AN ALMOST (BUT NOT QUITE) NAÏVE SEMANTICS FOR COMPARATIVES

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1. THE ALMOST (BUT NOT QUITE) NAIVE SEMANTICS FOR DP COMPARATIVES

1.1. NAIVE THEORY OF MEASURES:

Base semantics on a 'naïve' ontology of degrees and measures: the one used in the sciences.
-**measure scales** based on the set of real numbers, equipped with order, supremums and infimums, and arithmetic.
-**measure functions,** like *height in inches* which assign to one individuals (in a world at a time at a place,…) one height in inches.

-No measure relations in which I am many heights simultaneously (Heim and others)
-No extents of Tallness and anti-extents of Shortness (von Stechow, Kennedy)
-No conceptual reconstructions of scales and measures (Kamp, McConnel-Ginet,Klein,…)

1.2. NAÏVE SEMANTICS OF MEASURES:

- Fred is taller than Susan if there is a difference in height between them, in Fred's favor.
- Fred is tall if there is a difference in height between Fred's height and a contextual standard, in Fred's favor.

Following von Stechow: **adjective** *tall-Ø* and **comparative** *tall-er* are not defined in terms of each other, both are defined in terms of **dimension** *tall*. (Against Bartsch and Vennemann, Kamp, Klein,…)

1.3. *NOT QUITE* MEANS: SEMANTICS NEED NOT BE NAÏVE.

> **The Principle BPR: (Bach, Partee, Rooth):**
> Interpret everything as low as you can, but not so low that you will regret it later.

Ideal semantics for degree phrases:
-*three* denotes 3.
-Keep the denotations of degree expressions at the level of degrees, predicates of degrees, etc. for as long as you can.

Example:
We want to define the meaning of *very* in *very tall* as an operation on the degree meaning of *tall*:

Degree meaning: The set of degrees higher than the standard for minimal Tallness.
Adjectival meaning: The set of individuals who are Tall in the context.

This requires type shifting principles, most importantly:

Let d be the type of individuals, t of truth values, δ of degrees.
Let α be a predicate of degrees of type $<\delta,t>$
Let M be a measure function (with world parameter specified) of type $<d,\delta>$

TYPE SHIFT: Compose with measure function

α	$\rightarrow$	$\alpha \circ M$	(where $\alpha \circ M = \lambda x.\alpha(M(x))$)
$<\delta,t>$		$<d,t>$	

$\lambda\delta.\ \delta > 3$	$\rightarrow$	$\lambda x.\ M(x) > 3$
The set of degrees bigger than three		The set of individuals with measure bigger than three

1.4. MEASURE ONTOLOGY

r is the type of (real) **numbers**.	**R**
m is the type of **measures.**	**Primitives:** H(eight), Age, …
u is the type of **measure units.**	**Primitives:** m(eters), "(inches), …
δ is the type of **degrees**.	

A degree is a triple **<r,u,m>** where r is a real number, u a unit, m a measure, and u is an appropriate unit for m.

General convention:
mnemonic superscrips denote the relevant element of a tuple:
example: $<29\ ,\ "\ ,\ H>^{\mathbf{r}} = 29$; ($^{\mathbf{r}}$ for *real value*)

Measure functions:
-$H_{"}$ *height in inches* is a (partial) function which assigns to an index w (world, time) and an individual x a triple $H_w(x) = <r\ ,\ ",\ H>$, where r is a real number.
-This functions is convertible into the function H_m, height in meters.
-Up to convertability of units, there is, *per measure*, only one measure function.

SCALES

s is the type of **scales**.

Given measure M, unit u, context k.

A **basic scale** $S_{M,u,k}$ is a tuple:

$S_{M,u,k}$ = <M, D, $>_M$, $\sqcup_M$, $-_M$, M_u, $LOW_{M,u,k}$ $HIGH_{M,u,k}$> where:

1. M is the measure that S is based on.
2. D = {<r,u,M>: r $\in$ **R**} the domain of relevant degrees.
3. $>_M$ (bigger than), $\sqcup_M$ (supremum), $-_M$ (subtraction) specify the **direction** of the scale. These notions are lifted from **R**:
   e.g. <p,u,M> $>_M$ <q,u,M> iff n $>_{\mathbf{R}}$ m

[Note: $>_{\mathbf{M}}$ is lifted from $>_{\mathbf{R}}$ and not $<_{\mathbf{R}}$. This means that $\sqcup_M$ corresponds to $\sqcap_{<\mathbf{R}}$]

4. M_u is the measure function.
5. $LOW_{M,u,k}$, $HIGH_{M,u,k}$ $\in$ D and $HIGH_{M,u,k}$ $>_M$ $LOW_{M,u,k}$

Let $S_{M,u,k}$ be a basic scale.

The **converse scale for** $S_{M,u,k}$, $\mathbf{S_{M,u,k}}^{\mathbf{c}}$ is given by:

$S_{M,u,k}{}^{c}$ = <M, D, $>_M{}^{c}$, $\sqcup_M{}^{c}$, $-_M{}^{c}$, M_u, $LOW_{M,u,k}{}^{c}$ $HIGH_{M,u,k}{}^{c}$>, where

1. Measure, domain, and measure function of the converse scale are the same as those of the basic scale.
2. The **directional** notions are the converse notions:
   - <**n**,u,M> $>_M{}^{c}$ <**m**,u,M> iff <**m**,u,M> $>_M$ <**n**,u,M>
   - $\sqcup_M{}^{c} = \sqcap_M$
   - <**n**,u,M> $-_M{}^{c}$ <**m**,uM> = <**m**,uM> $-_M$ <**n**,u,M>
3. Contextual low and high are converted:
   $LOW_{M,u,k}{}^{c}$ = $HIGH_{M,u,k}$; 5. $HIGH_{M,u,k}{}^{c}$ = $LOW_{M,u,k}$

Also here I will use mnemonic superscripts.

The context k may provide for a measure like H(eight) a default unit, which I will call $\Delta_{M,k}$, or Δ for short.

1.5. THE ALMOST (BUT NOT QUITE) SEMANTICS

1. *tall* has an interpretation as a **measure,** *short* does not.
2. *tall* and *short* have interpretations as **dimensions** (in essence, **scales**):

   tall denotes the **basic scale** of Height.
   short denotes the **converse scale** of Height.

3. The meaning of adjectives *tall/short* and comparatives *tall/short* are derived from their **dimensional** meanings.

4. *more* denotes the **difference function** (subtraction)
   less denotes the **converse of the difference function**

5. **Function composition.**

   Generalized function composition
   $f \circ g \quad = \quad \lambda x_n \ldots \lambda x_1.f(g(x_1,\ldots,x_n))$

   (Bring function g down to the input type for f, by applying it to variables, apply f to the result, and abstract over all the variables used.)

The heart of the semantics for comparatives is the following principle:

> **A one place number/degree predicate combines with a difference function to form a two place number/degree relation.**

SAMPLE DERIVATION
STEP 1:

-PRED$_{num}$: null-predicate of numbers $Ø_{a\ bit}$, with semantic meaning ***a bit*** (or ***some***)

$Ø_{\mathbf{a\ bit}} \rightarrow \lambda r.r >_{\mathbf{R}} 0$ (the set of positive real numbers)

