2.1. TENSE LOGIC AND BRANCHING TIME

2.1.1. Basic Tense Logic

In Priorian Tense Logic we have a language L_T built from atomic formulas with $\neg$, $\wedge$, $\vee$, $\rightarrow$ and the tense operators

P	it is true at some point in the past that
H	it is true at every point in the past that
F	it is true at some point in the future that
G	it is true at every point in the future that

A **frame for** L_T is a pair $\mathbf{T} = \langle T, < \rangle$ where $<$ is a binary relation on non-empty set T.
A **model for** L_T is a pair $\mathbf{M} = \langle \mathbf{T}, F\rangle$ where $\mathbf{T}$ is a frame for L_T and
F: ATFORM $\times$ T $\rightarrow$ {0,1} is an interpretation function.

We define $[\![\varphi]\!]_{M,t}$ the truth value of φ in M at point of time t.

1. If $\varphi \in$ ATFORM then $[\![\varphi]\!]_{M,t} = F_t(\varphi)$
2. $[\![\neg\varphi]\!]_{M,t} = 1$ iff $[\![\varphi]\!]_{M,t} = 0$, etc.
3. $[\![P\varphi]\!]_{M,t} = 1$ iff for some $t' < t$: $[\![\varphi]\!]_{M,t'} = 1$
4. $[\![H\varphi]\!]_{M,t} = 1$ iff for every $t' < t$: $[\![\varphi]\!]_{M,t'} = 1$
5. $[\![F\varphi]\!]_{M,t} = 1$ iff for some $t' > t$: $[\![\varphi]\!]_{M,t'} = 1$
6. $[\![G\varphi]\!]_{M,t} = 1$ iff for every $t' > t$: $[\![\varphi]\!]_{M,t'} = 1$

We define:

$[\![\varphi]\!]_{M} = 1$ iff for every $t \in T$: $[\![\varphi]\!]_{M,t} = 1$	truth in model **M**
$[\![\varphi]\!]_{T} = 1$ iff for every F: $[\![\varphi]\!]_{\langle \mathbf{T},F\rangle} = 1$	truth on frame **T**

Let ***F*** be a class of frames

$[\![\varphi]\!]_{\boldsymbol{F}} = 1$ iff for every $\mathbf{T} \in \boldsymbol{F}$: $[\![\varphi]\!]_{\mathbf{T},t} = 1$	truth in a class of frames

Logical validity is truth in the class of all frames.

Propositional logic

Definition schemas:	$(\varphi \vee \psi) =_{df} (\neg\varphi \rightarrow \psi)$
	$(\varphi \wedge \psi) =_{df} \neg(\neg\varphi \vee \neg\psi)$
Modus Ponens:	From φ and $(\varphi \rightarrow \psi)$ derive ψ
Axiom Schemas:	$(\varphi \rightarrow (\psi \rightarrow \varphi))$
	$((\varphi \rightarrow (\psi \rightarrow \chi)) \rightarrow ((\varphi \rightarrow \psi) \rightarrow (\varphi \rightarrow \chi)))$
	$((\neg\psi \rightarrow \neg\varphi) \rightarrow (\varphi \rightarrow \psi))$

Minimal Tense Logic

Propositional logic +

Definition schemas:	$H\varphi =_{df} \neg P\neg\varphi$
	$G\varphi =_{df} \neg F\neg\varphi$
Generalization:	If you can derive φ you can derive $G\varphi$ and you can derive $H\varphi$
Axiom schemas:	$G(\varphi \rightarrow \psi) \rightarrow (G\varphi \rightarrow G\psi)$
	$H(\varphi \rightarrow \psi) \rightarrow (H\varphi \rightarrow H\psi)$
	$\varphi \rightarrow HF\varphi$
	$\varphi \rightarrow GP\varphi$

Fact 1: Minimal tense logic is complete: an inference is tense logically valid iff it is derivable in minimal tense logic.
Fact 2: Irreflexivity and asymmetry of < are not tense logically definable.
Fact 3: A tense logical inference is valid on the class of all frames iff it is valid on the class of all asymmetric frames.

With this we restrict ourselves to asymmtric frames.

Tense logical formula φ **expresses** that the order has property P iff
φ is true on all frames where $<_T$ has property P, and
false on all frames where $<_T$ doesn't have property P.

Fact 4: $G\varphi \rightarrow GG\varphi$ (or equivalently $PP\varphi \rightarrow P\varphi$) expresses that the order is transitive
Fact 5: $F\varphi \rightarrow G(F\varphi \vee \varphi \vee P\varphi)$ expresses that the order is right linear
$P\varphi \rightarrow H(F\varphi \vee \varphi \wedge P\varphi)$ expresses that the order is left linear

Linear Tense Logic:
Minimal tense logic +
$G\varphi \rightarrow GG\varphi$
$F\varphi \rightarrow G(F\varphi \vee \varphi \vee P\varphi)$
$P\varphi \rightarrow H(F\varphi \vee \varphi \wedge P\varphi)$

Fact 6: Linear orders and non-branching orders are tense-logically indistinguishable.

Fact 7 $G\varphi \rightarrow F\varphi$ expresses that the order is right continuing
$H\varphi \rightarrow P\varphi$ expresses that the order is left continuing
$F\varphi \rightarrow FF\varphi$ expresses that the order is dense.
Fact 8 Within the class of linear orders:
$(P(\varphi \vee \neg\varphi) \wedge HF\varphi) \rightarrow (\varphi \vee F\varphi)$ expresses left discreteness
$(F(\varphi \vee \neg\varphi) \wedge GP\varphi) \rightarrow (\varphi \vee P\varphi)$ expresses right discreteness
$H(H\varphi \rightarrow \varphi) \rightarrow H\varphi$ expresses wellfoundedness
Fact 9 $(P\varphi \wedge PH\neg\varphi) \rightarrow P(GP\varphi \wedge H\neg\varphi)$ expresses left continuity
$(F\varphi \wedge FG\neg\varphi) \rightarrow F(HF\varphi \wedge G\neg\varphi)$ expresses right continuity

Example: Fact 8:
$P(\varphi \vee \neg\varphi)$ expresses at t that t has a predecessor.
$HF\varphi$ says at t that for every predecesser of t, $F\varphi$ is true. If, t has predecessors, but not a direct predecessor, then you can satisfy this, while making φ true at t and all t's sucessors.
If t does have a direct predecessor $t-1$, then $F\varphi$ is onlt true at $t-1$ if $\varphi \vee F\varphi$ is true there.

Example: Fact 9:
The principles express the existence of bounds.
Thus if in the past φ starts out false, but is true somewhere, then there is a past point t where φ is first true: φ is true at t, but false at all predecessors of t: $H\neg\varphi$ says that φ is false at all redecessors of t, and $GP\varphi$ says that $P\varphi$ is true in t all successors of t.

2.1.2. Modal Logic

In Modal logic we have a language L_M built from atomic formulas with $\neg$, $\wedge$, $\vee$, $\rightarrow$ and the modal operators

$\Box$ it is necessarily true that

$\Diamond$ it is possibly true that

A **frame for** $L_<$ is a pair $\mathbf{W} = \langle W, R \rangle$ where R is a binary relation on non-empty set W.
A **model for** T_L is a pair $\mathbf{M} = \langle \mathbf{W}, F \rangle$ where $\mathbf{W}$ is a frame for T_M and
F: ATFORM $\times$ W $\rightarrow$ {0,1} is an interpretation function.

We define $[\![\varphi]\!]_{M,w}$ the truth value of φ in M in possible world w.

1. If $\varphi \in$ ATFORM then $[\![\varphi]\!]_{M,w} = F_w(\varphi)$
2. $[\![\neg\varphi]\!]_{M,w} = 1$ iff $[\![\varphi]\!]_{M,w} = 0$, etc.
3. $[\![\Diamond\varphi]\!]_{M,w} = 1$ iff for some v: R(w,v) and $[\![\varphi]\!]_{M,v} = 1$
4. $[\![\Box\varphi]\!]_{M,w} = 1$ iff for every v: if R(w,v) then $[\![\varphi]\!]_{M,v} = 1$

We define:

$[\![\varphi]\!]_M = 1$ iff for every $w \in W$: $[\![\varphi]\!]_{M,w} = 1$ truth in model **M**
$[\![\varphi]\!]_{\mathbf{W}} = 1$ iff for every F: $[\![\varphi]\!]_{\langle \mathbf{W},F\rangle} = 1$ truth on frame **W**

Let ***F*** be a class of frames

$[\![\varphi]\!]_{\boldsymbol{F}} = 1$ iff for every $\mathbf{W} \in \boldsymbol{F}$: $[\![\varphi]\!]_{\mathbf{W},t} = 1$ truth in a class of frames

Logical validity is truth in the class of all frames.

Mimimal modal logic: K

Propositional logic +
Definition schemas: $\Box\varphi =_{df} \neg\Diamond\neg\varphi$
Generalization: If you can derive φ you can derive $\Box\varphi$
Axiom schemas: $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$

Fact 1: $\Box\varphi \rightarrow \varphi$ expresses that R is reflexive
T = K + $\Box\varphi \rightarrow \varphi$

Fact 2: $\Box\varphi \rightarrow \Box\Box\varphi$ expresses that R is transitive
$\mathbf{S_4}$ **= T +** $\Box\varphi \rightarrow \Box\Box\varphi$

Fact 3: $\Diamond\Box\varphi \rightarrow \varphi$ expresses that R is symmetric
$\mathbf{S_5} = \mathbf{S_4}$ + $\Diamond\Box\varphi \rightarrow \varphi$

Hence: S_5 characterizes the class of all frames where R is an equivalence relation.

2.1.3. Time-dependent modality.

In temporal-modal logic we have language $L_{M\times T}$ with operators P, H, F, G, $\Box$, $\Diamond$.
We will talk about branching time in the next subsection. In the present setting, time is **linear** time,

A **frame** for L_{M+T} is a structure **T×W** = <T,< , W, R> where
<T,< > is a **linear** frame for L_T, W is a non-empty set of worlds, and
for every $t \in T$: $<W,R_t>$ is a frame for L_M

A **model** for L_{M+T} is a pair < **T×W**,F> with **T×W** a frame for L_{M+T} and
F: ATFORM $\times$ W $\times$ T $\rightarrow$ {0,1} an interpretation function.

We define $[\![\varphi]\!]_{M,w,t}$ the truth value of φ in M in possible world w at time t.

