:

The extravagant function assumption:
Assume that there are **abstract** objects (not in the domain of normal quantification D), and assume the abstract existence of an abstract object u, such that ADORE(x,u) iff
$\neg\exists y \in D{:}ADORE(x,y)$ (Fine 1984 discussed such abstract objects).
Assume that functions can map objects onto concrete or abstract objects, functions that map arguments onto abstract objects like u are **extravagant functions**.
Normally, quantification over functions will not range over extravagant functions, because the values are semantically restricted by noun phrase information to non-extravagant values. **Precisely in our case** the noun phrase information (the restriction of the value to women) doesn't get to restrict the functions in question.

With this, I assume that:

> **the statement $\exists f[ADORE(x,f(x))]$ is trivially true, and hence drops out.**

Or we can derive the effect through a grammatical constraint: the lexical information of the head noun in the relative clause construction does not constrain even indirectly the second argument of the relation in relative clause, this can be seen as a violation of a semantic form of the theta-criterion (cf. Landman 2000). And we can assume that this is why the clause drops out.

[**Tier 2-before**]^1 = $\lambda x.BOY(x) \wedge \sigma(\lambda y.WOMAN(y) \wedge C(x,y)){\neq}\bot$
The set of boys for whom there is a woman they stand uniquely in relation C to.

In sum:

the woman that DET boys adore

T_1: $\lambda f.\forall x \in dom(f): f(x) = \sigma(\lambda y.WOMAN(y) \wedge C(x,y)) \wedge DET(BOY,\lambda x.ADORE(x,f(x)))$

[**Tier 2-after**]^1: $\lambda x.BOY(x) \wedge ADORE(x,\sigma(\lambda y.WOMAN(y) \wedge C(x,y))$

[**Tier 2-before**]^1: $\lambda x.BOY(x) \wedge \sigma(\lambda y.WOMAN(y) \wedge C(x,y))\neq\bot$

The proposal is: **there is a first tier operation of existential closure with domain restriction which takes BOTH tiers as input:**

EXISTENTIAL CLOSURE WITH DOMAIN RESTRICTION

$\lambda P.\exists f \in T_1$: $\mathbf{[T_2]^1 \subseteq dom(f)} \wedge P(f)$

Since we had an ambiguity at the second tier we now get an ambiguity at the first tier:

the woman that DET boys adore

$\lambda P.\exists f[\forall x \in dom(f): f(x) = \sigma(\lambda y.WOMAN(y) \wedge C(x,y)) \wedge$
$DET(BOY,\lambda x.ADORE(x,f(x))) \wedge$
$\lambda x.BOY(x) \wedge \sigma(\lambda y.WOMAN(y) \wedge C(x,y))\neq\bot \subseteq dom(f) \wedge$
$P(f)$

$\lambda P.\exists f[\forall x \in dom(f): f(x) = \sigma(\lambda y.WOMAN(y) \wedge C(x,y)) \wedge$
$DET(BOY,\lambda x.ADORE(x,f(x))) \wedge$
$\lambda x.BOY(x) \wedge ADORE(x,\sigma(\lambda y.WOMAN(y) \wedge C(x,y)) \subseteq dom(f) \wedge$
$P(f)$

Note that we are not identifying the domain of f with the set $[T_2]^1$. We are just requiring that f at least assigns all the objects in that set a value. In evaluating examples we will take a main clause predicate and assume a distributive analysis: P(f) will be interpreted as:

$\forall x \in dom(f): P(f(x))$

What we are thus evaluating is which objects f(x) are **minimally** claimed by the sentence to have property P. With this in mind there is a relevant observation:

$[\text{Tier 2-after}]^1 \subseteq [\text{Tier 2-before}]^1$

This means that for the purpose of evaluating which objects must be **minimally** in, we must check the reading based on **Tier 2-after**. That is, we would only consider the set based on **Tier 2-before,** if the set based on **Tier r-after** is empty. Because, if the sentence is ambiguous, the reading that uses the smaller domain restriction is the more relevant one.