-DIF$_{num}$: *more* and *less* denote the **difference function and its converse** (resp)**:**

more $\rightarrow \lambda m\lambda n.\ (n -_{\mathbf{R}} m)$

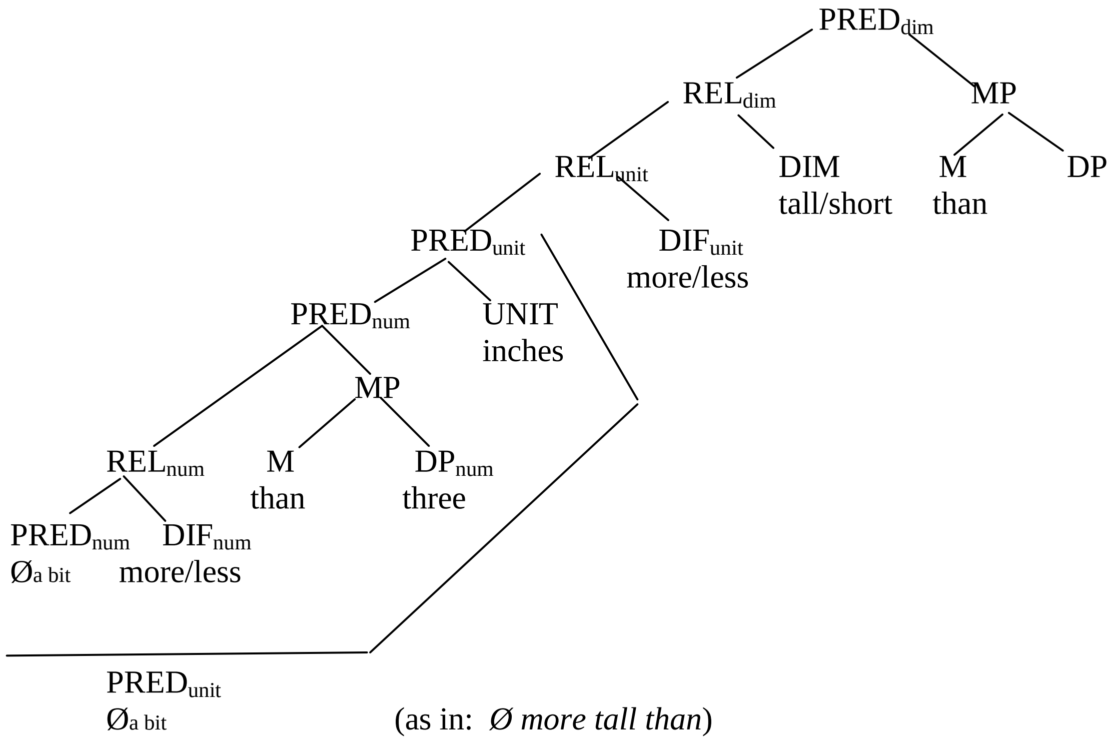
less] $\rightarrow \lambda m\lambda n.\ (m -_{\mathbf{R}} n)$

STEP 2: COMPOSITION: REL$_{num}$

$Ø_{a\ bit}$ *more* $\rightarrow \lambda r.r > 0 \circ \lambda m\lambda n.\ (n - m)$
$= \lambda m\lambda n.\ (n - m) > 0 \quad = \quad >_{\mathbf{R}}$

$Ø_{a\ bit}$ *less* $\rightarrow \lambda r.r > 0 \circ \lambda m\lambda n.\ (m - n)$
$= \lambda m\lambda n.\ (m - n) > 0 \quad = \quad <_{\mathbf{R}}$

I will assume a structure that follows the following semantic composition:



STEP 3: PRED_{num}: **APPLY** the meaning of the REL_{num} derived to 3:

more than three $\rightarrow$ $\lambda m.\ m >_{\mathbf{R}} 3$

less than three $\rightarrow$ $\lambda m.\ m <_{\mathbf{R}} 3$

STEP 4: $\text{PRED}_{\text{unit}}$: **LIFT** the numerical predicate and " (*inch*) to predicates of degrees, semantically adjoin the latter to the first. (Formulation of the obvious operations omitted.):

more than three inches $\rightarrow$ $\lambda\delta.\delta^{r} > 3 \wedge \delta^{u} = ''$

the set of degrees whose numerical value is bigger than three and whose unit is inch

less than three inches $\rightarrow$ $\lambda\delta.\delta^{r} < 3 \wedge \delta^{u} = ''$

the set of degrees whose numerical value is bigger than three and whose unit is inch

Also: null degree predicate $Ø_{\mathbf{a\ bit}}$:

$Ø_{\mathbf{a\ bit}}$ $\rightarrow$ $\lambda\delta.\delta^{r} > 0 \wedge \delta^{u} = \Delta$

(A wrinkle: context or grammar must here pick the correct measure for the derivation)

STEP 5: DIF_{unit}: *more* and *less* denote **scale dependent functions of subtraction and its converse:** ($^{-}$ is a mnemonic superscript)

more $\rightarrow$ $\lambda s.s^{-}$

The function that maps every scale onto its subtraction operation.

less $\rightarrow$ $\lambda s.(s^{c})^{-}$

The function that maps every scale onto the subtraction operation of its converse scale.

STEP 6: DIM: *tall* and *short* are **dimensions:** in essence they denote **scales,** in practice functions from units to scales:

tall $\rightarrow$ $\lambda u.S_{H,u,k}$

The function than maps unit u onto the **basic** scale $S_{H,u,k}$

short $\rightarrow$ $\lambda u.S_{H,u,k}{}^{\mathbf{c}}$

The function that maps unit u onto the **converse** scale $S_{H,u,k}{}^{\mathbf{c}}$

DERIVATION SKETCH:

STEP 7: REL_{unit}: **compose** $PRED_{unit}$ and DIFunit (*more/less*):
REL_{unit}: f**unction from scales into two-place relations between degrees**.

STEP 8: APPLY DIM *tall*/*short* to the unit derivable from REL_{unit}
(form meanings $S_{H,'',k}$ or $S_{H,\Delta,k}$ for *tall* and the converse scales for *short*)

STEP 9: REL_{dim}: **APPLY STEP 7 TO STEP 8.**

Result: relations between degrees of type $<\delta,<\delta,t>>$

After reduction:

$[_{RELdim}$ *more than three inches more tall than*] $\rightarrow$ $\lambda\delta_2\lambda\delta_1\in D_{H,''}$: $\delta_1{}^r > \delta_2{}^r + 3$
$[_{RELdim}$ *less than three inches more tall than*] $\rightarrow$ $\lambda\delta_2\lambda\delta_1\in D_{H,''}$: $\delta_1{}^r < \delta_2{}^r + 3$
$[_{RELdim}$ *more than three inches less tall than*] $\rightarrow$ $\lambda\delta_2\lambda\delta_1\in D_{H,''}$: $\delta_1{}^r < \delta_2{}^r - 3$
$[_{RELdim}$ *less than three inches less tall than*] $\rightarrow$ $\lambda\delta_2\lambda\delta_1\in D_{H,''}$: $\delta_1{}^r > \delta_2{}^r - 3$
$[_{RELdim}$ Ø *more tall than*] $\rightarrow$ $\lambda\delta_2\lambda\delta_1\in D_{H,\Delta(H,k)}$: $\delta_1{}^r > \delta_2{}^r$
$[_{RELdim}$ Ø *less tall than*] $\rightarrow$ $\lambda\delta_2\lambda\delta_1\in D_{H,\Delta(H,k)}$: $\delta_1{}^r < \delta_2{}^r$

FACT: *β more short* is equivalent to *β less tall*

E.g.: *at least three inches* ***more short*** *than* = *at least three inches* ***less tall*** *than*

Relations between individuals: composition with the measure function
(twice in the derivation):

(1) *Fred is taller than Susan* $H_{''}(\text{Fred})^r > H_{''}(\text{Susan})^r$
(2) *Fred is more than three inches taller than Susan* $H_{''}(\text{Fred})^r > H_{''}(\text{Susan})^r + 3$
(3) *Fred is less than three inches taller than Susan* $H_{''}(\text{Fred})^r < H_{''}(\text{Susan})^r + 3$

(4) *Fred is shorter than Susan* $H_{''}(\text{Fred})^r < H_{''}(\text{Susan})^r$
(5) *Fred is more than three inches shorter than Susan* $H_{''}(\text{Fred})^r < H_{''}(\text{Susan})^r - 3$
(6) *Fred is less than three inches shorter than Susan* $H_{''}(\text{Fred})^r > H_{''}(\text{Susan})^r - 3$

Case (3) (and 6): compare (7):

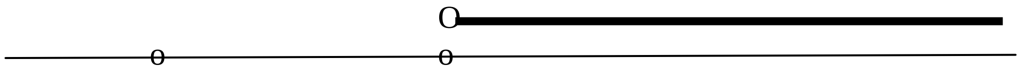
(7) A. Is John taller than Mary?
B. I don't know. But I *do* know that he is less than two centimeters taller than Mary. You see, Mary is 1.63. And I happen to know that John was rejected by the police because of his height, and they only accept people 1.65 and up.

This discourse is felicitous and compatible with John being **smaller** than Mary, supporting the interpretation given in (3).