1. If $\varphi \in$ ATFORM then $[\![\varphi]\!]_{M,w,t} = F_{w,t}(\varphi)$
2. $[\![\neg\varphi]\!]_{M,w,t} = 1$ iff $[\![\varphi]\!]_{M,w,t} = 0$, etc.
3. $[\![P\varphi]\!]_{M,w,t} = 1$ iff for some $t' < t$: $[\![\varphi]\!]_{M,w,t'} = 1$
4. $[\![H\varphi]\!]_{M,w,t} = 1$ iff for every $t' < t$: $[\![\varphi]\!]_{M,w,t'} = 1$
5. $[\![F\varphi]\!]_{M,w,t} = 1$ iff for some $t' > t$: $[\![\varphi]\!]_{M,w,t'} = 1$
6. $[\![G\varphi]\!]_{M,w,t} = 1$ iff for every $t' > t$: $[\![\varphi]\!]_{M,w,t'} = 1$
7. $[\![\Box\varphi]\!]_{M,w,t} = 1$ iff for every $v \in W$: if $R_t(w,v)$ then $[\![\varphi]\!]_{M,v,t} = 1$
8. $[\![\Diamond\varphi]\!]_{M,w,t} = 1$ iff for some $v \in W$: $R_t(w,v)$ and $[\![\varphi]\!]_{M,v,t} = 1$

We define:

$[\![\varphi]\!]_M = 1$ iff for every $w \in W$ for every $t \in T$: $[\![\varphi]\!]_{M,w,t} = 1$	truth in model **M**
$[\![\varphi]\!]_{\mathbf{T}\times\mathbf{W}} = 1$ iff for every F: $[\![\varphi]\!]_{<\mathbf{T}\times\mathbf{W},F>} = 1$	truth on frame **T**×**W**

Let ***F*** be a class of frames

$[\![\varphi]\!]_{\boldsymbol{F}} = 1$ iff for every $\mathbf{W} \in \boldsymbol{F}$: $[\![\varphi]\!]_{\mathbf{W},t} = 1$	truth in a class of frames

Logical validity is truth in the class of all frames.

The modal operators $\Box$ and $\Diamond$ can model a variety of time dependent modal notions, as in:

(1) a. Last year, it was not possible to publish this book, but this year it is possible.

Tenses, temporal framing adverbs and Partee's Problem

Partee's Problem

Let O be the proposition that I turned off the stove.
I turned off the stove $\rightarrow$ P(O)
P(O) is true at t iff $\exists$t'[t' < t $\wedge$ O is true at t']

Problem: *I didn't turn off the stove*
Possibility 1: P($\neg$O) $\exists$t'[t' < t $\wedge$ O is false at t']
Problem: this is trivially true.

Possibility 2: $\neg$P(O) $\neg\exists$t'[t' <t $\wedge$ O is true at t']
Problem: this is too strong: it means: *I never ever turned off the stove*.

Events versus states:

States: *live in Amsterdam*
(1) I lived in Amsterdam in 1992 Compatible with: I still do.
✓I lived in Amsterdam in 1992 and, in fact, I never left, I still live there.
Events: *write a book*
(2) I wrote a book in 1992 Not compatible with: I am still writing it.
#I wrote a book in 1992, and, in fact, I never finished it, I am still writing it.
Progressive is stative:
(2) I was writing a book in 1992
✓I was writing a book in 1992, and, in fact, I never finished it, I am still writing it.

The past (-ed) and perfect (have –ed) interact differently with states and event (Hinrichs 1984)
If P is a stative property PAST(P) says:
there is a P-state s and a past interval i such that the running time of s **overlaps** i.
If s is a state of me living in Amsterdam, then (1) expresses that the running time of this state **overlaps** a past interval. This is compatible with that state still going on.

If P is an eventive property PAST(P) says:
there is a P-event e and a past interval i such that the running time of e **is included in** i.
If e is a book writing event with me as agent, then (1) expresses that the running time of this event is included in a past interval. But e is not an event of me being engaged in book writing (a progressive event), e is an event in which I write a book, so at the end of the event, there is a book. This means that by the end of the past interval the bookwriting event e is over, so it no longer continues beyond that interval.

We now give a semantics for tense and temporal location adverbs.
(the form is derived from Condaravdi 2002, but simplified a bit.)

W is the set of worlds.
I is the set of time intervals.
EV = E $\cup$ S, where E $\cap$ S = Ø. EV is the set of eventualities (term coined by Emmon Bach)
E is the set of events.
S is the set of states.

τ: EV $\times$ W $\rightarrow$ I is a partial function, the *temporal trace function* (Link 1986)
$\tau_w(e)$ is the running time of e in w, the interval at which e goes on in w.

(Extensional) properties: sets of entities
P $\subseteq$ EV: eventuality property
P $\subseteq$ E: eventive property
P $\subseteq$ S: stative property
P $\subseteq$ I: temporal property

Eventuality semantics:
- VPs are interpreted as eventuality properties, eventive or stative.
-Temporal operators (eg. termporal framing adverbs) map these onto temporal properties.
-Tenses map eventuality or temporal properties onto truth values.

[More precisely: intensional version of this:
Intensional properties are functions from worlds into sets of entities.
-VPs: intensional eventuality properties
-Temporal operators: are functions from intensional intensionality properties to intensional temporal properties.
-Tenses map intensional properties onto propositions, functions from worlds to truth values, ie. sets of worlds.
-proposition p is true in w iff w $\in$ p]

Neo Davidsion semantics: (Davidson 1967, Parson 1990, Landman 2000)
Eventuality semantics + thematic roles:
Thematic roles, like Ag (agent), Th (theme) are partial functions from eventualities to entities (like individuals): Ag: E $\rightarrow$ D. Thematic roles specify the participants of events.
If e is a singing event then Ag(e) is the person who sings, and Th(e) is what is sung.

Semantic representations of VPs (including, for simplicity, the subject):
Fred write a book $\rightarrow \lambda e.\mathit{write}_w(e) \wedge Ag(e)=Fred \wedge \mathit{book}_w(Th(e))$
The set of events that are (in w) writing events, whose agent is Fred and whose theme is (what is in w) a book.

Fred live in Amsterdam $\rightarrow \lambda s.\mathit{live}_w(e) \wedge Th(e)=Fred \wedge In(s) \sqsubseteq_{space}$ Amsterdam
The set of strates that are (in w) states of living, of which the theme is Fred and of which the location is spatially part of Amsterdam.

Temporal semantics following Condarovdi.
Condarovdi defines relation $AT_w[P,i]$ property P goes on in w at i:
In AT we include the aspectual differences between states and events:

$$AT_w[P,i] = \begin{cases} \exists e[e \in P \wedge \tau_w(e) \subseteq i] & \text{if P is eventive} \\ \exists s[s \in P \wedge \tau_w(s) \text{ O } i] & \text{if P is stative} \\ i \in P & \text{if P is temporal} \end{cases}$$

PRES = $\lambda P.AT_w[P,\mathbf{now}]$
PAST = $\lambda P.\exists j[j < \mathbf{now} \wedge AT_w[P,j]]$

PRES and PAST take properties and map them onto truth values.

We impose a *chronology of days* on time, and we assume that *yesterday* denotes the interval **yesterday** which is $\mathbf{day_{now}} - 1$, which means that **yesterday** < **now**.

We interpret the temporal adverb *yesterday* as YESTERDAY:

$$\text{YESTERDAY} = \begin{cases} \lambda P \lambda i.AT_w[P,i \cap \mathbf{yesterday}] & \text{if P is a non-temporal property} \\ \bot & \text{otherwise} \end{cases}$$

YESTERDAY maps a set of eventualities (events or states) P onto the set of intervals i such that P goes on in w at i $\cap$ **yesterday**.

YESTERDAY maps non-temporal properties onto temporal properties.
YESTERDAY does not accept propositions as input.

We see how it works:

(1) *Yesterday I was in Amsterdam*

A is a stative property, the set of states of me being in Amsterdam.

We argue: The semantics is built from three ingredients: ,

Yesterday *I be*+past *in Amsterdam*
YESTERDAY PAST A

There are potentially two orders of application:
(1) YESTERDAY(PAST(A))
(2) PAST(YESTERDAY(A))

But order (1) is impossible:
PAST(A) is a truth value, not a property
YESTERDAY must apply to a property,

hence (1) YESTERDAY(PAST(A)) is not defined.

So only derivation (2) is possible:

Yesterday I was in Amsterdam $\rightarrow$ PAST[YESTERDAY[A]]

And this *is* possible because:
-A is a stative property
-YESTERDAY[A] is a temporal property
-PAST[YESTERDAY[A]] is a truth value

We work out the semantics:

YESTERDAY(A) =
$\lambda i.\ AT_w[A, i \cap \mathbf{yesterday}] =$
$\lambda i.\exists s[s \in A \wedge \tau_w(s)\ O\ (i \cap \mathbf{yesterday})]$ =

We apply PAST to this and get:

$\exists j[j < \mathbf{now} \wedge AT_w[\lambda i.\exists s[s \in A \wedge \tau_w(s)\ O\ (i \cap \mathbf{yesterday})],j]\ ]$ =

$\exists j[j < \mathbf{now} \wedge j \in \lambda i.\exists s[s \in A \wedge \tau_w(s)\ O\ (i \cap \mathbf{yesterday})]]$ =

$\exists j[j < \mathbf{now} \wedge \exists s[s \in A \wedge \tau_w(s)\ O\ (j \cap \mathbf{yesterday})]]$ =

Given that **yesterday** < **now** we simplify this to:

$\exists s[s \in A_w \wedge \tau_w(s)\ O\ \mathbf{yesterday}]$

Yesterday I was in Amsterdam $\rightarrow \exists s[s \in A \wedge \tau_w(s)\ O\ \mathbf{yesterday}]$
There is a state of me being in Amsterdam located in w at an interval that overlaps yesterday.

This is compatible with that state continuing to hold.

(2) *Yesterday I wasn't in Amsterdam*

We assume that negation is just truth conditional negation:
not $\rightarrow \neg$

Then negation must apply at the level where we have derived a truth value, and hence we have for (2) only the derivation:

Yesterday I wasn't in Amsterdam $\rightarrow$ **not**(PAST[YESTERDAY[A]])

Yesterday I wasn't in Amsterdam $\rightarrow \neg\exists s[s \in A \wedge \tau_w(s)$ O **yesterday**]

There is no state of me being in Amsterdam located in w whose running time in w overlaps with **yesterday**.