So, let's look at example (4a):

(4) a. [Radio host on the day before Christmas] The woman that most American men spend Chistmas with is right now standing in the kitchen making the traditional Christmas pudding. [*their mother*]

Which women are **required** by (4a) to stand in the kitchen making the traditional Christmas pudding, on the assumption that C is the mother relation?

[**Tier 2-after**]^1 = λy.A-MAN(x) $\wedge$ SPENDCHRWITH(x,σ(λyMOTHER(y,x)))

On this analysis (4a) does **not** require that everybody's mother stands in the kitchen.
It does **not** require that every American's mother stands in the kitchen.
It does **not** require that every American has a mother.
It does **not** require that the mother of every American who has a mother stands in the kitchen.
It requires that the mother of every American who has a mother and spends Christmas with her stands in the kitchen, (and it requires that the Americans who have a mother that they spend Christmas with are most Americans).

Interestingly, then, the reading we derive is like a **dependent e-type reading:**

(4b) Most american men spend Christmas with their mother. The mother of *the American men who spend Christmas with their mother* is standing in the kitchen right now.

Of course, if the quantifier is EVERY, you get more:

(11) The book that every religious jew carries with him has golden lettering, *his prayerbook.*

Here too the domain includes at least the religious jews that carry their prayerbook with them, but the semantic requirement induced by EVERY is that this is all the religious jews. In this cases, there **is** a requirement that every religious jew uses a prayerbook (C), carries it with him, and they all have golden lettering. While other individuals **may** be in the domain as well, this is not required. In particular, there is no requirement that any muslim is in the domain of the function (in 11).

Why inclusion rather than identity? The matter is subtle, but concerns examples like (15):

(15) Mitterand had an affair with the woman that most americans wouldn't dare have an affair with (because of legal threats). *his secretary.*

Mitterand is chosen here only because everybody knows he is French. If the domain of the function would be **just** the Americans who wouldn't dare to have an affair with their secretary, Mitterand couldn‘t have an affair with **his** value of the function. On the present analysis, he could.

For interpretations using **tier 2-before** look at example (16):

(16) The woman that no Fin spends Christmas with doesn’t dare to tell him that she misses him at Christmas. *his mother*.

The **Tier 2-after**-restriction would be:
$\lambda x.FIN(x) \wedge SPENDCHRWITH(x,\sigma(\lambda yMOTHER(y,x))$

Clearly, by the meaning of NO, this set is empty. Hence, the reading for (16) we get, is the reading with the **Tier 2-before**-restriction:

$\lambda x.FIN(x) \wedge \sigma(\lambda yMOTHER(y,x)) \neq \perp$
The fins that have a mother.

-(16) doesn't tell you that all fins have a mother,
-(16) tells you that no fin spends christmas with his mother, and for those fins that have a mother, thir mother doesn’t dare to tell them that she misses them at Christmas.

Conclusion:
-In functional readings we find by preference an e-type domain restriction on the function, **a restriction of the domain to include the smallest set derived from the second tier.**
-If this set is empty, as it is when the determiner is NO, **the wider set derived from the second tier is used**.

You may ask: Why not reduce this phenomenon to e-type anaphora (just stick in an invisible anaphor). *Au contraire:*

E-type restrictions we find with discourse anaphora (cf. Evans 1972, Kadmon 1987).
I have discussed here cases of e-type restrictions without discourse anaphora.
Jacobson, *many papers*, proposes a functional analysis of anaphora in general.
If anaphora involve functions, these functions may require domain restrictions similar to what we have seen here.
This opens up the possibility of reducing the e-type restrictions found on discourse anaphora to the domain restriction mechanism proposed here.

ADDING *ONLY*

Consider the following cases:

(17) a. The woman that every boy adores is his mother.
b. The *only* woman that every boy adores is his mother.
c. The *only* two women that every boy adores is his mother.

We have, so far, given a weak interpretation to (17):
(17) requires a contextual relation C that assigns to every boy a woman who is in context the only woman who stands in relation C to that boy. This means that (17) is not (necessarily) telling us that every boy only **adores** one woman.

We may quibble about the adequacy of this for (17a), but it is surely **inadequate** for (17b,c). And , in fact, applying the strategy of interpreting *only* on the value of the function is not going to get us the right interpretation.