2. PREDICTIONS FOR DP COMPARATIVES

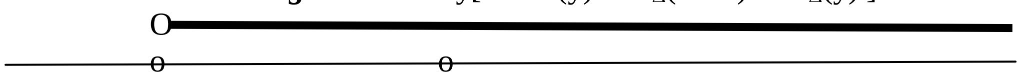
2.1. QUANTIFICATIONAL DP COMPLEMENTS

John is taller than *every girl*. $\forall y[\text{GIRL}(y) \rightarrow H_{\Delta}(\text{John})^r > H_{\Delta}(y)^r]$



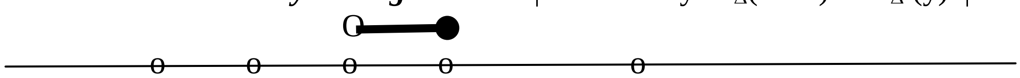
$H_{\Delta}(g_1)^r$ ………………$H_{\Delta}(g_n)^r$ **John is taller than the tallest girl**.

John is taller than *some girl*. $\exists y[\text{GIRL}(y) \wedge H_{\Delta}(\text{John})^r > H_{\Delta}(y)^r]$



$H_{\Delta}(g_1)^r$ ………………$H_{\Delta}(g_n)^r$ **John is taller than the shortest girl.**

John is taller than *exactly three girls*. $|\text{GIRL} \cap \lambda y.\ H_{\Delta}(\text{John})^r > H_{\Delta}(y)^r| = 3$

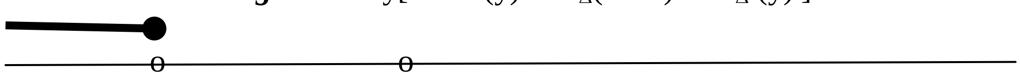


$H_{\Delta}(g_1)^r$ $H_{\Delta}(g_2)^r$ $H_{\Delta}(g_3)^r$ $H_{\Delta}(g_4)^r$…….. $H_{\Delta}(g_n)^r$

John is taller than the shortest three girls, but not taller than any other girls.

(Many theories have problems getting this)

John is taller than *no girl*. $\neg\exists y[\text{GIRL}(y) \wedge H_{\Delta}(\text{John})^r > H_{\Delta}(y)^r]$

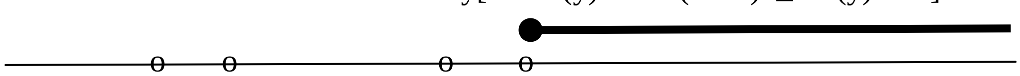


$H_{\Delta}(g_1)^r$ …………$H_{\Delta}(g_n)^r$ **John's height is at most that of the shortest girl.**

(Stilted, because English prefers auxiliary negation, but felicitous.)

John is at least two inches taller than *every girl*.

$\forall y[\text{GIRL}(y) \rightarrow H_{''}(\text{John})^r \geq H_{''}(y)^r + 2]$

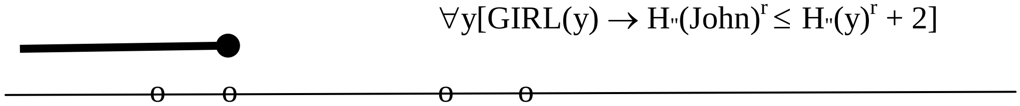


$H_{''}(g_1)^r$ +2 …………$H_{''}(g_n)^r$ +2

John's height is at least the height of the **tallest** girl plus two inches.

John is at most two inches taller than *every girl*.

$\forall y[\text{GIRL}(y) \rightarrow H_{''}(\text{John})^r \leq H_{''}(y)^r + 2]$



$H_{''}(g_1)^r$ +2…………$H_{''}(g_n)^r$ +2

John is at most the height of the **shortest** girl plus two inches.

Cf. the following valid inference:

a. John is at most two inches taller than every girl.
b. Mary is the shortest girl.
c. Hence, John is at most two inches taller than Mary.

2.2. DP-COMPARATIVES AND POLARITY

DP-comparatives **seem** to allow polarity sensitive (PS) items and seem to be downward entailing (DE):

(1) a. Mary is more famous than *anyone*.
b. (1) Mary is more famous than John *or* Bill.
(2) Hence, Mary is more famous than John.

Hoeksema 1982:

1. *anyone* in (1a) and *or* in (1b) allow **free choice interpretations** (FC).
Hence: the facts in (1) are consequences of FC interpretations, not PS interpretations.
(Certainly DP-comparatives allow FC-*any*: only FC-*any* can be modified by *almost*
(2) Mary is more famous than *almost anyone*.)

2. **DP comparatives are not downward entailing:** (3) is **invalid**:

(3) (1) John is more famous than *Mary*.
(2) Mary is a girl.
(3) Hence, John is more famous than *every girl*.

3. Dutch has PS items that are not FC items, and these are not felicitous in DP comparatives.

Hoeksema: *ook maar iemand* is PS but not FC (cf. FC item *wie ook maar*):

(4) a. **Ik leen geen boeken uit aan *ook maar iemand*.** — **DE context:**
I lend no books out to *ook-maar*-someone — **PS felicitous**
I don't lend books to anyone

b. **#Dat kan je *ook maar iemand* vragen.** — **Modal context:**
That can you *ook-maar*-someone ask — **PS infelicitous**
That, you can ask anyone

c. **Dat kan je *wie dan ook* vragen.** — **Modal context:**
That can you who-*dan ook* ask — **FC felicitous**
That, you can ask anyone

PS items are felicitous in CP comparatives but not in DP comparatives:

(5) a. **Marie is beroemder dan *ook maar iemand* ooit geweest is.** — **CP comparative**
Marie is more famous than *ook-maar*-someone ever been is — **PS felicitous**
Marie is more famous than anyone has ever been.

b. **#Marie is beroemder dan *ook maar iemand*.** — **DP comparative**
Marie is more famous than *ook-maar*-someone — **PS infelicitous**
Marie is more famous than anyone

c. **Marie is beroemder dan *wie dan ook*.** — **DP comparative**
Marie is more famous than who-*dan ook* — **FC felicitous**
Marie is more famous than anyone

Hoeksema's claim can be strengthened by looking at stressed é*nige*. As a plural, not-necessarily stressed item *enige* means *a few*, and is not at all a polarity item:

(6) Ik heb hem *enige boeken* uitgeleend.
I lent him *a few* books.

But as a singular, stressed element, é*nige* is a PS item, and it means *any*, PS *any*, and nor FC *any*:

(7)	a.	**Ik leen geen boeken uit aa**n ***énige filosoof*.** I lend no books to any philosopher	**DE context** **PS felicitous**
	b.	**#Dat kan je *énige filosoof* vragen.** That, you can ask any philosopher.	**UE modal context:** **PS infelicitous**

And we find that ***énige*** is infelicitous in DP comparatives:

(8) a.	**Marie is beroemder dan é*nige filosoof* ooit geweest is.** Marie is more famous than an philosopher has ever been.	**CP comparative** **PS felicitous**
b.	**#Marie is beroemder dan é*nige filosoof*** Marie is more famous than any philosopher	**DP comparative** **PS infelicitous**

Comment: (5b) and (8b) **improve** in felicity if we tag on them a **FC appositive phrase**:

(8) a. Marie is beroemder dan *ook maar iemand*, ***wie dan ook***.
b. Marie is beroemder dan *énige filosoof*, ***welke je ook maar kiest.***
whichever one you choose.

This supports: FC is licensed in DP-comparatives, PS is not.

Prediction of the almost (but not quite) naïve semantics for DP comparatives (following Hoeksema 1982):

Montague's generalization applies to DP comparatives:

The DP complement of an extensional transitive verb/DP-comparative relation **takes semantic scope over** the meaning of the transitive verb/comparative relation.

Consequently:

DP-comparatives are not downward entailing on their DP complement argument, and polarity items are not licensed.

CONCLUSIONS FOR THE SEMANTICS OF DP COMPARATIVES:

-Hoeksema 1982 was on the right track for DP comparatives (not to reduce them to CP comparatives, but treat them semantically on a par with extensional transitive verbs).
-His theory can be 'modernized' in a type shifting semantics of degrees with composition.
-The almost (but not quite) naïve theory adds to this an analysis of **converse orders** and **converse operations**.
-The resulting theory smoothly predicts the right interpretations for quantificational complements and makes the right predictions about polarity items in the complement.