This is the correct result.

(3) a. *I turned off the stove yesterday*.
b. *I didn't turn off the stove yesterday*.

I turned off the stove yesterday $\rightarrow$ PAST(YESTERDAY(O))

YESTERDAY(O) = $\lambda i.\exists e[e \in O \wedge \tau_w(e) \subseteq (i \cap \mathbf{yesterday})]$ =

PAST(YESTERDAY(O)) = $\exists j[j < \mathbf{now} \wedge \exists e[e \in O \wedge \tau_w(e) \subseteq (j \cap \mathbf{yesterday})]]$

I turned off the stove yesterday $\rightarrow \exists e[e \in O_w \wedge \tau_w(e) \subseteq \mathbf{yesterday})]$

There is an event of me turning off the stove whose running time is a subinterval of **yesterday**.

Since the turning off the stove event is temporally included in a past interval, it can't continue to the present.

I didn't turned off the stove yesterday $\rightarrow \neg\exists e[e \in O_w \wedge \tau_w(e) \subseteq \mathbf{yesterday})]$

There isn't an event of me turning off the stove whose running time is a subinterval of **yesterday**.

In both cases, we get the right results.

Partee's puzzle

I turned off the stove: PAST(O)

PAST(O) = $\exists j[j < \mathbf{now} \wedge AT_w[O,j]]$ =
$\exists j[j < \mathbf{now} \wedge \exists e[e \in O \wedge \tau_w(e) \subseteq j]]$

I turned off the stove $\rightarrow \exists j[j < \mathbf{now} \wedge \exists e[e \in O \wedge \tau_w(e) \subseteq j]]$

There is an interval j before **now** that temporally includes an event of me turning off the stove.

This is just the same as P(O), with P the priorian operator. So, not surprisingly we get:

I didn't turn off the stove $\rightarrow \neg\exists j[j < \mathbf{now} \wedge \exists e[e \in O \wedge \tau_w(e) \subseteq j]]$

0There is no interval j before **now** that temporally includes an event of me turning off the stove.

And *I didn't turn off the stove* gets a meaning that is too strong: *I never turned off the stove*.

Diagnosis and solution strategy:
We want to assign the correct meaning to *I didn't turn off the stove*
and we want to block assigning the incorrect meaning to *I didn't turn off the stove*.

[We assume that *I didn't turn off the stove* doesn't semantically mean *I never turned off the stove*.
Of course, pragmatically there may well be situations where it is understood to mean the 0latter.
Thus if the accused says to the judge under oath: *I didn't break into this appartment*, he may hope that the judge takes this as a statement meaning *I never broke into this appartment*, rather than what he literally says: *I didn't break into this appartment at the time that you are asking me about* [*since I did a week earlier…*].

We take as our lead three *observations*:
1. There are no problems when the temporal framing adverbial *yesterday* is there.
2. In the derivation with *yesterday* PAST applies to a temporal property rather than an eventuality property.
3. In the derivation with the never reading, PAST applies directly to an eventuality property.

Idea: -Block the derivation under observation 3.
-Replace this by a derivation modelled on the derivation in observation 1.

Step 1: Tenses operate on temporal properties only

PRES = $\lambda P.\ \mathbf{now} \in P$	P a variable over temporal properties
PAST = $\lambda P.\exists j[j < \mathbf{now} \wedge j \in P]$	P a variable over temporal properties

This makes no difference for PAST[YESTERDAY[O]],
but it does make a difference for PAST[O], which is no longer well defined, because O is not a temporal property.

This means that we have a ***type mismatch*** in the derivation of *I turned off the stove*:
Without further assumptions, the grammar cannot assign a felicitous meaning to *I turned off the stove*: PAST + O cannot be resolved.

Step 2: we resolve the mismatch by assuming and *implicit* operation C which maps eventuality properties onto temporal properties and interpreting as follows:

I turned off the stove $\rightarrow$ PAST(C(O))

For the semantics of of C, we take YESTERDAY as our model.

We assume a ***contextually given*** interval **c.** Intuitively, **c** is the reference interval of the conversation: it is the time interval that is big enough to include the events we are talking about.
The dynamics of conversation may shift what interval is relevant.
Pragmatically, we assume that **c** is flexible enough that it gets updated if there is a semantic conflict. That is, a present tense statement may require that **c** includes **now**.

If it didn't so far, then we tacitly accommodate it, so as to make the current assertion pragmatically felicitous.

In the new semantics we don't really need the AT relation, so we incorporate this directly in the semantics of framing adverb interpretation YESTERDAY and implicit operation C:

Let P be a variable over eventuality properties.

$$\text{YESTERDAY} = \lambda P.\begin{cases} \lambda i.\exists e[e \in P \wedge \tau_w(e) \subseteq (i \cap \textbf{yesterday})] & \text{if } P \subseteq E \\ \lambda i.\exists e[e \in P \wedge \tau_w(e)\ \mathrm{O}\ (i \cap \textbf{yesterday})] & \text{if } P \subseteq S \end{cases}$$

$$C = \lambda P.\begin{cases} \lambda i.\exists e[e \in P \wedge \tau_w(e) \subseteq (i \cap \mathbf{c})] & \text{if } P \subseteq E \\ \lambda i.\exists e[e \in P \wedge \tau_w(e)\ \mathrm{O}\ (i \cap \mathbf{c})] & \text{if } P \subseteq S \end{cases}$$

In the new theory, derivation PAST(O) is no longer available, hence we will not derive the never-interpretation, unless we can derive it via C (we can't).

(1) *I turned off the stove* $\rightarrow$ PAST(C(O))

$C(O) = \lambda i.\exists e[e \in P \wedge \tau_w(e) \subseteq (i \cap \mathbf{c})]$
$PAST(C(O)) = \exists j[j < \textbf{now} \wedge \exists e[e \in P \wedge \tau_w(e) \subseteq (j \cap \mathbf{c})]] =$
$\exists i[i \subseteq \mathbf{c} \wedge i < \textbf{now} \wedge \exists e[e \in P \wedge \tau_w(e) \subseteq i]]$

I turned off the stove $\rightarrow \exists i[i \subseteq \mathbf{c} \wedge i < \textbf{now} \wedge \exists e[e \in P \wedge \tau_w(e) \subseteq i]]$
Contextual interval **c** includes a past interval that includes the running time of some event of me turning off the stove.

I didn't turn off the stove $\rightarrow \neg\exists i[i \subseteq \mathbf{c} \wedge i < \textbf{now} \wedge \exists e[e \in P \wedge \tau_w(e) \subseteq i]]$
Contextual interval **c** includes no past interval that includes the running time of some event of me turning off the stove: turning off the stove events are absent of the past stretch of contectual interval **c**.

Both of these are the correct readings. So we now derive the correct reading for *I didn't turn off the stove*, and we no longer derive the *never* reading.

Discourse theory of reference intervals. The relevant intervals are often given by discourse.

The difference between states and events shows up in (1)

(1) a. I was asleep when Jane knocked.	Overlap
b. I woke up when Jane knocked	After

(2) a. I wasn't asleep.	Requires contextually given reference interval
b. I wasn't asleep when Jane knocked	Reference interval fixed in discourse

See the volume on tense and aspect of Linguistics and Philosophy 1984.

2.1.4. Branching Time and historical necessity

Thomason, Richmond, 1984, ‘Combinations of Tense and Modality,’ in: Guenthner and Gabbay (eds) *Handbook of Philosophical Logic*, Kluwer, Dordrecht.
von Kutchera, Franz, 1997, ‘T × W Completeness,’ in *Journal of Philosophical Logic* 26
Condoravdi, Cleo, 2002, ‘Temporal interpretation of modals,’ in Beaver et. al. (eds) *The*
Thomason, Richmond and Anil Gupta, 1980, ‘A theory of conditionals in the context of branching time,’ in *Philosophical Review* 89.
Zanardo, Alberto, 2006, ‘Quantification over sets of possible worlds in branching time semantics,’ in *Studia Logica* 82

Thomason builds on unpulished work by Hans Kamp (Kamp ms. 'Historical necessity')

Historical necessity is concerned with the interpretation of the the future tense *will* and its relation to determinism.

Aristotle's sea battle:Determinism:

(1) Either a sea batlle will take place or it won't take place tomorrow.
(2) If a sea battle will take place tomorrow it is now already true that it will.
(3) If a sea battle won't take place tomorrow it is now already true that it won't.
(4) Hence the future is already determined.

Gilbert Ryle: A soldier standing straight up in the trenches, arguing to his mates:
(1) Either there is a bullet in the enemy's gun with my name on it, or there isn't.
(2) If there is, it will get me whether or not I duck.
(3) If there isn't it won't get me whether or not I duch.
(4) Hence I can just as well stay standing up.

If we take the principles of Priorian tense logic, we see that one of the axioms of minimal tense logic is:

$\varphi \rightarrow HF\varphi$

φ itself can be a modal formula, so we have:

$F\varphi \rightarrow HFF\varphi$

Assuming transitivity, we can conclude:

$F\varphi \rightarrow HF\varphi$
If φ is going to be true, then it was always true that φ was going to be true.

This seems to go rather a long way towards justifying the premises of determinism that make Aristotle's argument go through.

Branching time: the future is open.
Ockhamist branching time: the future tense is a temporal-modal operator

An **Ockhamist frame for branching time** is a structure $\mathbf{T_W} = <\mathbf{T}, W_{\mathbf{T}}>$ where

1. $\mathbf{T} = <T,<>$ is a left-linear partial order
2. W_T, the set of worlds, complete possible histories, is the set of all branches in T.

Worlds, then, are maximal chains in **T**. The set of worlds running through point t, W_t is the set of branches b such that $t \in b$.

We will here consider only what is called the Peircean variant of Ockhamist branching time models. The only relevant clause is that for Fφ:

$$[\![F\varphi]\!]_{M,t} = 1 \text{ iff for every } w \in W_t\text{: there is a } t'\in w\text{: } t' > t \text{ and } [\![\varphi]\!]_{M,t'} = 1$$

Fφ is true at t iff for every world, every branch running through t, at some point φ is true.

(1) a. I will go to Innisfree.
b. I will go to Innistree next week.

Thus, (1a) is true at t iff every world through t contains a point later than t where I go to Innisfree.
(1b) is true at t iff every world through t contains a point in the interval *next-week* where I go to Innisfree.