Should we say, after all, that *only* tells us that it is not just that there is a function with the right properties, but there is **only one such function**? This would, of course, bring us back to the natural function approach.

But in a setting with partial functions this is not a very plausible approach: if the functions we derive are very partial and their identity conditions contextual and subtle, it is not at all plausible to assume that even in the domain of contextually relevant, natural functions, there is one and only one function that satisfies the requirements. It seems to me that in a normal case, given domain unclarity, there are by necessity many such functions.

So I don't think that it is the **function** of which *only* says that there is only one that satisfies the requirements. This is where the relation C comes in again. While it is not plausible to assume that there is exactly one function that satisfies the requirements, it is **quite possible** to assume that there is, in context, **only one contextual relation** C that satisfies the requirements. If so, we can assume that *only* adds a constraint to this effect:

EXISTENTIAL CLOSURE WITH DOMAIN RESTRICTION AND *ONLY*:
Given: **Tier 1$_{\mathbf{C}}$** and **Tier 2$_{\mathbf{C}}$.** We form:

$$\lambda P.\exists f \in \textbf{Tier } \mathbf{1_C}\text{: } [\textbf{Tier } \mathbf{2_C}]^1 \subseteq \text{dom}(f) \wedge P(f) \wedge$$
$$\forall C' \in \mathbf{C}\text{: } \exists\, f \in \textbf{Tier } \mathbf{1_C}\text{: } [\textbf{Tier } \mathbf{2_{C'}}]^1 \subseteq \text{dom}(f) \rightarrow$$
$$C \upharpoonright [\textbf{Tier } \mathbf{2_C}]^1 = C' \upharpoonright [\textbf{Tier } \mathbf{2_{C'}}]^1$$

(17b) The only woman that every boy adores is his mother.

On this analysis, what (17b) says is the following:
-There is a contextual relation C and a function f, which maps every argument x in its domain onto the woman who uniquely stands in relation C to x, such that every boy adores its f-value, this function is contextually identical to the mother function for the boys, and:

in **C**, C **is the only relation, up to [Tier 2]**[1], for which there is a function f which maps every argument x in its domain onto the woman who uniquely stands in relation C to x, such that every boy adores its f-value.

Thus, in the above example, if every boy adores only his mother, the mother relation **and** the adore relation would both satisfy the requirements, and, arguably, they are both in **C**. If we require strict identity this would be a problem, since they are not the same relation. But they are the same relation on $[T_2]^1$, that's why we require identity on $[T_2]^1$.

On the other hand, if there are boys that adore more than one woman, we cannot exclude there being relations C and C' in **C** that both satisfy the requirements without being identical on the set of boys.
And certainly, if every boy adores both his mother and his sister in law, then **surely** there are two relations in **C** satisfying the requirements that are not identication on the set of boys, and the sentence is false.

The proposal, then, is that *only* does **not** semantically constrain the value of the function, **nor** the function itself, **but** the underlying relation.

If we assume that, we may well think about changing our analysis of the definite article *the* as well. **Externally**, *the only* and *the* would have basically the same meaning (*only* being an intensifier). In fact, if we assume that *the* allows **both** derivation strategies, internal and external, we can see the function of *only* as to **disambiguate** in favor of te stronger external meaning.

PART THREE: GROSU AND KRIFKA RELATIVES

(1) *The* brilliant mathematician that *some* of these boys might one day become, might one day prove the Riemann Hypothesis.

Salient properties:
1. The gap in the relative clause is in non-argument position.
2. The external determiner must be definite (*the*).
3. The relative clause is an **intensional modifier** on the head noun (not intersective, the sentence is **not** about actual brilliant mathematiciant that have the property that some of these boys might one day become them.).
4. The subject DP inside the relative clause can be anything:

(2) The caring father that *none of these men* ever was might have prevented his son from becoming a junkie.

5. The modal in the main clause is most naturally interpreted as 'resumptive' wrt the modal in the relative clause: he proves the Riemann hypothesis in a world where he has become a brilliant mathematician.
6. The temporal phrase in the main clause is most naturally interpreted as 'dependent' (on the time expressed in the relative clause): he proves the Riemann hypothesis at a time **later** than the time where he has become a brilliant mathematician.
7. The sentence **presupposes** that some of these boys might one day become a brilliant mathematician.
8. The sentence **expresses** that, given that presupposition, each of the boys who might one day become a brilliant mathematician, might become a brilliant mathematician and one day prove the Riemann hypothesis.