3. CP COMPARATIVES

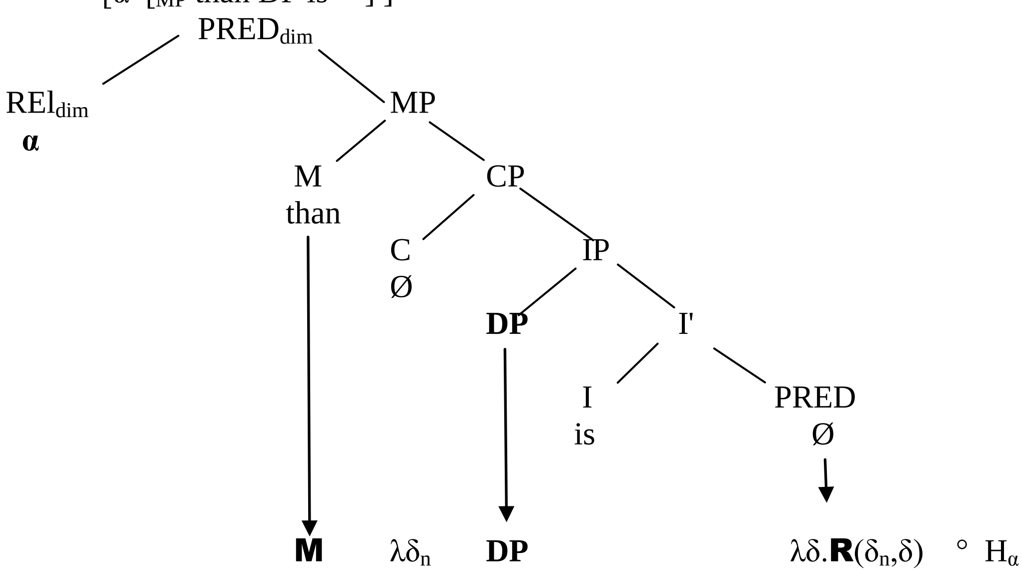
3.1. GENERAL SEMANTICS FOR CP COMPARATIVES

Terminology: DP-comparative and CP-comparative in (1): **comparative correlates**. (

(1) a. John is taller than DP

b. John is taller [$_{CP}$ than DP is –]

[α [$_{MP}$ than DP is –]]



-CP: Operator gap construction: semantically interpreted (variable binding)
CP level: Abstraction over a degree variable (δ_n) introduced in the gap.

-Gap: predicate gap. With BPR: **degree predicate** ($\lambda\delta.\mathbf{R}(\delta_n,\delta)$ for some relation **R**), shifted to a predicate of individuals with composition with the measure function.

GENERAL SEMANTICS FOR COMPARATIVE COMPLEMENTS:

[α [$_{MP}$ than DP is –]]

α + **M** ($\lambda\delta$. **DP** ($\lambda x.\ \delta\ \mathbf{R}\ H_\alpha(x)$))

1. What is relation R?

2. What is operation M?

Von Stechow:	**R**	=	(identity)
	M	$\sqcup_<$	(supremum, maximalization operation)
Heim	**R**	$\lambda\delta_2\lambda\delta_1.\ 0 < \delta_1^r \leq \delta_2^r$	(monotonic closure down)
	M	$\sqcup_<$	(supremum, maximalization operation)

The Naïve (but clever) Theory (→Schwarzschild and Wilkinson)

R	**α**	(the external comparative relation)
M	$\lambda P.P$	(identity)

3.2. VON STECHOW'S SUPREMUM THEORY.

PROBLEM: (Schwarzschild and Wilkinson):

Maximalization theories predict unnatural readings and fail to predict natural readings.

(1) John is taller than some girl is –

von Stechow: $H_\Delta(\text{John}) >_H \sqcup_<(\lambda\delta.\exists x[\text{GIRL}(x) \wedge \delta = H_\Delta(x)])$

Wrong meaning: John is taller than the **tallest girl**

(2) John is taller than every girl is –

Heim: $H_\Delta(\text{John}) >_H \sqcup_<(\lambda\delta.\forall x[\text{GIRL}(x) \rightarrow 0 < \delta^r \leq H_\Delta(x)^r])$

Wrong meaning: John is taller than the **shortest girl**

von Stechow: $H_\Delta(\text{John}) >_H \sqcup_<(\lambda\delta.\forall x[\text{GIRL}(x) \rightarrow \delta = H_\Delta(x)])$

John is taller than **the** degree to which every girl is tall.

Unwarranted presupposition: all girls have the same degree of height

PROPOSED SOLUTION (Larsons, Von Stechow):

Give the CP internal DP scope over the comparative.

Larsons: standard scope mechanism; von Stechow: non-standard scope mechanism

PROBLEM 1:

This requires **systematic scoping of all kinds of DPs out of the CP**, which is problematic . Cf. scoping out of relative clauses, wh-clauses, or even propositional attitude complements: in all other CPs scoping out is severely restricted.

PROBLEM 2: **Intensional contexts (Schwarzschild and Wilkinson)**

(3) John is taller than [$_{CP}$ Bill believes that every girl in Dafna's class is –.]

$\lambda\delta.\ \text{BELIEVE}(\text{BILL}, {}^\wedge\forall x[\text{GIRL-IN-DC}(x) \rightarrow \delta^r > H_\Delta(x)^r)])\ \ (H_\Delta(\text{John}))$

John's actual height has the property that Bill believes it to be bigger than what he thinks is the height of what he thinks is the tallest girl in Dafna's class.

-No presupposition that Bill believes that the girls have the same height.

-von Stechow: scope *every girl* over the comparative.

-But *every girl* takes narrow scope under *believe*, which is inside the comparative.

Problem 3: **Polarity items:**

Unlike DP-comparatives, CP-comparatives allow polarity items inside the CP-complement:

(4) **Marie is beroemder dan énige filosoof en énige psycholoog ooit geweest zijn.**

Marie is more famous than any philosopher and any psychologist have ever been.

-No presupposition that any philosopher has the same degree of fame as any psychologist.

- von Stechow: scope *enige filosoof en enige psycholoog* over the comparative.

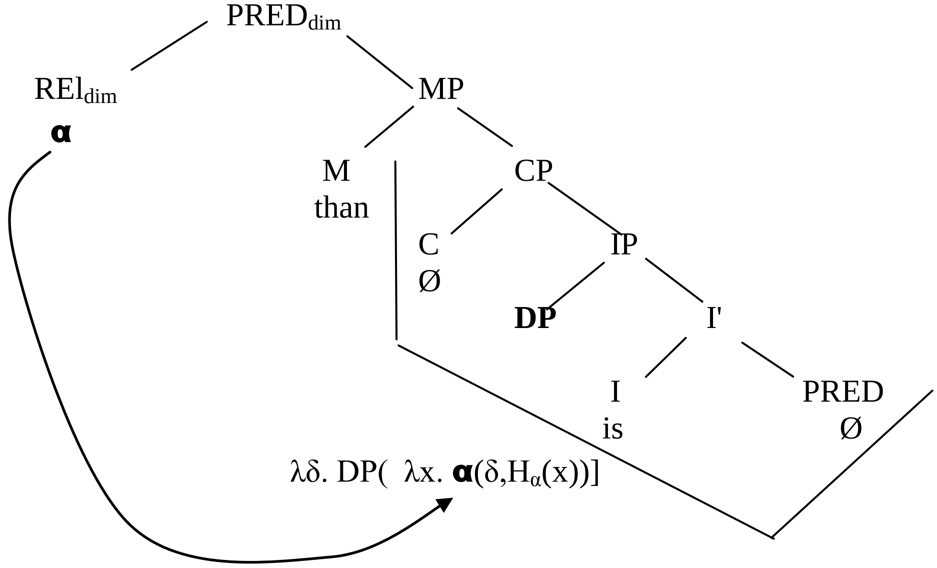
-But conjunction of PS items: **not licensed if scoped over the comparative.**

CONCLUSION (Schwarzschild and Wilkinson):

Supremum theories like von Stechow's (and Heim's) are untenable.