In branching time the past and present are settled, the future is open.
The futurate modal operator *will* (F) expresses that certain statements can be settled even though they are futurate. Thus, (1a), on this analysis expresses that it is already settled at a point of time in the future, I go to Innisfree (meaning that I am ignoring possible futures where I don't go as not accessible).
The idea about the sea battle is that we do not accept that *there will be a sea battle* is settled, some future worlds have a sea battle in them, some don't. Look at a world that has a sea battled in at at t' > t. Is it true at every moment in the past of t' that Fφ holds there?
No, because Fφ actually doesn't hold at t.
So the principle of minimal tense logic we accepted before, we don't accept now.

From a semantic perspective there is a rather serious compositionality problem with Peirciean Ockhamist semantics. We accept the insights of the semantics for (1a,b), but are now concerned with the proper semantics for (2a,b):

(1) a. I will go to Innisfree.	$F\varphi$
b. I will go to Innistree next week.	$F_{\text{next week}}(\varphi)$
(2) a. I won't go to Innisfree.	
b. I won't go to Innistree next week.	

What should the meaning of (2a,b) be?

$\neg F\varphi$	Some world through φ has no future moment where I go to Innisfree
$\neg F_{\text{next week}}(\varphi)$	Some world through φ has no moment in its interval *next week* where I go to Innisfree

The problem is that these clauses are too weak. The modal $F\varphi$ expresses at t that it is settled at t that φ happens at a future moment. With that, $\neg F\varphi$ expresses at to that it is not yet settled that φ happens at a future moment. But (2a) doesn't naturally express that, it makes a much stronger claim.

The alternative would be to interpret (2a,b) as:

$F\neg\varphi$	Every world through t has a future point where $\neg\varphi$ is true
$F_{\textit{next week}}(\neg\varphi)$	Every world through t has a point in its interval *next week* where $\neg\varphi$ is true

The problem is that these formulas are so weak as to be practically tautologies: nobody was suggesting that it was going to be *sea battle* from now to Eternity, so anybody would accept $F\neg\varphi$, the same with $F_{\textit{next week}}(\neg\varphi)$, the issue is not that its sea battle *all* next week. So certainly $F\neg\varphi$ is not what (2a,b) should express.
Of course, we know what we want (2a,b) to express in this framework:

$G\neg\varphi$	No world through t has a future point where φ is true
$G_{\textit{next week}}(\neg\varphi)$	No world through t has a point in the interval *next week* where φ is true

The problem is a compositionality problem: we don't want *won't* and *will not* to be independent lexical items. we want to build their meaning with *will* and *not*. But that seems impossible.

The logic of Historical Necessity takes a different view. It assumes that the future tense is itself not a modal operator but a tense operator, and, for us, just the Priorian operator.
This means that the principle of minimal tense logic $\varphi \rightarrow HF\varphi$ is accepted. Even worse, we just assume linear time here. So obviously, we seem to *want* to accept that principle.
Clearly, we must be trying to solve the determinism problem in a different way. And we are.

If the future tense is not itself a modal, we will need to make assumptions about how modal interpretations come about. For this to be expressible, we introduce the *modal* operator of Historical necessity. We come back to the present discussion later.

Historical necessity

For technical reasons in formulating the axioms we introduce unrestricted logical necessity, and use the closed box for it. The open box we use for the necessity that we are interested in here, historical necessity:

$\Box$: it is historically necessary that
$\blacksquare$: it is logically necessary that

In branching time the past and present are settled, the future is open.
Historical necessity is a very local notion of necessity, it is concerned with what is, at a certain point of time, already settled and what is still open.
Historical necessity in this sense is not an epistemic modality, although it can, at times, be hard to distinguish the two, see Condoravdi's excellent discussion.

Historical necessity is a form of metaphysical modality (possibilities about what the world is like, could be like, could have been like), but a very local form. If p has happened in w at t, p is a metaphycal reality, but not necessarily a metaphysical necessity. But p is settled at t, and hence an historical necessity: the past and present are settled, the future is open. Some aspects of the future at t may already be settled at t as well, even though they haven't happened yet: what is metaphysically possible at t in combination with what has happened so far determines what is and is not open at t. For instance, let c be a normal coin, with an edge that is too narrow to stand on, and let c be spinning in the air at t. Then
F*lands-heads*(c) and F*lands-tails*(c) are open at t, since we can assume that it isn't yet settled at t that the coin will land heads at some future time or tails at some future time, but we can assume that F(*lands-heads*(c) $\vee$ *lands-tails*(c)) **is** settled at t: a coin that is spinning in the air at t, must come down at some point.
This means that at t the following is true:
$\Diamond$F*lands-heads*(c) $\wedge$ $\Diamond$ F*lands-tails*(c) $\wedge$ $\Box$F(*lands-heads*(c) $\vee$ *lands-tails*(c))

Thus:

φ is historically necessary at t if φ is true at t regardless of what the future is like.

Since the past is settled and, as time passes, more and more issues get settled, and what is settled does not get unsettled, the issues that are open **decrease monotonically with time**. And this means that, for historical necessity, the accessible worlds decrease monotonically with time.

Frames for Historical necessity are T×W frames we introduced for time-dependent modality above, but with modal accessiblity relation $\sim_t$:

A **branching time frame** for L is a structure **T×W** = $<T,<, W, R>$ where

$<T,<>$ is a **linear time** frame and for every $t \in T$: $<W,\sim_t>$ is a **modal** frame with $\sim_t$ a **right monotone decreasing equivalence relation** on W

v $\boldsymbol{\sim_t}$ w means: v and w share the same modal past at t: what was true and possible in v at moments of time before t is the same as what was true and possible in w at moments of time before t.

for world w and time t: $[w]_{\sim_t}$ is the set of worlds accessible to w that have the same past as w up to t.

$\sim$ is **right monotone decreasing** iff for every $t_1, t_2 \in T$ for every $w \in W$:

$$\text{if } t_1 < t_2 \text{ then } [w]_{\sim_{t_2}} \subseteq [w]_{\sim_{t_1}}$$

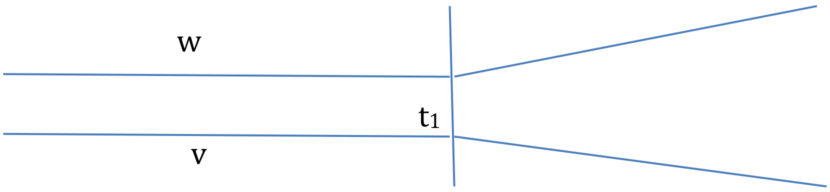
equivalently: for every $t_1, t_2 \in T$ for every $v \in W$: if $t_1 < t_2$ and $v \sim_{t_2} w$ then $v \sim_{t_1} w$

In the context of branching time, we think of worlds as **complete possible histories**.
In the T $\times$ W approach to branching time, it is not the time itself that branches (T is a linear order): the branching is encoded in the structure of $\sim$.

A **branching time model** is a structure **M** = <**T×W**, i> where

1. **T×W** is a branching time frame
2. i: ATFORM × W × T → {0,1} such that:
   for every $\varphi \in$ ATFORM, $\forall w,v \in W\ \forall t_1, t_2 \in T$:
   if $w \sim_{t_2} v$ and $t_1 \leq t_2$ then $i_{w,t_1}(\varphi) = i_{v,1}(\varphi)$

If you have two histories w_t and v_t and you look at the *same* past moment t_1 from t_2, then the same basic facts hold at t_1 in histories w_t and v_t



.
The constraint on atomic formulas enforces the principles of branching time.
Let $\varphi \in$ ATFORM, t_2 be the present time and w the real world.
We look first at the past.
Assume that $t_1 < t_2$ and $i_{w,t_1}(\varphi)=1$. This means that φ is true in this world at some past time.
Then for all $v \in [w]_{\sim_{t_1}}$: $i_{v,t_1}(\varphi)=1$, by the constraint.
Since $[w]_{\sim_{t_2}} \subseteq [w]_{\sim_{t_1}}$, this means that in all worlds v that are accessible from w **at the present time** t_2, φ is true in v at past time t_1.

Now we look at the future.
Assume that $t_3 > t_2$ and that $i_{w,t_2}(\varphi)=1$
Assume that $w \sim_{t_2} v$ and that $i_{v,t_3}(\varphi)= 0$
This is perfectly possible, because it is not required that $w \sim_{t_3} v$. Precisely not: as time passes the set of alternatives shrinks, and if φ becomes true in the real world at t_3, then **at that time,** world v fails to be an alternative for w.

$[\![\varphi]\!]_{M,w,t} = 1$

1. If $\varphi \in$ ATFORM then: $[\![\varphi]\!]_{M,w,t} = i_{w,t}(\varphi)$
2. $[\![\neg\varphi]\!]_{M,w,t} = 1$ iff $[\![\varphi]\!]_{M,w,t} = 0$, etc. for $\wedge, \vee, \rightarrow$
3. $[\![P\varphi]\!]_{M,w,t} =$ for some $t' < t$: $[\![\varphi]\!]_{M,w,t'} = 1$
4. $[\![F\varphi]\!]_{M,w,t} =$ for some $t' > t$: $[\![\varphi]\!]_{M,w,t'} = 1$
5. $[\![\Box\varphi]\!]_{M,w,t} =$ for all $v \in W$: if $v \sim_t w$ then $[\![\varphi]\!]_{M,v,t} = 1$
6. $[\![\blacksquare\varphi]\!]_{M,w,t} =$ for all $v \in W$: $[\![\varphi]\!]_{M,v,t} = 1$

Truth in M is true in M at every w and t.
Validity is truth in M for all models M

Branching time logic
Linear time + S5 for □ and for ■ +

Axiom schemas: $■\varphi \to □\varphi$ Logical necessity entails historical necessity

$P□\varphi \to □P\varphi$ Historical necessity is right monotone decreasing

$P■\varphi \to ■P\varphi$
$F■\varphi \to ■F\varphi$ Logical necessity is temporally constant

Gabbay irreflexivity: From $■(\neg\varphi \wedge G(\varphi)) \to \psi$ derive ψ, if φ is an atomic formula that does not occur in φ

Kamp's axiom: $□\varphi \vee □\neg\varphi$ if φ is an atomic formula
von Kutchera shows that the above logic (without Kamp's axiom) is complete with respect to the class of branching time frames (without the restriction on the interpretation function to atomic formulas).