Properties (1) and (2) are generally found with 'third kind' relatives:

(3)   a. *The* three books that there were - at the table have gone.
b. John will never be *the* caring doctor that his father was -

(8) is the central property for our purposes here. The sentence expresses that each of the **actual present boys** who **might one day** become a brilliant mathematician might one day prove the Riemann hypothesis. This means that **at the level** at which the complex subject DP combines with the main predicate it must be derivable information:
-what are the actual present boys
-for each of these boys, what are **the worlds accessible in** $\mathbf{w_0}$ where they become a brilliant mathematician
-for each of these boys and worlds, what is the **time** at which they do so in those worlds.

We have three quantifiers inside the relative clause: over times, over worlds and over individuals, and for each of these quantifiers **the set that the quantifier lives on must be accessible still at the higher level.**
Hence, we have, once again, a problem for the classical analysis, **a second tier phenomenon.**

That there is such a problem can be shown rather directly in this case.
Look at examples (4):

> (4) The frogs that at most 20 princesses kissed – were light green.

If there are only 10 princesses, then every frog has the property that at most 20 princesses kissed it, and (4) is equivalent to (5):

> (5) The frogs were light green.

I.e. the relative clause is trivial and imposes no restriction.

Now consider (6):

> (6) The frogs that at most 20 princes had turned into –, were indeed light green.

We assume that there are only 10 princes (Why do we say, *at most 20*? Well, to remind ourselves that by general principles derivable from the Theory of Magic, there cannot be more than than 20 frog-princes at the same time). The point is this: in this context, (6) does **not** say the same as (5), but (in this context) it says the same as (7):

> (7) The frogs ***that a prince had turned into*** were light green.

The puzzle is: **the relative clause imposes a real restriction despite the fact that its classical content is trivial.** It is this restriction that goes beyond the bounds of the classical theory.

We will analyze sentences of schema (8):

(8) The brilliant mathematician that DET boys might one day become, might one day prove the Riemann Hypothesis.

TWO TIER ANALYSIS. Step 1. The relative clause
-The whole relative is an intensional modifier of the head noun.
-Intensional modifiers (like intensional adjectives) are not intersective, but are functions on the intension of the head noun.
-We have a gap in predicate position.

The natural assumption is to assume that the gap is a **property** gap (intensional). If relativization abstracts over a property variable, this will make the relative clause an intensional modifier.

Intensional property:
brilliant mathematician → **Tier 1,Tier 2**: $\lambda t \lambda w \lambda x.\mathbf{BM}(x,w,t)$
gap → **Tier 1,Tier 2**: $\mathbf{P} \in \mathrm{VAR}_{<t,<w,<e,t>>>}$

We have **three quantifiers**, schematically:
one time[Fut]: **Tier 1**: $\lambda P \exists t > t_0 \text{: } P(t)$
Tier 2: $\lambda t.t > t_0$

might: **Tier 1**: $\lambda P.\exists v \in ACC_{w0} \text{: } P(v)$
Tier 2: $\lambda v.\ v \in ACC_{w0}$

DET(BOY): **Tier 1**: $\lambda P.\ DET(BOY_{w0t0},P)$
Tier 2: $BOY_{w0,t0}$

We put these together as expected by now, apply the relative clause to the head noun and derive at the NP level:

brilliant mathematician that det boys might one day become
Tier 1: $DET[BOY_{w0t0},\lambda x.\exists t > t_0 \text{: } \exists v \in ACC_{w0} \text{: } BECOME(\mathbf{BM}(x,v,t))]$
Det boys might one day become a brilliant mathematician (**proposition**)
Tier 2: $\lambda t \lambda v \lambda x.BOY_{w0t0}(x) \wedge t > t_0 \wedge v \in ACC_{w0} \wedge BECOME(\mathbf{BM}(x,v,t)$
The set of triples of actual boys b, future times t, accessible worlds w such that b become a brilliant mathematician at t in w (intensional property)

TWO TIER ANALYSIS. Step 2. Presuppositional maximalization

We are at the **same point** as we were in the functional relative clauses where we used an operation on both tiers. Here too a tier 1 operation which takes both tiers as input operates. The operation is called **presuppositional maximalization**.