3.3. THE NAÏVE (BUT CLEVER) THEORY:

$[\alpha\ [_{MP}$ than DP is –]]



α is interpreted inside the comparative CP as part of the interpretation of the gap.

The *clever* intuition:

(5) John is *taller than* Mary/ every girl is –

Everybody else: The CP denotes a set of degrees to which Mary, every girl is tall.
Schwarzschild and Wilkinson: The CP denotes the set of degrees **bigger than** Mary's height/every girl's height.
This is the clever bit.

PREDICTIONS:

1. Correct readings:

CP-comparatives that have correlates have the same interpretation as their correlates: (1) – (2) get the correct readings.

2. No unwanted presuppositions: *in situ* readings for (2), (3) and (4) do not presuppose the same height, belief of same height, or the same degree of fame. Consequently, **no scopin**g out of the CP is necessary to get the correct readings.

3.4. TWO PROBLEMS FOR THE NAÏVE (BUT CLEVER) THEORY

PROBLEM ONE: downward entailing DPs in CP-comparatives.

Hoeksema, Rullmann: Downward entailing DPs in CP-complements are infelicitous.

(1) a. Mary is taller than nobody.
b. #Mary is taller than nobody is –.
c. #Mary is taller than nobody ever was –.

Cf, also

(2) a. #Mary is more famous than John isn't –.
b. #Mary is more famous than John will never be –.
c. #Bill is taller than at most three girls ever were –.
d. #Bill is at least two inches taller than nobody ever was.
e. #Bill is at most two inches taller than nobody ever was.
f. #Bill is exactly two inches taller than nobody every was.

-(1a) is felicitous, if stilted, and *nobody* has a wide scope reading:
Nobody is such that Mary is taller than them., i.e. Mary is the shortest.
-(1b) and (1c) are baffling.

What does (1c) mean? Are you trying to say that nobody ever was as tall as Mary is?
Mary's height has boldly gone where nobody's height has ever gone before?
It is not clear how (1c) can mean **that** compositionally, if it should mean that at all!

My own judgement: My brain is trying several interpretation strategies simultaneously and gets hopelessly muddled.

The naïve (but clever) theory makes a wrong prediction:
(1b) means the same as (1a).

PROBLEM TWO: Polarity items in CP-comparatives.

PS items are felicitous in CP-comparatives, but CP comparatives are not downward entailing (Schwarzschild and Wilkinson):

(3) a. John is more famous than *Mary* is –.
b. Mary is a girl.
c. Hence, John is more famous than *every girl* is.

The naïve (but clever) theory has no stage in the semantic derivation on which to hang the difference in polarity licensing between CP comparatives and DP comparatives.

4. THE DIMENSIONAL SUPREMUM THEORY OF CP COMPARATIVES

4.1. DIMENSIONAL SUPREMUMS

Back to the almost (but not quite) naïve theory of DP comparatives.

Let us use **+** for elements that derive from the **basic scale** S_H,

– for elements that derive from the **converse scale** $S_H^{\mathbf{c}}$:

We stipulate: $>_R$ is **+**, $<_R$ is **–**.

For simplicity and clarity I will in the following discussion ignore cases involving $=_R$.
(The discussion is easily amended to include these cases.)

more *than three inches* ***more tall*** *than*	$\delta_1^r >_R \delta_2^r + 3$
+ + +	+
less *than three inches* ***more tall*** *than*	$\delta_1^r <_R \delta_2^r + 3$
– + +	–
less *than three inches* ***more short*** *than*	$\delta_1^r >_R \delta_2^r - 3$ (= **less** than three inches **less tall**)
– + –	+ - - +

The relation α derived by the semantics consist of a comparison relation $>_R$.$<_R$. $=_R$ and a differential +3, –3. Let us call this relation the **R-relation of α.**

FACT: For α derived above, the R-relation of α is +(–) iff the number of – elements used in the semantic composition is even (odd).

(This means that the notion of **R-relation of α** can be defined derivationally.)

DIMENSIONAL ORDER AND DIMENSIONAL SUPREMUM OF α

Let α be derived as above

The dimensional order for α, $>_\alpha$, and the dimensional supremum for α, $\sqcup_\alpha$ are given by:

$$>_\alpha \;=\; \begin{cases} >_H & \text{if the } \textbf{R-relation of } \alpha \text{ is } + \\ <_H & \text{if the } \textbf{R-relation of } \alpha \text{ is } - \end{cases}$$

$$\sqcup_\alpha \;=\; \sqcup_{>\alpha}$$

Examples:

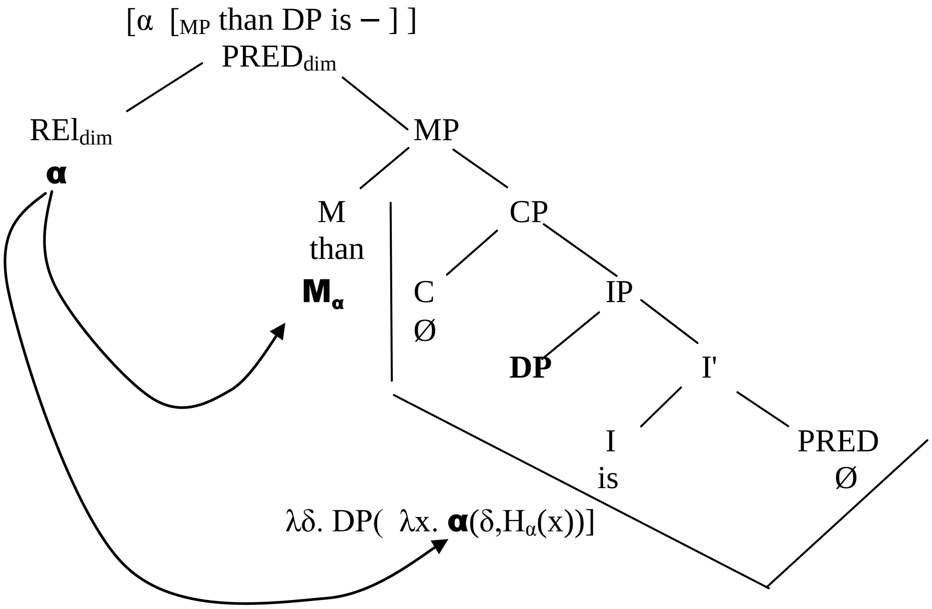
more *than three inches* ***more tall*** *than*

$\alpha = \delta_1^r >_R \delta_2^r + 3$ $\sqcup_\alpha = \sqcup_H$ (which corresponds to $\sqcap_R$)

less *than three inches* ***more tall*** *than*

$\alpha = \delta_1^r <_R \delta_2^r + 3$ $\sqcup_\alpha = \sqcap_H$ (which corresponds to $\sqcup_R$)

4.2 THE DIMENSIONAL SUPREMUM THEORY



-Naïve (but clever) theory:

-comparison relation α is interpreted inside the comparative CP.

-**Dimensional supremum theory**: comparison relation α has a **double** effect:

- comparison relation α is interpreted inside the comparative CP.
- comparison relation α determines the interpretation of the head M as operation $\mathbf{M}_\alpha$.

DIMENSIONAL SUPREMUM AS A PRESUPPOSITIONAL CHECK OPERATION $\mathbf{M}_\alpha$

$$\mathbf{M}_\alpha(\beta) = \begin{cases} \beta & \text{if } \sqcup_\alpha(\beta) \text{ is defined} \\ \text{undefined} & \text{otherwise} \end{cases}$$

$\mathbf{M}_\alpha$ is a **presuppositional** version of the identity function:

- $\mathbf{M}_\alpha$ has no **semantic** effect on input set of degrees β, $\mathbf{M}_\alpha(\beta)$ has **the same meaning as** β,
- $\mathbf{M}_\alpha$ **presupposes** that the dimensional supremum of α of set of degrees β **exists**.

I assume here that $\sqcup_\mathbf{R}$ and $\sqcap_\mathbf{R}$ are **operations on R,** and since ∞ and $-\infty$ are not in **R**, for our purposes here, $\sqcup_\mathbf{R}(X)$ and $\sqcap_\mathbf{R}(X)$ are **undefined when infinite**.