Fact: $P□\varphi \to □P\varphi$ is true at a frame iff ~ is right monotonically decreasing
Proof:

1. if ~ is right monotonically decreasing $P□\varphi \to □P\varphi$ is true on the frame.

Assume: $[\![P□\varphi]\!]_{w,t_2} = 1$
Then for some t_1: $t_1 < t_2$ and $[\![□\varphi]\!]_{w,t_1} = 1$
Then for some t_1: $t_1 < t_2$ and for all $v \in [w]_{\sim t1}$: $[\![\varphi]\!]_{v,t_1} = 1$
Since $[w]_{\sim_{t_2}} \subseteq [w]_{\sim_{t_1}}$, it follows that:

$$\text{for all } v \in [w]_{\sim_{t_2}}: [\![\varphi]\!]_{v,t_1} = 1$$

Hence for all $v \in [w]_{\sim_{t_2}}$: $[\![P\varphi]\!]_{v,t_2} = 1$
Hence $[\![□P\varphi]\!]_{w,t_2} = 1$
(The accessible worlds at t_1 are accessible *pasts* at t_2)

If ~ is not right monotonically decreasing, we can make a counterexample to $P□\varphi \to □P\varphi$.
let $t_1 < t_2$ and $z \in [w]_{\sim_{t_2}}$ but $z \notin [w]_{\sim_{t_1}}$

Make φ true at t_1 in every world $v \in [w]_{\sim_{t_1}}$ and false at every other moment of time (for every world). Clearly, $P□\varphi$ is true in w at t_2.
But for world z there is no time in the past of t_2 such that φ is true, so $P\varphi$ is false in z at t_2.
Hence $□P\varphi$ is false in w at t.

The sea battle in Historical Necessity

Aristotle's sea battle:

(1) Either a sea batlle will take place or it won't take place tomorrow.
(2) If a sea battle will take place tomorrow it is now already true that it will.
(3) If a sea battle won't take place tomorrow it is now already true that it won't.
(4) Hence the future is already determined.

φ = A sea battle takes place
We assume: $\varphi \in$ ATFORM

- $\Box F\varphi$ A sea battle is unavoidable

$[\![\Box F\varphi]\!]_{w,t} = 1$ iff $\forall v \sim_t w\ \exists t' > t$: $[\![\varphi]\!]_{v,t'} = 1$

We assume that in our world w at the present time t this is false, because there is an accessible world v with the same past at t as w in which at no time t' in the future of t $[\![\varphi]\!]_{v,t'}=1$.

- $\Box G\neg\varphi$ A sea battle will not happen

$[\![\Box\neg F\varphi]\!]_{w,t} = 1$ iff $\forall v \sim_t w\ \forall t' > t$: $[\![\varphi]\!]_{v,t'} = 0$

This too we assume to be false in w at t, because there also is an accessible world z with the same past at t as w in which at some time t' in the future of t $[\![\varphi]\!]_{v,t'}=1$.

- $\Box(F\varphi \vee \neg F\varphi)$ A sea battle will happen or won't happen

$[\![\Box(F\varphi \vee \neg F\varphi)]\!]_{w,t} = 1$ iff $\forall v \sim_t w[\ \exists t' > t$: $[\![\varphi]\!]_{v,t'} = 1$ or $\forall t' > t$: $[\![\varphi]\!]_{v,t'} = 0]$

This is perfectly true.

What is not true is the reasonable expression of Aristotle's conditional:

- $F\varphi \to \Box F\varphi$
If a sea battle will happen it is now already true that it will happen.

We interpret this as:

If a sea battle will happen, it is unavoidable.

$[\![F\varphi \to \Box F\varphi]\!]_{w,t} = 1$ iff either $\neg\exists t' > t$: $[\![\varphi]\!]_{w,t'} = 1$ or $\forall v \sim_t w\ \exists t' > t$: $[\![\varphi]\!]_{v,t'} = 1$

This is not true, because it may well be that $t'>t$ and $[\![\varphi]\!]_{w,t'} = 1$, but for some $v \sim_t w$:
$\neg\exists t' > t$: $[\![\varphi]\!]_{v,t'} = 1$

So the argument for determinism doesn't hold in branching time (well, not surprisingly, because it's why we assume branching time in the first place).

The negation problem.

(1) a. I will go to Innisfree. $F\varphi$
b. I will go to Innistree next week. $F_{\text{next week}}(\varphi)$
(2) a. I won't go to Innisfree. $\neg F\varphi$
b. I won't go to Innistree next week. $\neg\, F_{\text{next week}}(\varphi)$

We assume the future tense is not a modal operator but a tense operator. This has the big advantage that the examples in (2) have unproblemantically the correct temporal meaning:

(1a) is true in w at t if at some point t' in the future of t I go to Innisfree at t' in w.
(2a) is true in w at t if that is not the case, i..e. if at no point t' in the future of t I go to Innisfree at t' i n w.

(1b) is true in w at t if at some point t' in the interval *next week* I go to Innisfree at t' in w.
(2a) is true in w at t if that is not the case, i..e. if at no point t' in the interval *next week* I go to Innisfree at t' i n w.

To which we can add unproplematically the tense relation presented earlier (see Condaravdi).

What about the modal interpretations?

The set of alternatives $[w]_{\sim t}$ tells you what is settled at t and what is not. We can think of this set of alternatives as being determined by a set of reasons: I go to Amsterdam this week because I have a ticket a seat reservation a reason to go there and nothing holding me, etc…
The fact that w is included in $[w]_{\sim t}$ tells us that these reasons are *de facto* sound reasons. Pragmatically, this may be a bit too strong. I try to deduce my claims about the future from sound reasons, but ultimately I cannot be guaranteed that my reasons are really correct.
For this reason it may be too strong to *require* that w be one of the worlds in the set of alternatives. For this reason we introduce a slightly weaker notion that is not committed to the real world being in the set of alternatives.

We introduce a function S (for *settledness*) which maps every time t onto a relation S between worlds. We let $S_{t,w} = \{v \in W: S_t(w,v)\}$. We define $S_{t,w}$ in terms of $[w]_{\sim_t}$ by:

For every $t \in T$, $w \in W$: $S_{t,w} = [w]_{\sim_t}$ or $S_{t,w} = [w]_{\sim_t} - \{w\}$

Thus, S differs from ~ in that it is open whether w is accessible or not: in some worlds the evidence is sufficient to include w, in others you don't, in general you're not sure.
We introduce a third universal modal operator $\boxtimes$ to correspond to S:

$[\![\boxtimes\varphi]\!]_{M,w,t} = 1$ iff for every $v \in S_{w,t}$: $[\![\varphi]\!]_{M,v,t} = 1$

The modal operator $\boxtimes$ makes a claim about the alternatives for w, but not about w itself. Hence, $\boxtimes$ is weaker than $\Box$, and that is pragmatically useful.
Thus, when I assert $\boxtimes\varphi$, I am saying that, according to the generalizations encoded in the accessibility relation, φ is settled. But I am not saying that this means that φ actually happens

in w, because, the generalizations encoded in the accessibility relation may turn out to be wrong after all. So, $\boxtimes$ is a pragmatically reasonable variant of $\Box$.

With this we assume a principle of *pragmatic modal weakening of the future tense*:

Pragmatic modal weakening of the future tense: Interpret $F\varphi$ as $\boxtimes F\varphi$

Instead of taking our futurate statement to be a claim that a *future fact* holds (which we don't really have access too), we can weaken it to a claim that this future fact follows – as far as we can tell - from *present* concerns (encoded in the accessibility relation) about what we regard as settled.

For (1a) and (1b) this means that we can reformulate the claim made at t that at t' in the future (or in the interval *next week*) I actually go to Innisfree, to the claim that in every accessible world in $S_{w,t}$ I go to Innisfree at some point in the future (or at some point in the interval *next week*): according to the accessibility relation, my going to Innisfree in the future is already predetermined.

The important difference with Ockhamist time is that we can assume exactly the *same* pragmatic account of the cases in (2):

We can reformulate the claim made at t that at no point t' in the future (or in the interval *next week*) I actually go to Innisfree to the claim that in no accessible world in $S_{w,t}$ I go to Innisfree at any point in the future (or at any point in the interval *next week*): according to the accessibility relation, my not going to Innisfree in the future is already predetermined.

This means that the future is a tense and not a modal, but we have a general pragmatic principle that gives us modal interpretations of the future tense, and in fact, the correct modal interpretations. I think that this is an important semantic insight, and that it supports the division of labour that the logic of Historical Necessity proposes.

Branching time: Counterfactuals

The semantic of branching time and historical necessity is relevant for the semantics of counterfactuals.

(2) If she hadn't left me I wouldn‘t be so miserable now.

Stalnaker 1968, Stalnaker and Thomason 1968 and Lewis 1974 give a modal semantics for counterfactuals as variably strict conditionals.
In the simplest possible terms, Stalnaker assumes the following semantics for counterfactuals: Let $\varphi \rhd \psi$ be a counterfactual conditional.

$$[\![\varphi \rhd \psi]\!]_{M,w} = 1 \text{ iff } [\![\psi]\!]_{M,\mathbf{select}(\varphi,w)} = 1$$

Here **select** is a function that maps w and φ onto the closest world to w where φ is true. (within a contextually given set of alternatives).
This, of course, requires an ordering of worlds in terms of closeness. Lewis 1975 assumes an ordering of overall similarity.

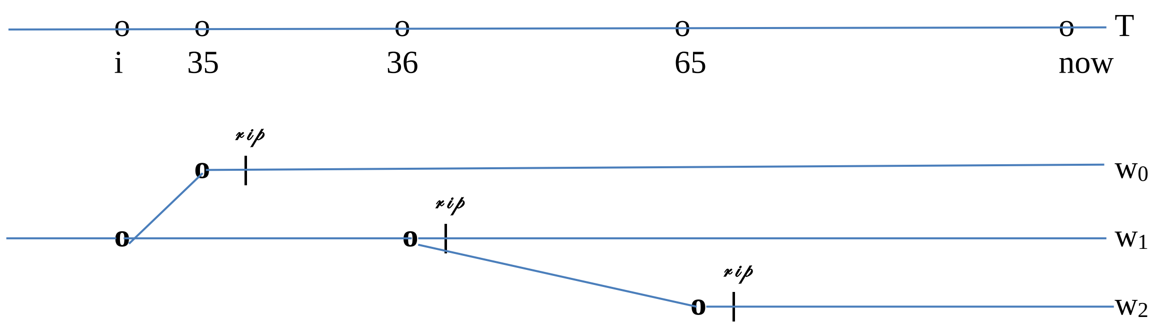
The variable strictness of the semantics avoids inferences like the following:

(3) a. If Mozart hadn't died at 35 he would have written many more operas
    b. If Mozart hadn't died at 35, but at 36, he would have written many more operas.