The new first tier meaning is actually constructed from the relational second tier, with the propositional first tier acting as a presuppositional constraint.

I define the following operation:

$$\sigma_{\varphi}(P) = \begin{cases} \sigma(P) & \text{if } \varphi \\ \perp & \text{otherwise} \end{cases}$$

PRESUPPOSITIONAL MAXIMALIZATION:
Form <**Tier 1**, **Tier 2**> form:
$\sigma_{\textbf{Tier 1}}(*\textbf{Tier 2})$

Suppressing technical details: this operation derives **presuppositional property:**

$$\sigma_{\textbf{Tier 1}}(*\textbf{Tier 2}) = \begin{cases} \textbf{Tier 2} & \text{if } \textbf{Tier 1} \\ \perp & \text{otherwise} \end{cases}$$

$$\begin{cases} \lambda t \lambda v \lambda x.\text{BOY}_{w0t0}(x) \wedge t > t_0 \wedge v \in \text{ACC}_{w0} \wedge \text{BECOME}(\textbf{BM}(x,v,t)) \\ \quad \text{if DET}[\text{BOY}_{w0t0}, \lambda x.\exists t > t_0\text{: } \exists v \in \text{ACC}_{w0}\text{: BECOME}(\textbf{BM}(x,v,t))] \\ \perp \quad \text{otherwise} \end{cases}$$

We can now follow Grosu and Krifka to complete the DP semantics.
We have derived as the interpretation of the complex noun an intensional individual (a property): **Tier 2 (under presupposition Tier 1)**.
But we need a **predicate** interpretation here. This is derived, as usual, with type shifting operation IDENT from the Partee triangle:

$\lambda z.z =$ **Tier 2** (pres. Tier 1)

This means that the conplex noun interpretation is already definite, and that means that the only determiner available is σ, *the*. We derive:

σ($\lambda z.z =$ **Tier 2**) (**pres. Tier 1**) = **Tier 2 (pres Tier 1)**

(i.e. the same as we had above, but now a DP)

SHIFTING THE DP TO A DISTRIBUTIVE BINARY GENERALIZED QUANTIFIER

We turn, to get a distributive interpretation of the main predicate (*might one day prove the Riemann hypothesis*) the DP derived into a distributive binary generalized quantifier:

Let α be the interpretation of the DP
$\alpha \rightarrow \lambda \mathbf{R}.\forall x \in [\alpha]^1: \mathbf{R}(x,\alpha)$

R is of type < <t<w<e,t>>>, <e,t>>.
Thus, this is the type that we should derive the main predicate *might one day prove the Riemann hypothesis* at.

THE ANALYSIS OF THE MAIN CLAUSE PREDICATE:

We start out with the verb phrase:
prove the Riemann hypothesis $\rightarrow \lambda t \lambda w \lambda x.\text{PROVE}(x,\text{RH},w,t)$

We have the same temporal and modal expressions as in the relative clause:
might and *one time*.

Just as in the functional readings we needed to introduce a functional variable, we need to introduce here a **property** variable. In this case, since we have a property to start out with, there is a natural type shifting principle available that introduces a property variable, namely,

INTERSECTIVE MODIFIER SHIFT: $\alpha \rightarrow \lambda \mathbf{P}.\lambda t \lambda w \lambda x.\mathbf{P}(x,w,t) \wedge \alpha(x,w,t)$

This shift can, in principle take place at three levels:
a. *might one day* SHIFT(*prove the Riemann hypothesis*)
b. *might* SHIFT(*one day prove the Riemann hypothesis*)
c. SHIFT(*might one day prove the Riemann hypothesis*)

-As it turns out, shift *a* is possible, but interferes generally with the aspectual properties of the predicate in the relative clause.
(Shift *a* is unproblematic if the predicate in the relative clause is stative but doesn't give the most natural reading if the predicate is not as in our example: On the a-shift the boys must prove the Rieman hypothesis in the worlds where they become at some time a brilliant mathematician **at the same time** as they become a brilliant mathematician, which is unnatural.)
-I stipulate that the c-shift is not available: it would produce what I think are unavailable readings.
So I will assume a derivation with the b-shift.