$\mathbf{M}_\alpha$: like a definiteness operation, but without the type change from <d,t> to d:

$\mathbf{M}_\alpha(\beta)$ **presupposes that** the unique object $\sqcup_\alpha(\beta)$ exists, even if it doesn't **denote** it.

CONSEQUENCE:

> **If β_{CP} and β_{DP} are comparative correlates and the meaning of β_{CP} is defined, then β_{CP} and β_{DP} have the same meaning.**

1a) and (1b) are predicted to be equivalent, whenever (1b) is defined:

(1) a. John is α than DP
b. John is α than [$_{CP}$DP is –]

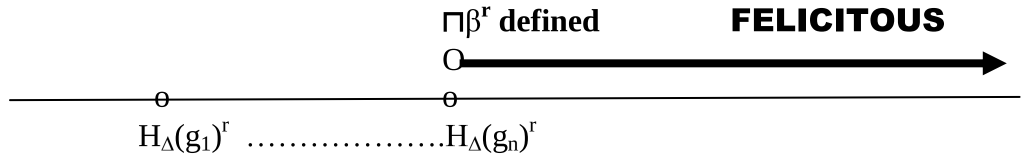
Thus: the dimensional supremum theory of CP comparatives is a presuppositional variant of the naïve (but clever) theory.

4.3. DOWNWARD ENTAILING EXPRESSIONS INSIDE CP COMPARATIVES

(1) **John is taller than *every girl* is –**

taller: $\alpha = >_H$ $\sqcup_\alpha = \sqcup_H$ derived from $\sqcap_{\mathbf{R}}$

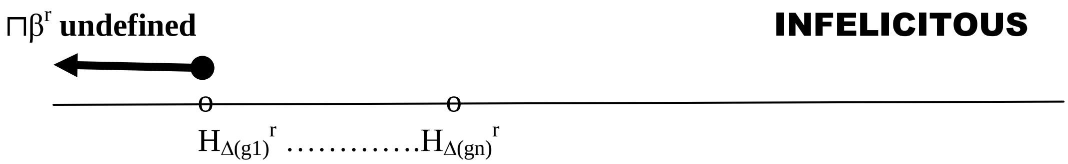
CP: *than every girl is* – $\beta = \lambda\delta.\forall y[\mathrm{GIRL}(y) \rightarrow \delta^r > H_\Delta(y)^r]$



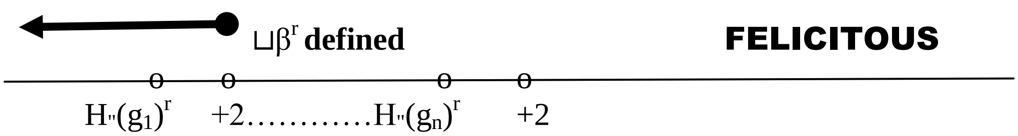
(2) **#John is taller than *no girl* is –**

taller: $\alpha = >_H$ $\sqcup_\alpha = \sqcup_H$ derived from $\sqcap_{\mathbf{R}}$

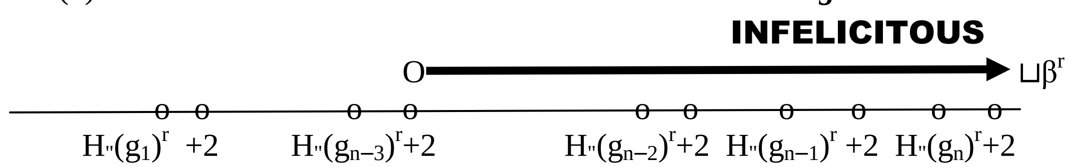
CP: *than no girl is* – $\beta = \lambda\delta.\neg\exists y[\mathrm{GIRL}(y) \wedge \delta^r > H_\Delta(y)^r]$



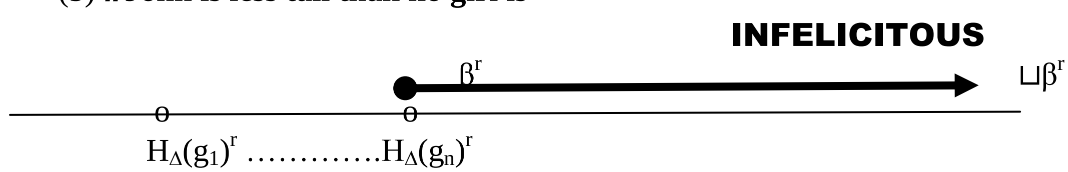
(3) **John is at most two inches taller than *every girl* is –.**



(4) **#John is at most two inches taller than *at most three girls* are –.**



(5) **#John is less tall than no girl is –**



CONSEQUENCE: Downward entailing noun phrases in the CP complements are predicted to be infelicitous.

4.4. POLARITY ITEMS IN THE DIMENSIONAL SUPREMUM THEORY

(1) Mary is more famous than John *ever* was.

FAME: F_Δ : individuals × moments of time → degrees of fame (measured in units of Δ).

[$_{CP}$ *John ever was* $-_{famous}$]
$\lambda\delta.\ \exists t[\delta >_F F_\Delta(\text{John},t)]$
The set of degrees δ for which there is a time t such that δ is bigger on the scale of fame than John's degree of fame at t.

We assume, with Kadmon and Landman 1993, that the semantic effect of *ever* is **widening**. Thus, you may have said (3A), and I reply (3B):

(2) A. Mary is more famous than John is now.
B. Mary is more famous than John *ever* was.

WIDENING IN CP COMPARATIVES

The widening is temporal, and involves a wide and a narrow interpretation:

(3) a. $\lambda\delta.\ \exists t[t \in \textbf{WIDE}\ \text{SET} \wedge \delta >_F F_\Delta(\text{John},t)]$
b. $\lambda\delta.\ \exists t[t \in \textbf{NARROW}\ \text{SET} \wedge \delta >_F F_\Delta(\text{John},t)]$

Scalar comparison construction:
Standard assumption: widening is not unconstrained, but is along the scale.

$\mathbf{max_{JOHN,F}}$ is the **moment of time** where John's fame is maximal
(for simplicity assume that there is one such time).

Pragmatic assumption: there must be a **point** to using *ever*, and hence to widening.
So:

Implicature: $\mathbf{max_{JOHN,F}} \notin$ NARROW SET

Simplest assumption:
Widening done by *ever* just **adds** $\mathbf{max_{JOHN,F}}$ to the narrow set:

Widening: WIDE SET = NARROW SET ∪ { $\mathbf{max_{JOHN,F}}$}

Interpretation of the CP:

[$_{CP}$ *John ever was* $-_{famous}$]
$\lambda\delta.\ \exists t[t \in \text{NARROW SET} \cup \{\mathbf{max_{JOHN,F}}\} \wedge \delta >_F F_\Delta(\text{John},t)]$

By the assumptions made, John's degree of fame at $\mathbf{max_{JOHN,F}}$ is higher than John's degree of fame at any of the points of time in the narrow set: so:

[$_{CP}$ *John ever was* $-_{famous}$]
$\lambda\delta.\ \delta >_F F_\Delta(\text{John},\mathbf{max_{JOHN,F}})$

Given these assumptions, in a natural context (1) is interpreted as:

(1) Mary is more famous than John *ever* was.
$F_\Delta(\text{Mary})^r >_F F_\Delta(\text{John},\mathbf{max_{JOHN,F}})^r$
Mary's degree of fame(now) is higher than John's degree of fame when it was maximal.

This is an adequate account of the meaning of (1).

THE LICENSING OF THE PS ITEM.

Kadmon and Landman: a PS item is licensed if **widening leads to strengthening** at the level of the closest relevant operator the PS item is in the scope of.

STRENGTHENING IN CP COMPARATIVES

Main assumption: **strengthening is defined at the level of the comparative scale (**relation between elements of type δ)

SCALAR STRENGTHENING:

On scale S_M: δ_1 **strengthens** $\boldsymbol{\delta}_2$ iff $\delta_1 \geq_M \delta_2$
On scale $S_M{}^{\mathbf{c}}$: δ_1 **strengthens** $\boldsymbol{\delta}_2$ iff $\delta_1 \leq_M \delta_2$

DP comparatives:

-No grammatical level where the interpretation of the DP is of type δ. (type d or <<d,t>,t>)
(Composition with the measure function takes place in the comparison relation, not in its object.)