Intuitively, the inference from (3a) to (3b) is invalid, which is predicted by variable strict semantics. Let's assume that the closest world where Mozart didn't die at 35 is one where he didn't catch the infection at the masoric meeting that killed him three weeks later, and in this world, Mozart lived on and wrote many more operas. (3a) is true. This does not entail that in the closest world where he didn't die at 35 but at 36 he lived on and wrote many more operas.

Thomason and Gupta point out that branching time and historical necessity is the natural setting for counterfactuals. While Stalnaker's account is fully modal, the counterfactuals are mixed temporal-modal and are concerned with what is settled and what is not.

We can draw the following picture:



In w_0, the real world, Mozart dies when he does at 35.
Point of time i is the point at the masoric meeting where person x hasn't yet coughed in his direction. at this point worlds w_0, w_1 and w_2 are still open. So $w_0, w_1, w_2 \in [w_0]_{\sim i}$
The counterfactual $\varphi \rhd \psi$ tells us to go back to the point where φ (*Mozart dies at 35*) was still open, i.e. to point i. The worlds where *Mozart doesn't die at 35* that are open at i are worlds w_1 and w_2. We assume that the Stalnaker selection function must select a world within this set as the closest, and we assume that the function selects world w_1.
Then, as before, we block the unwanted inference.

A bit more discussion

$[\![\varphi \rhd \psi]\!]_{M,w,t} = 1$

1. Go back in w to t_1 where $\neg\varphi$ is not yet settled.
2. In the closest world v where $[\![\varphi]\!]_{v,t_1} = 1$ also $[\![\psi]\!]_{v,t_1}$

But note: there is possible ambiguity about what $\neg\varphi$ is!

(3) a. If Mozart hadn't died at 35 he would have written many more operas

A: $\neg\varphi$ = If Mozart hadn't DIED at 35
   Instruction: go back to t_1 where he didn't get the virus
B: $\neg\varphi$ = If Mozart hadn't died AT 35

Instruction: go back to $t_2 < t$ where the effect is slower and carries him over his birthday
Not settled in A: Mozart is about to die
Not settled in B: Mozart will die before his birthday

In A: the closest world could well eb one in which he wrote more operas
In B: he doesn't write more operas in the closest world

So: *Suppose Mozart hadn't died* and *Suppose Mozart hadn't died at 25* are in the context different propositions, and the antecedent can be read as either.

This means that variable strictness of the conditional comes in (in part) via *contextual selection of what proposition to regard as unsettled at a reasonable contextual past time*.

Past predominence

Thomason and Gupta point out that *closeness* is not just a modal notion, but is itself a modal-temporal notion that depends on the notion of historical necessity (i.e. on issues of settledness).
They propose a principle of *past predominence*:

> **Past predominence:**
> In determining how close $<t_1,w_1>$ and $<t_2,w_2>$ are , past closeness predominates over future closeness.

Thomason and Gupta discuss the following case.
Look at conditional (4):

(4) If this button had been pushed, there would have been a nuclear holocaust.

We go back to the time where it was still open whether or not the button would have been pushed. We compare worlds where the button **is** pushed at a later time.
Let us assume that the relevant worlds are two worlds, w_1 where the disconnection team manages to disconnect the button in time, and w_2 where they don't.
The truth of (1) depends on which of these worlds is the closest to w_0, the real world.
Now, if you compare these worlds in terms of overall similarity to w_0 then, since a nuclear holocaust has not taken place, clearly we should decide that w_2 is closer to our world than w_1.
But that seems incorrect:

> What is relevant is **not** the overall similarity of the worlds, but which **initial stretch** is more similar to w_0: the stretch of w_1 up to the pressing of the button, or the stretch of w_2 up to the pressing of the button.

The semantics of counterfactuals brings you back to a branching point, where the antecedent is still an option, , say, the point where the finger was hovering over the button, or any of the points in recent history, when there was a political crisis. In in left-linear models the past *up to that evaluation point* is fixed. This means that *if* at that point the button was connected to the Device – and we assume it was - you cannot at that point go to a world where it actually isn't connected *at that time*.

This means that the only worlds that you *can* consider in which the button is pressed and the nuclear holocaust doesn't happen are worlds in which there pops up just before the pressing a deus-ex-machina cord-cutter who snaps the cord just before pressing: the button is pressed, and nothing happens. Such world we can call an *opera seria world.* The opera seria world is *overall* closest to our current world. But the deus-ex-machina device that makes this world possible, the cord-cutting event, is extremely unlikely at the branching point.

Dowty 1979, following work by Hans Kamp introduces a notion of an *inertia world*, a world in which nothing unexpected happens. A version of this may well be useful here. We determine a branching point: the finger hovering over the button. At this point we restrict ourselves to worlds where the antecedent event – the button pressing is going to happen. But we want to restrict ourselves to worlds that are, apart from the fact that something happens that actually didn't happen, are inertia worlds wrt. the real world in that all the *independent* events that were already set in motion at that time continued to develop the way they did before, and *no independent events* suddenly popped out of nowhere.
Thus, we imagine the finger hovering over the butten in indecision, and it went the other way from the way it actually went. The worlds in which *that* happened (plus all the events that made it possible, like the correct muscle actions neuron actions, etc.) and everything else stayed the same are the alternatives to be considered. This will exclude opera seria worlds as alternatives, unless there is direct reason to assume them (as there would be, of course, in Holywood scenarios, where the Team-that-is-to-save-the-world was *already* working relentlessly to have a cord-cutter on the scene).

2.1.5. Lettting the time branch

The T×W frames discussed above have a single linear time-axis, the branching effects of time are achieved via the worlds and the equivalence relation on the worlds.
There is quite a lot of study in the philosophical literature of structures where worlds are not required to have the same time axis and even structures in which worlds are defined in a branching time frame.

A **Kamp frame for branching time** is a structure $\mathbf{T} \times \mathbf{W} = \langle \boldsymbol{T}, W, \sim \rangle$ where:

1. W is a non-empty set of worlds, complete possible histories.
2. **T** is a function from worlds to temporal orders such that;
   for every $w \in W$, $\mathbf{T}_w = \langle T_w, <_w \rangle$, a linear order.
3. For every $t \in \cup\{T_w: w \in W\}$, $W_t = \{w: t \in T_w\}$
   For every $t \in \cup\{T_w: w \in W\}$: $\sim_t$ is an equivalence relation on W_t
4. If $w_1 \sim_t w_2$ then $\{t' \in T_{w1}: t' <_{w_1} t\} = \{t' \in T_{w2}: t' <_{w_2} t\}$
5. If $w_1 \sim_t w_2$ and $t' <_{w_1} t$ then $w_1 \sim_{t'} w_2$

$[\![\varphi]\!]_{M,w,t}$ as above with the restriction that $w \in W_t$

In a Kamp frame worlds the past is linear, and worlds that are temporal alternatives at t necessarily share the same past, but not the same future.

A **Bundle frame for branching time** is a structure $\mathbf{T_W} = <\mathbf{T}, W_T >$ where

1. $\mathbf{T} = <T,< >$ is a left-linear partial order
2. W_T, the set of worlds, complete possible histories, is **a subset of** the set of all branches in T.
3. For every $t \in T$ there is a $w \in W_T$: $t \in w$

Obviously, T×W frames are a special kind of Kamp-frames.
Obviously Ockamist frames are a special kind of Bundle frames.
Zanardo shows that also Kamp-frames are a special kind of Bundle frames, so this may be the most general useful notion.

Thomason expresses that the T×W model with its fixed linear time axis goes against his philophical intuitions. The model doesn't allow the structure of time to vary across possible worlds. So, for instance, if we assume that time is not right-continuing, then there are no worlds in which time is right-continuing and time will inevitably come to an end. Kamp's model allows this to vary: in some worlds time may end, while in others it continues.
There is an intermediate position:

A **liberated branching time frame** is a branching time frame $<T, <, W, \sim >$ where

1. for each $w \in W$: $\mathbf{T}_w = <T_w,<_w>$ is the restriction of **T** to an initial segment of T.
2. restriction on $w_1 \sim_t w_2$: $t \in T_{w_1} \cap T_{w_2}$

Here we still have one linear order, but it may stop earlier in some worlds than in others.

This would presumably still go against Thomason's philosophical intuitions, because it doesn't allow time to have fundamentally different structures in different worlds.
For instance, if the issue is open whether time is discrete or dense, then maybe we want it to be discrete in some worlds and dense in others.
I am not so sympathetic to that, unless we were to find that **in the past** time has been alternatingly dense and discrete. In that case we may have to prepare for the *possibilty* of a discrete stage happening in the near future.

Still, this stays playing around with philosophical intuitions, which is perfectly legitimate for philosophy, but in semantics we have other constraints. The question is whether such differences in the structure of time are detectable in natural language semantics, or whether the assumptions of natural language metaphysics favor a principle of **homogeneity**:

Modal homogeneity:
The structure of time looks the same wherever you stand in time.
For every $w_1, w_2 \in W$ $<\{t' \in T: t' \leq_{w_1} t\},\leq > \simeq <\{t' \in T: t' \leq_{w_2} t\},< >$ and
$<\{t' \in T: t' \geq_{w_1} t_1\},\leq > \simeq <\{t' \in T: t' \geq_{w_2} t_2\},> >$

Temporal homogeneity: (discussed in van Benthem 1984 *The Logic of Time*)
The structure of time looks the same wherever you stand in time.
For every $t_1, t_2 \in T$ $<\{t' \in T: t' \leq t_1\},\leq > \simeq <\{t' \in T: t' \leq t_2\},< >$ and
$<\{t' \in T: t' \geq t_1\},\leq > \simeq <\{t' \in T: t' \geq t_2\},> >$

Empirically, the issue of a common structure of time versus branches should show up in counterfactuals, but it is very hard to construct covincing arguments here.