Step one: combine *one day* with *prove the Riemann hypothesis* (composition)

one day prove the Riemann hypothesis
$\lambda t_0 \lambda w \lambda x.\exists t > t_0$: PROVE(x,RH,w,t)

Step two: shift this predicate with intersective modifier shift:

one day prove the Riemann hypothesis
$\boldsymbol{\lambda}\mathbf{P}\lambda t_0 \lambda w \lambda x.\ \mathbf{P}(x,w,t_0) \wedge \exists t > t_0$: PROVE(x,RH,w,t)

Step three: combine *might* with this (composition):

might one day prove the Riemann hypothesis
$\boldsymbol{\lambda}\mathbf{P}\lambda t_0 \lambda w_0 \lambda x.\ \exists v \in ACC_{w0}$: $\mathbf{P}(x,w,t_0) \wedge \exists t > t_0$: PROVE(x,RH,w,t)

We now link the tense and the modality. There are two possibilities:

1. **independent**: set tense to t_0/modality to w_0 (present/real world) (= apply to t_0/w_0)
2. **P-dependent:** since **P** is a variable over relations between individuals, worlds and times, the world/time can be set depending on **P**, i.e. dependent on some world/time in the relevant coordinate of **P**.

Fact about modals: modals with the same accessibility relation resist dependent interpretations (unlike modal logic, natural language doesn't like expressions like *must may* φ)
Default for modals: independent.

On the other hand, dependent interpretations are **natural** for **tenses**. And that is what we are interested in here.

Step four:
Set modality to independent.
Set tense to P-dependent.

might one day prove the Riemann hypothesis
$\boldsymbol{\lambda}\mathbf{P}\lambda x.\ \exists t_0 \in [\mathbf{P}]^3$: $\exists v \in ACC_{\mathbf{w0}}$: $\mathbf{P}(x,w,t_0) \wedge \exists t > t_0$: PROVE(x,RH,w,t)

Step five: combine the dirstributive generalized quantifier with the predicate.

The brilliant mathematician that DET boys might one day become might one day prove the Riemann hypothesis.
$\forall x \in [\boldsymbol{\alpha}]^1$: $\exists t_0 \in [\boldsymbol{\alpha}]^3$: $\exists v \in ACC_{\mathbf{w0}}$: $\boldsymbol{\alpha}(x,w,t_0) \wedge \exists t > t_0$: PROVE(x,RH,w,t)
where $\alpha = \lambda t \lambda v \lambda x.BOY_{w0t0}(x) \wedge t > t_0 \wedge v \in ACC_{w0} \wedge$ BECOME(**BM**(x,v,t))
if DET[BOY_{w0t0},$\lambda x.\exists t > t_0$: $\exists v \in ACC_{w0}$: BECOME(**BM**(x,v,t))]

SEMANTICS OF THE MAIN PREDICATE : THE FAST VERSION

The subject denotes a relation:

$\lambda t \lambda w \lambda x.\text{BOY}(x,w_0,t_0) \wedge w \in \text{ACC}_{wo,t0} \wedge t > t_0 \wedge \text{BECOME-BrilMath}(x,w,t)$
if DET boys might become a brilliant mathematician.
undefined otherwise.

The main predicate is going to denote a predicate of such relations:
The semantics will build up the following predicate, with brief annotations about how if comes about:

$\lambda R.$ ← set of relations
$\forall x \in [R]^1$: ← dependent distributivity operator
$\exists t \in [R]^3$: ← dependent tense
$\exists w \in \text{ACC}_{wo.t0}$:
might
$R(x,w,t) \wedge$
$\exists t' > t$: ProofRieHyp(x,w,t')
one day prove the Riemann hyp.

standard shift from rel to <rel,rel> introduces the relational variable.