Hence, scalar strengthening is irrelevant and PS items are not licensed in DP comparatives.

CP comparatives:

-No grammatical level where the interpretation of the CP is of type δ.

-But the presuppositional check operation brings in an interpretation of type δ as a presupposition: $\sqcup_{\alpha}(\beta)$ of type δ.

Assumption: The polarity item is licensed if widening leads to strengthening at the presuppositional level $\sqcup_{\alpha}(\beta)$

This means that we check whether (2a) strengthens (2b):

(2) a. $\sqcup_{>F}(\lambda\delta.\delta >_F F_{\Delta}(j,\mathbf{max}_{,JOHN,F}))$
b. $\sqcup_{>F}(\lambda\delta.\exists t[t \in \text{NARROW SET} \wedge \delta >_F F_{\Delta}(\text{John},t)])$

$\sqcup_{>F}$ corresponds to $\sqcap_{\mathbf{R}}$, hence:

$$\sqcup_{>F}(\lambda\delta.\delta >_F F_{\Delta}(j,\mathbf{max}_{,JOHN,F})) \quad = \quad F_{\Delta}(j,\mathbf{max}_{,JOHN,F})$$

Let $\mathbf{min}_{narrow,JOHN,F}$ be the time in NARROW where **John's fame is minimal in comparison to the other times in the narrow set**.
Obviously, the infimum of the degrees of John's fame corresponding to the times in the narrow set is $F_{\Delta}(j,\mathbf{min}_{narrow,JOHN,F})$.

$$\sqcup_{>F}(\lambda\delta.\exists t[t \in \text{NARROW SET} \wedge \delta >_F F_{\Delta}(\text{John},t)]) \quad = \quad F_{\Delta}(j,\mathbf{min}_{narrow,JOHN,F})$$

Thus, we are checking whether (3a) strengthens (3b):

(3) a. **wide degree** $F_{\Delta}(j,\mathbf{max}_{,JOHN,F})$
b. **narrow degree** $F_{\Delta}(j,\mathbf{min}_{narrow,JOHN,F})$

Clearly, $F_{\Delta}(j,\mathbf{max}_{JOHN,F}) \geq_F F_{\Delta}(j,\mathbf{min}_{narrow,JOHN,F})$, hence **(3a) strenghtens (3b)**:

CONSEQUENCE:
The polarity item *ever* is licensed in the CP comparative (1):

(1) Mary is more famous than John *ever* was.

We look at (4):

(4) Mary is less famous than John *ever* was.

-Widening involves $\mathbf{min}_{\mathbf{JOHN,F}}$, the time where John's fame was minimal
The pragmatic assumption is that John's fame at $\mathbf{min}_{\mathbf{JOHN,F}}$ is smaller than all the times in the narrow set.

The supremums we get here are as in (5):

(5) a. $\sqcup_{<F}(\lambda\delta.\delta <_F F_\Delta(\text{John},\mathbf{min}_{\mathbf{JOHN,F}})) = \mathbf{min}_{\mathbf{JOHN,F}}$
b. $\sqcup_{<F}(\lambda\delta.\exists t[t \in \text{NARROW SET} \wedge \delta <_F F_\Delta(\text{John},t)]) = \mathbf{max}_{\mathbf{narrow,JOHN,F}}$

The relevant dimension is the converse dimension, i.e., we are in $S_F^{\,c}$, so we use $S_F^{\,c}$-**strengthening** as defined above, and we see that indeed:

$\mathbf{min}_{\mathbf{JOHN,F}} \leq_F \mathbf{max}_{\mathbf{narrow,JOHN,F}}$ **(5a) strengthens (5b)**

CONSEQUENCE:
The polarity item *ever* is licensed in the CP comparative (4):

(4) Mary is more famous than John *ever* was.

CONCLUSIONS FOR THE SEMANTICS OF CP COMPARATIVES:

The dimensional supremum theory is a presuppositional variant of the naïve (but clever) there.
-For CP comparatives that have a DP correlate, when defined, the CP comparative has the same meaning as its DP correlate.
-But negation and downward entailing noun phrases inside the CP complement make the CP comparative undefined (hence infelicitous), unlike their DP correlate (if they have one).
-Polarity licensing is defined directly on the level of the comparative scale and is sensitive to the direction (+/−) of the dimensional order involved.
-On the dimensional supremum theory, the type of the scale is invoked directly at the presuppositional level invoked by the operation M_α, and polarity items inside the CP complement are licensed at that level.

APPENDIX 1: INTRANSITIVE MEASURE PHRASES

$[_{\text{measure}}$ *tall* $] \rightarrow H$
The **measure meaning** of *tall* combines with *more than three inches* to give a degree predicate (after composition with the measure function):

$[_{\text{PRED}}$ *more than three inches tall*$]$ $\quad\rightarrow\quad \lambda x.\ H_{",w}(x)^r > 3$

(1) a. Wiplala is more than three inches tall.
b. $H_{",w}(\text{Wiplala})^r > 3$
c. **#**Wiplala is more than three inches short.

No interpretation for (1c), because there is no lexical item $[_{\text{measure}}$ *short* $]$ with meaning H.

Trivial account. But better than von Stechow/Kennedy.
von Stechow/Kennedy: (1c) is infelicitous because the human brain cannot convert supremums correctly, hence the human brain uses faulty mathematics.

APPENDIX 2: ADJECTIVES

Dimensions with default unit:
$[_{\mathbf{DIM}}$ *tall*$]$ $\quad\rightarrow\quad S_{H,\Delta,k}$
$[_{\mathbf{DIM}}$ *short*$]$ $\quad\rightarrow\quad S_{H,\Delta,k}{}^{\mathbf{c}}$

Adjective formation:
$\text{dim} \rightarrow \lambda\delta.\ \delta\ \text{dim}^{>}\ \text{dim}^{\mathbf{HIGH}}$

$[_{\mathbf{ADJ}}$ *tall*$]$ $\quad\rightarrow \lambda\delta.\ \delta^r > (\text{HIGH}_{H,\Delta,k})^r$ $\qquad$ ($tall^{>} = >_H \quad tall^{\mathbf{HIGH}} = \text{HIGH}_{H,\Delta,k}$)
$[_{\mathbf{ADJ}}$ *short*$]$ $\quad\rightarrow \lambda\delta.\ \delta^r < (\text{LOW}_{H,\Delta,k})^r$ $\qquad$ ($short^{>} = <_H \quad short^{\mathbf{HIGH}} = \text{LOW}_{H,\Delta,k}$)

(1) Borremans is tall iff $H_\Delta(\text{Borremans})^r > (\text{HIGH}_{H,\Delta,k})^r$

Borremans is tall iff his height in Δ exceeds the contextual height value, which is likely (dependent on k, of course), since Borremans is a giant.

(2) Wiplala is short iff $H_\Delta(\text{Wiplala})^r < (\text{LOW}_{H,\Delta,k})^r$

Wiplala is short iff his height in Δ is below the contextual height value, which is likely (dependent on k), since Wiplala is a wiplala, and wiplalas fit in your coat pocket.

APPENDIX 3. MEASURES INSIDE CP COMPARATIVES

(1) The tower is *taller* than it is – *wide*.

The naïve assumption for CP comparatives with internal measures:
Wide inside the CP comparative is a measure: $[_{\text{measure}}$ *wide*$] \rightarrow W_\Delta$

Prediction: We find for measure phrases inside the complement of CP comparatives the same contrast as we found for *five feet tall/#short*.

(2) a. The gate is *higher* than it is *wide*
b. #The gate is *higher* than it is *narrow*
c. The gate is *lower* than it is *wide*.
d. #The gate is *lower* than it is *narrow*.

a. The gate is *wider* than it is *high*.
b. #The gate is *wider* than it is *low*.
c. The gate is *narrower* than it is *high*.
d. #The gate is *narrower* than it is *low*.

CP interpretation with internal measures:

External comparative α makes measure function $\mathbf{M_\alpha}$ available.

be α than DP is – $[_{\text{measure}}$ ***wide*]**
$\lambda\delta.\ DP(\lambda x.\ \alpha(\delta,\ \mathbf{[W_\Delta \rightarrow M_\alpha]}(x)\ )$ (In our examples: $\mathbf{[W_\Delta \rightarrow H_\Delta]}$)

$\mathbf{[W_\Delta \rightarrow H_\Delta]}$ is the result of **converting** $\mathbf{W_\Delta}$ to a measure function that maps individuals onto heights degrees.