The reason is that there is interference with other notions that clearly **are** modally variable,

So, if I say on the 24^{th} of February (5):

(5) If today had been the 28^{th} of October it would have been my birthday.

we are instructed to go to a point where it is not yet fixed that *today is not the 28th of October*. (It is actually not so clear how you can do that.)
But we are playing here with the difference between what McTaggert called the A-series and the B-series: in the B-series time passes through dates (the 28^{th} of October) and is ordered 'objectively' by <, while in the A-series time passes from past to present to future (from yesterday to today to tomorrow), and is contextually focussed on the present. Obviously, the tagging of the two series onto each other is open for modal variation: the truth of (5) does not require us to go to a point where the 24^{th} of Februari is the 28^{th} of October.
And even when I say (6)

(6) If today had been yesterday, I would have been in Amsterdam.

we can use the tagging of the A-series onto the B-series: an A-series concept like *yesterday* denotes, with respect to the B-series, a function from contexts to intervals of time (i.e. a temporal individual concept), mapping the present speech context onto the intervals of time that corresponds to the day before the present day.
As always for individual concepts there is an ambiguity between the expression *yesterday* denoting the function or denoting the **value** of the function. The intended reading of (6) can be paraphrased as:

(7) If we were to switch from contect c_0, where *today*(c_0) = 24^{th} of February, to a context c_1 where *today*(c_1) = *yesterday*(c_0), I would have been in Amsterdam at *today*(c_1).

This does not seem to require changing the order of time at all. Most temporal examples that I can think of can be analyzed away along these lines. You have to go to examples like (8) to get in the ball park of changing temporal orders:

(8) If today were stil today and yesterday had been yesterday a year ago, then I would be a year younger than I am now.

But my native speaker at home is very uncomfortable about (8) (and so am I) and regards it as a stretching play with the temporal expressions (and with stretchmarks), rather than a statement that is within the normal range of meaning of these expressions. (8) requires today and yesterday to be fixed to, say, the 24^{th} and 23^{rd} of Februari 2012 and going to a world where between these two dates a whole year is fitted in. Clearly, this requires the temporal order to vary from world to world.

It seems to me that if the evidence for temporal variation is at best of *this* nature, we better stay with one order, and think of the philosophical discussion as stretching the semantics.

There are good practical reasons to stay with one order for semantical applications. Temporal adverbials of the A-series and the B-series pick out intervals in past, present, and future, and require a metric comparability of the branches anyway: that is, future branches must pick out a region of future time which counts as *today in two weeks* or *2017*. That is, if we allow time to branch, we must impose a metric which makes future stretches of time on different branches comparable, so that our temporal adverbials can pick out a region of time on different branches.

This can be done, of course, but it can also be avoided, by using one fixed temporal order (or initial segments of one and the same order) for different worlds.

Even though the logics involved are called *tense logics* they are not really about natural language tenses. For illuminating discussion of natural language tenses and their relation to modality, see Condoravdi.

2.1.6 Intervals and Aspect: Landman and Rothstein, summary of basic notions.

Eventualities.

Eventualities are states or events.

Frames: $\boldsymbol{E} = \langle E,\sim,I_T,<,\subseteq,W,\tau,D,TR\rangle$ with:

- $\langle E,\sim\rangle$ is a set of **eventualities** ordered by equivalence relation ~ (**cross-temporal identity**)
- $\langle I_T,<,\subseteq\rangle$ is the **interval structure** based on linear order $\langle T,<_T\rangle$
- W is a set of **possible worlds**
- τ, the **temporal trace function** is a partial function: $\tau: E\times W \rightarrow I_T$
- D is the **domain of individuals**
- TR, is the set of **thematic roles, Agent, Theme.,…** which are partial functions from events to event participants: Agent: $E \rightarrow D$

τ(e,w) is the running time interval of event e in world w if e goes on in w

Eventualities are temporal particulars that go on at one interval in a world.

Verbs, verb phrases, sentences: event types = sets of eventualities.

eat: $\lambda e.EAT(e)$ The set of eating events

eat a mango:

$\lambda e.EAT(e) \wedge MANGO(Theme(e)) =$

$\lambda e.EAT(e) \wedge Agent(e) \in D \wedge MANGO(Theme(e))$

The set of events whose agent is an individual and whose theme is a mango

Fred eat a mango

$\lambda e.EAT(e) \wedge Agent(e) = Fred \wedge MANGO(Theme(e))$

The set of events whose agent is Fred and whose theme is a mango

Fred ate a mango

$\lambda e.EAT(e) \wedge Agent(e) = Fred \wedge MANGO(Theme(e)) \wedge \tau(e,w_0) < \mathbf{now}$

The set of events whose agent is Fred and whose theme is a mango and that are located in this world in the past of now.

Cross-temporal identity

$e_1 \sim e_2$ if e_1 and e_2 count as one and the same eventuality, even if their running time is not the same.

(1) Fred was in Amsterdam *once* last month, from Sunday to Friday.

$s_1, s_2, s_3 \in \lambda s.\ \mathrm{LOCATE}(s,\mathrm{Fred}) \wedge \mathrm{Location}(s) \sqsubseteq \mathrm{Amsterdam}$
$\tau(s_3,w_0)$ = Sunday to Friday
$\tau(s_1,w_0)$ = Sunday
$\tau(s_2,w_0)$ = Friday
$s_1 \sim s_2 \sim s_3$

~-identity postulate: if $e_1 \sim e_2$ and $\tau(e_1,w)=\tau(e_2,w)$ then $e_1=e_2$

(2) Fred ate an artichoke *once* last week.

$e_1,\ldots e_{31} \in \mathrm{EAT}$
e_1: Fred eats the flesh off the first leave
e_2 = Fred eats the flesh off the first leave and then off the second leave
e_3= e_2+ and then off the third leave
…e_{30} = e_{29}+Fred eats half of the heart
e_{31} = e_{30}+Fred eats the other half of the heart

$e_1 \sim e_2 \sim e_3 \sim \ldots. \sim e30 \sim e_{31}$

$i \preccurlyeq j$: i **is an initial subinterval of** j

$e_1 \preccurlyeq e_2$: e_1 **is an initial stage of** e_2

$e_1 \preccurlyeq e_2$ iff for every world $w \in W$: if $\tau(e_2,w) \neq \perp$ then $\tau(e_1,w) \preccurlyeq \tau(e_2,w) \wedge e_1 \sim e_2$

Events have onsets, states hold at points.

VP with event type α
V_α is the event type of the verbal head of VP.

eat a mango $\alpha = \lambda e.\mathrm{EAT}(e) \wedge \mathrm{MANGO}(\mathrm{Theme}(e))$
$V_\alpha = \lambda e.\mathrm{EAT}(e)$

$e \in \alpha$ has a V_α-onset iff there is an event $o(e,V_\alpha) \in V_\alpha$ such that:

1. $O(e,V_\alpha) \preccurlyeq e$
2. For every $w \in W$: if $\tau(O(e,V_\alpha),w) \neq \perp$ the $\tau(O(e,V_\alpha),w)$ is not a **point.**
3. if $e' \prec O(e,V_\alpha)$ then $e' \notin V_\alpha$

Onset constraint: If VP is an eventive predicate with event type α based on V_α then every event e in α has a V_α-onset.

Intuition: The eat-onset of an event of eating a mango is the first bit of activity that counts itself as eating and that will develop into the eating of that mango.

Activities: if $e \in \alpha$ then $O(e,V_\alpha) \in \alpha$ (the first bit of waltzing counts as waltzing)
Accomplishments: if $e \in \alpha$ then $O(e,V_\alpha) \in V_\alpha$, but not necessarily $O(e,V_\alpha) \in \alpha$
(the first bit of eating a mango counts as eating, but not itself as eating a mango).

States:	Fred was in Amsterdam, the Garbage stank.
Activities:	Fred waltzed, Fred pushed a cart
Accomplishments:	Fred ate a mango, Fred wrote a book
Achievements:	Fred arrives at the station, Fred was born.

Stative homogeneity (subvinterval property)

Lexical requirement: Stative event types are homogenous:

stative event type α is **homogenous** iff every state in α is homogenous with respect to α

Let s be a state and α a stative event type and $s \in \alpha$:

s **is homogenous with respect to** α iff for every world $w \in W$: if $\tau(s,w) \neq \perp$ then:
$\forall i \subseteq \tau(s,w)$: $\exists s' \in \alpha$: $s' \sim s$ and $\tau(s',w)=i$

The set of states α of Fred being in Amsterdam is homogenous, because every state in it is (postulated to be) homogenous with respect to α: if s is such a state that goes on in this world, then a state of Fred being in Amsterdam, cross-temporally identical to s goes on at every subinterval of the running time of s, **including the points in that interval**.
States go on at points.

Incremental homogeneity

Lexical requirement: Activity event types (like the event type of *eat*) are homogenous:

eventive event type α is **homogenous** iff every event in α is homogenous **with respect to α and V_α**

Let α be an event type and let $e \in \alpha$
Let V_α be the corresponding verbal event type and let $O(e,V_\alpha) \in V_\alpha$, the onset of e.

e **is homogenous with respect to** α and V_α iff for every $w \in W$: if $\tau(s,w) \neq \perp$ then:
$\forall i$[if $\tau(O(e,V_\alpha),w) \preccurlyeq i \prec \tau(e,w)$ then $\exists e' \in \alpha$: $e' \sim e$ and $\tau(e',w)=i$

e is homogenous wrt to α and V_α iff the onset of e is in α and for every intitial subinterval i of $\tau(e,w)$, which extends the time of the onset, there is an initial stage e' of e with i as running time such that $e' \in \alpha$.

Fact: Activity event types are homogenous, activity events are incrementally homogenous (for *eat* $\alpha = V_\alpha$).
Accomplishment event types are not homogenous, accomplishment events are not incrementally homogenous.

for an hour: modifier of event types (maps event types onto event types):

for an hour$_w(\alpha) = \lambda e \in \alpha$: $\text{duration}_{hour}(\tau(e,w))=1$

Presuppositional semantics of *for an hour*:

for an hour:

$$\lambda\alpha.\begin{cases}\textbf{for an hour}_{\mathrm{w}}(\alpha) & \text{if } \textbf{for an hour}_{\mathrm{w}}(\alpha) \text{ is (non-trivially) homogenous.} \\ \perp \text{ otherwise} & \end{cases}$$

Landman and Rothstein 2012, part 1 and 2

2.2. VAGUENESS AND COMPARATIVES

Kamp 1975: Two theories about adjectives, in: Keenan (ed) *Semantics for Natural Language*
Fine 1975: Truth, vagueness and logic, in: *Synthese* 30
Kamp 1981: The paradox of the heap, in: Monnich (ed), *Aspects of Philosophical Logic*
Klein 1980: A semantics for positive and comparative adjectives, in: *Linguistics and Philosophy* 4
Kamp and Partee 1995: Prototype theory and compositionality, in: *Cognition* 57
Galit Sassoon 2007: *Vagueness, Gradability and Typicality*, Diss. TAU
Papers by Sassoon on her webpage.
Robert van Rooy, papers on vagueness on his webpage.