This statement., simplified asserts and presupposes:

The brilliant mathematician that DET boys might one day become might one day prove the Riemann hypothesis.

1. **Presupposition**:
$\text{DET}[\text{BOY}_{w0t0},\lambda x.\exists t > t_0: \exists v \in \text{ACC}_{w0}: \text{BECOME}(\mathbf{BM}(x,v,t))]$
(det boys might one day become a brilliant mathematician)

2. **Assertion**:
$\forall x[\text{BOY}_{w0t0}(x) \wedge \exists t>t_0 \wedge \exists v \in \text{ACC}_{w0}: \text{BECOME}(\mathbf{BM}(x,v,t)) \rightarrow$
$\exists t[t > t_0\ \exists v \in \text{ACC}_{\mathbf{w0}}: \text{BECOME}(\mathbf{BM}(x,v,t)) \wedge \exists t_1 > t: \text{PROVE}(x,RH,v,t_1)$

For every boy in BOY_{w0t0} for which there is a future time and accessible world where he becomes a brilliant mathematician, there is a future time and accessible world where he becomes a brilliant mathematician and later prove the Riemann hypothesis.

Or simpler: every one of the boys that might one day become a brilliant mathematician, **might one day become a brilliant mathematician and then** one day prove the Riemann hypothesis.

Note that I have done the R-shift **not** at the lowest possible level (= shift *prove the RH*), but one level higher up. The first is not impossible (it is what Grosu and Krifka do in the version of the paper I have seen), but interferes with the aspect of the verb.
That is, it is innocent when the verb is stative (as in Grosu and Krikfa's examples), but gives the less plausible reading, when the predicate is an achievement (as with *become*).

That is: on the most plausible reading, the sentence does **not** say that they might prove the RH **at the moment that** they become a brilliant mathematician.

As a consequence, on the more plausible reading I deriveit is not **semantically** guaranteed that that at the time they prove the Riemann hypothesis, they are brilliant mathematicians, though in natural contexts this would be pragmatically inferred (simply that it is unlikely that they will prove the RH in, say, a state of mental deterioration).

And this is **good,** I think, because you don’t **want** to require cotemporality in the **semantics** because of examples like the following:

(9) The fifty year old that you will one day become will in the years to follow think with melancholy about her youth.

The thinking is, by the semantics, not thinking of a fifty year old, but of someone who acquired at fifty a property that produced melancholy in later years. The a-shift would produce wrong results here, it’s the b-shift we want.

MORAL:

In building up the relative clause meaning you need to keep track of the sets that the quantifiers live on (two tier semantics): the actual boys, the future times, the accessible worlds where they become brilliant mathematicians.
With that, the main clause predicate can be interpreted as
MODALLY RESUMPTIVE (= the worlds introduced in the main clause predicate ARE the same words already introduced in the relative clause)
and as
TEMPORALLY DEPENDENT (= the times introduced in the main clause predicate are NOT the same as the times introduced in the relative clause, BUT are quantificationallyt dependent on the times introduced in the relative clause).

Such dependencies cannot be expresses in the classical theory, except by sticking in everywhere invisible e-type pronouns, with a interpretation theory which is probably too unconstrained.

That is: it is perfectly ok to rely on pragmatics to derive, say, the most plausible reading of a sentence within the space of possible readings.
But there are cases where it wouldn't be crazy or incoherent or whatever if the sentence had reading φ, it just so happens that it doesn’t' (as in the functional domain restriction cases discussed above).

Such are cases where we would like the semantics to constrain the meaning so as to exclude the reading.

What we have seen in this talk are cases where the classical theory cannot impose such a semantic constraint, and hence would have to rely on the pragmatics.
To my mind the analysis would depend too much on the pragmatics, since the cases involved seem to be cases cases where it is not clear that a theory of pragmatic relevance and rationality could actually justify making the right restrictions.

In the two tier theory these dependencies become semantically available in (what I hope will turn out to be) a relatively restricted way: you can depend on what is recoverable from the modal temporal relation derived at the second tier.

One would need a theory of which operations can access second tier information when. It seems instructive to me, that in both cases of relative clauses discussed here the second tier access takes place at exactly the same stage of derivation.
But I do not have a theory about this.