In the case of width and height, the conversion is trivial:

$\mathbf{[W_\Delta \rightarrow H_\Delta] =} \lambda x.\ W_\Delta(x)\ [H/W]$
The function that differs **only** from W_Δ in that at each third element in the triple it has H instead of W.

(3) a. The tower is taller than it is – wide
b. $H_\Delta(\text{Tower}) >_H \mathbf{[W_\Delta \rightarrow H_\Delta]}(\text{Tower})$

If the tower is 20 meters tall and 4 meters wide, then

$H_m(\text{Tower}) = \langle 20,m,H\rangle$ $W_m(\text{Tower}) = \langle 4,m,W\rangle$
$[W_m \rightarrow H_m](\text{Tower}) = \langle 4,m,H\rangle$
$\langle 20,m,H\rangle >_H \langle 4,m,H\rangle$.

APPENDIX 4: INVERSE MEASURES

Converse antonym pairs:
tall: **basic scale** of measure H *short*: **converse scale** of measure H.

Inverse antonym pairs: *fat-thin*, *concave-convex*, *flat-sharp* (for musical keys).

Inverse antonyms: both elements of the pair have an interpretation as a measure:

$[_{\text{measure}}\ \textit{flat}] \rightarrow$ F $[_{\text{measure}}\ \textit{sharp}] \rightarrow$ S

Both (6a) and (6b) are felicitous:

(6) a. A major is *three notches sharp*. (###)
 b. F minor is *four notches flat*. (*bbbb*)

Both *sharp* and *flat* can occur inside the complement of CP comparatives, as in (7):

(7) a. F minor is *flatter* than A major is *sharp* *bbbb* **vs**. ###
 b. C-sharp minor is *sharper* than E-flat major is *flat* #### **vs.** *bbb*

Hence: **degrees of sharpness** <r,n,S> and **degrees of flatness** <r,n,F>

INVERSE MEASURES

Measures **A** and **B** are **inverse measures** iff
For every unit u appropriate for **A** and **B,** for every x, w:

$\mathbf{A_u}(x,w) = -\mathbf{B_u}(x,w)$

Flatness is negative sharpness, **sharpness is negative flatness**.

Example: $\mathbf{F_n}$(***F minor***) = <4,n,F>, hence $\mathbf{S}_n$(***F minor***) = <−4,n,S>

(11) a. F-minor is *seven notches flatter* than A-major is

F_n(***F-minor***) = <4,n,F> (*bbbb*) S_n(***A-major)*** = <3,n,S> (###).
 F_n(***A-major***) = <−3,n,F> (−*bbb*)

F_n(***F-minor***) $-_F$ F_n(***A-major***) = <4,n,F> $-_F$ <−3,n,F> = <7,n,F>
 bbbb −*bbb* *bbbbbbb*

. b. A-major is *seven notches sharper* than F-minor is

S_n(***A-major***) $-_S$ F_n(***F-minor***)= <3,n,S> $-_S$ <−4,n,S> = <7,n,S>
 ### −*####* *#######*

Measure conversion is **the same trivial substitution operation** as before:

$[S_n \rightarrow F_n] = \lambda x.(S_n(x)\ \ [F/S])$
$[F_n \rightarrow S_n] = \lambda x.(F_n(x)\ \ [S/F])$

(12) a. F minor is *flatter* than A major is *sharp* – *bbbb* **vs.** ###
 b. F_n(***F minor***) $>_F$ $[S_n \rightarrow F_n]$(***A major***)
 <4,n,F> $>_F$ <3,n,F/S>
 bbbb *bbb*

Comparison of inverse antonyms does not take place on a superscale but is a comparison between scales.

APPENDIX 5: THE INTERVAL THEORY OF SCHWARZSCHILD AND WILKINSON AND THE NAÏVE (BUT CLEVER) THEORY

DP_1 is α than DP_2 is –

1. We assume:
-Charity in correcting minor mistakes
-Charity in interpretation of undefined notions
-Charity in obvious extensions to other cases
2. We ignore vagueness (i.e. H_Δ(John) is a point degree, not an interval of degrees)
3. We ignore quantificational external subjects (i.e. DP_1 is, say, a proper name)

THEOREM: UNDER THE CONDITIONS 1-3, THE INTERVAL SEMANTICS OF SCHWARZSCHILD AND WILKINSON FOR DP_1 is α than DP_2 is – IS EQUIVALENT TO THE NAIVE (BUT CLEVER) THEORY.

What follows is an outline of how the proof goes.
Schwarzschild and Wilkinson:

(1) **DP_1 is β-taller than DP_2 is –.**
where β is a numerical predicate of the form:
at least two inches,
at most two inches,
exactly two inches,
Ø…..
$\exists j[\ DP_i \text{ is } j\text{-}\textit{tall} \wedge DP_2 \text{ is } \max(\lambda i.\ \boldsymbol{\beta}(j-i))\text{-}\textit{tall}\ ]$

There is a degree interval j such that DP_1 is j-*tall* and DP_2 is k-tall, where k is the maximal interval in the set of intervals i such that j–i is **β** (*at least two inches*, *at most two inches*…)

With all the assumptions under (1-3), the interval meaning for (1) can be massaged into the following meaning:

(2) **DP_1 is β-taller than DP_2 is –.**
$DP_1(\lambda x.\ \ [\lambda\delta.\ DP_2 \text{ is } \max(\lambda i.\ \boldsymbol{\beta}([\delta,\delta]-i))\text{-}\textit{tall}]\ (H_\Delta(x))$

With the principle BPR, and some more charity, we can derive the comparative meaning:

(3) **β-taller than DP is –.**
$\lambda\delta_n.(DP(\lambda x.\ \mathbf{R}(\delta_n, H_\Delta(x))$

where $\mathbf{R} = \lambda\delta_2\lambda\delta_1.\ \delta_2 \in \max(\lambda i.\ \boldsymbol{\beta}([\delta_1, \delta_1]-i))$

$\max(\lambda i.\ \boldsymbol{\beta}([\delta_1,\delta_1]-i))$ is defined in Schwarzschild and Wilkinson's paper.
Relevant for the theorem is the following central lemma:

Central Lemma: $\delta_2 \in \max(\lambda i.\boldsymbol{\beta}([\delta_1,\delta_1]-i))$ **iff** $\boldsymbol{\beta}([\delta_1,\delta_1]-[\delta_2,\delta_2])$
Proof: omitted

With the lemma, (3) is equivalent to (4):

(4) **β-taller than DP is –.**
$\lambda\delta_n.(DP(\lambda x.\ \mathbf{R}(\delta_n, H_\Delta(x))$

where $\mathbf{R} = \lambda\delta_2\lambda\delta_1.\ \boldsymbol{\beta}([\delta_1,\delta_1]-[\delta_2,\delta_2])$

Adequacy constraint: for point intervals, Schwarzschild and Wilkinson's notion ($\boldsymbol{\beta}_{sw}$) should be equivalent to the corresponding almost (but not quite) naïve notion (**β**) $\boldsymbol{\beta}_{\mathbf{sw}}([\delta_1,\delta_1]-[\delta_2,\delta_2])$ iff $\boldsymbol{\beta}(\delta_1 -_{\mathbf{H}} \delta_2)$

Hence (4) is equivalent to (5):

(5) **β-taller than DP is –.**
$\lambda\delta_n.(DP(\lambda x.\ \mathbf{R}(\delta_n, H_\Delta(x))$

where $\mathbf{R} = \lambda\delta_2\lambda\delta_1.\ \boldsymbol{\beta}(\delta_1 -_H \delta_2)$

This means that $\mathbf{R} = \boldsymbol{\beta} \circ -_H$, which is $\boldsymbol{\alpha}$ of the almost (but not quite) naïve theory**.**

Hence (5) is equivalent to (6):

(6) **The Naïve (but Clever) Theory**
β-taller than DP is –.
$\lambda\delta_n.(DP(\lambda x.\ \boldsymbol{\alpha}(\delta_n, H_\Delta(x))$