Vagueness and many valued logic (discussion in Kamp 1975)

We are interested in borderline vaguenss between φ and ¬φ, so not between blue and red, but between red and not red. In three valued semantics we assume that predicates have a positive extension $[\![red]\!]^+$, the objects that are unproblematically red and $[\![red]\!]^-$, the objects that are unproblematically not-red.
So:

$$[\![red]\!]^+ \cup [\![red]\!]^- \subseteq D$$
$$[\![red]\!]^+ \cap [\![red]\!]^- = \emptyset$$

In three valued semantics with three values 1,0 ⊥ (undefined) there are two options for the connectives: Weak Kleene, in which undefined wins out, and Strong Kleene, in which it undefined is a remainder value.

Weak Kleene

∧	1	0	⊥
1	1	0	⊥
0	0	0	⊥
⊥	⊥	⊥	⊥

Strong Kleene

∧	1	0	⊥
1	1	0	⊥
0	0	0	0
⊥	⊥	0	⊥

Weak Kleene

∨	1	0	⊥
1	1	1	⊥
0	1	0	⊥
⊥	⊥	⊥	⊥

Strong Kleene

∨	1	0	⊥
1	1	1	1
0	1	0	⊥
⊥	1	⊥	⊥

Weak Kleene

→	1	0	⊥
1	1	0	⊥
0	1	1	⊥
⊥	⊥	⊥	⊥

Strong Kleene

→	1	0	⊥
1	1	0	⊥
0	1	1	1
⊥	1	⊥	⊥

Weak Kleene

Strong Kleene

$\neg$	1	0	$\perp$
	0	1	$\perp$

$\neg$	1	0	$\perp$
	0	1	$\perp$

Weak Kleene is used for sortal incorrectness: the idea being that the conjunction or disjunction of φ with a sortally incorrect statement is itself sortally incorrect.
(as in: φ and/or *I drank seventeen evil*, (where *evil* is an abstract mass noun).
Strong Kleeine is used for presuppositions.

Consequence:

If x is borderline *bald* then all of the following are undefined.

$\mathrm{BALD}(x) \wedge \neg\mathrm{BALD}(x)$
$\mathrm{BALD}(x) \vee \neg\mathrm{BALD}(x)$
$\mathrm{BALD}(x) \rightarrow \mathrm{BALD}(x)$

And if Buck and Chuck are both borderline bald, then the following is also undefined, even if Chuck is a bit more hairy than Buck:

$\mathrm{BALD}(\mathrm{Buck}) \rightarrow \mathrm{BALD}(\mathrm{Chuck})$

This is unsatisfactory.

Is *Fuzzy Logic* a solution?

Fuzzy Logic:

$[\![\varphi]\!] \in [0,1]$ (the real interval [0,1])
$[\![\varphi]\!] = 1 - [\![\varphi]\!]$
$[\![\varphi \wedge \psi]\!] = \min[[\![\varphi]\!],[\![\psi]\!]]$
$[\![\varphi \vee \psi]\!] = \max[[\![\varphi]\!],[\![\psi]\!]]$
Suggestion:
$[\![\varphi \rightarrow \psi]\!] = 1$ iff $[\![\varphi]\!] \leq [\![\psi]\!]$

If Buck is exactly on the border of bald and not bald, this gives for the tautologies and contradictions:

$[\![\mathrm{BALD}(x) \vee \neg\mathrm{BALD}(x)]\!] = 1/2$
$[\![\mathrm{BALD}(x) \wedge \neg\mathrm{BALD}(x)]\!] = 1/2$

Which is unsatisfactory.

For condionals look at:
$[\![\mathrm{BALD}(\mathrm{Buck})]\!] = 1/2$
$[\![\mathrm{SMART}(\mathrm{Chuck})]\!] = 2/3$
hence:

$[\![\mathrm{BALD}(\mathrm{Buck}) \rightarrow \mathrm{SMART}(\mathrm{Chuck})]\!] = 1$

Which is, obviously crazy. But if we define $\rightarrow$ in terms of $\neg$ and $\vee$ in the usual way, we get that

BALD(Buck) $\rightarrow$ BALD(Chuck)

doesn't come out as true, even if Chuck is more hairy than Buck.

Kamp's diagnosis: Three valued logic and Fuzzy Logic fail to deal with the compositional Boolean structure of the connectives.
Solution: Supervaluations (van Fraasen 1968).
Main idea: borderline vagueness in conditionals is similar to modality in conditionals.

Intuition: vagueness and precisification

φ is vague if there are still different different ways of making φ precise.

Buck is bald is true in s iff for every way w of making s precise *Buck is bald* is true
Buck is bald is false in s iff for every way w of making s precise *Buck is bald* is false

A **vagueness frame** is a structure **S** = <S, $\sqsubseteq$,W, D> where

1. S is a set of **possible standards of precision**
2. $\sqsubseteq$ is a relation of **sharpening of standards**
3. W, the set of possible worlds, is the set of all maximal elements in **S**: W = **max**(S)
4. D is a non-empty domain of individuals.
5. **Completeability:** for every $s \in S$ there is a $w \in W$: $s \sqsubseteq w$

$s_1 \sqsubseteq s_2$ means: s_2 is a sharpening of standard of s_1, s_1 is a relaxation of standard of s_2
Possible worlds are identified with possible totally sharp standards of precisition.
Every partial standard s can be sharpened to one or more totally sharp standards.

It will be useful to have two sets of operators in the language $\Box^{\uparrow}$, $\Diamond^{\uparrow}$ (quantifying over sharpenings) and $\Box^{\downarrow}$, $\Diamond^{\downarrow}$ (quantifying over relaxations).
With Kamp 1975 we will for simplicity of presentation assume a language L_V of predicate logic with only variables (i.e. you can add individual constants yourself) and the above operators.

A **vagueness model** is a structure **M** = <**S**, F^-, F^+ > where **S** is a vagueness frame and
F^+ and F^- are functions from standards and lexical items to interpretations, with the following constraints:
For every n-place predicate P and $s, s_1, s_2 \in S$, $w \in W$:

1. $F_s^-(P) \subseteq D^n$ and $F_s^+(P) \subseteq D^n$
2. $F_s^-(P) \cap F_s^+(P) = \emptyset$ (Non-overlap)
3. if $s_1 \sqsubseteq s_2$ then $F_{s1}^-(P) \subseteq F_{s2}^-(P)$
   and $F_{s1}^+(P) \subseteq F_{s2}^+(P)$ (Monotonicity)
4. $F_w^-(P) \cup F_w^+(P) = D^n$ (Totality)

We call $F_s^-(P)$ the **negative extension of** P in s and $F_s^+(P)$ the **positive extension of** P in s and define: $F_s^{\perp}(P) = D^n - (F_s^-(P) \cup F_s^+(P))$, the **gap of** P in s.

For one-place predicate P:

-$F_s^+(P)$ is the set of individuals that, according to standard of precisition s, definitely count as P

-$F_s^-(P)$ as the set of individuals that, according to standard of precisition s, definitely count as not-P

- $F_s^\perp(P)$ is the set of individuals that, according to standard of precisition s are borderline between P and not-P.

Note that the models I discuss here do not deal with **higher-order vagueness:** we only have three options: $-$, + or $\perp$, but the border between $\perp$ and + is not treated as vague.

The semantic constraints tell us:

1. Negative and positive extensions allow for gaps: we do not require in general that $F_s^-(P) \cup F_s^+(P) = D^n$, for $s \in S$. Hence it is possible that $F_s^\perp(P) \neq Ø$.
2. Negative and positive extensions are **consistent:** we do not allow $F_s^-(P)$ and $F_s^+(P)$ to overlap, for any $s \in S$.
3. Negative and positive extensions are **monotonic**: sharpening the standard of precision is interpreted as sharpening the standard of precisition for $F_s^\perp(P)$: from s_1 to s_2 elements may move from $F_{s1}^\perp(P)$ into $F_{s2}^-(P)$ or $F_{s2}^+(P)$.
4. Extensions are total in worlds.

There is, of course, another sense of sharpening standards of precisition whereby objects that at first counted as definite P's become, on a more precise criterium, borderline, or even non-P. This notion too can be studied in this framework.

Here you may want to introduce relation $\preccurlyeq_P^+$ (sharpening the criterium for P^+ where $s_1 \preccurlyeq_P^+ s_2$ entails that $F_{s2}^+(P) \subseteq F_{s1}^+(P)$ and $F_{s1}^-(P) \subseteq F_{s2}^-(P)$ (i.e. in this sense of sharpening, restricting the membership of P relaxes the membership of not-P).

With respect to the relation $\sqsubseteq$, worlds that stand in the $\preccurlyeq$-relation move not simply down in the relation $\sqsubseteq$, but down with respect to P^+ but up with respect to P^-.

See Galit Sassoon's dissertation and later work for much pertinent discussion, and a framework in which criteriums of sharpening are made explicit in the logical theory.

We will assume a semantics that is not innovative with respect to identity and quantification: quantification is over possible objects, identity is a total relation; Semantic complexities are hidden in an existence predicate which I will not discuss.

The semantics defines two relations simultaneously:

$$[\![\varphi]\!]_{M,s,g} = 1 \text{ and } [\![\varphi]\!]_{M,s,g} = 0$$

1. $[\![P(x_1,...,x_n)]\!]_{M,s,g} = 1$ iff $<g(x_1),\ldots,g(x_n)> \in F_s^+(P)$
   $[\![P(x_1,...,x_n)]\!]_{M,s,g} = 0$ iff $<g(x_1),\ldots,g(x_n)> \in F_s^-(P)$
2. $[\![(x_1 = x_2)]\!]_{M,s,g} = 1$ iff $g(x_1) = g(x_2)$
   $[\![(x_1 = x_2)]\!]_{M,s,g} = 0$ iff $g(x_1) \neq g(x_2)$
3. $[\![\neg\varphi]\!]_{M,s,g} = 1$ iff $[\![\varphi]\!]_{M,s,g} = 0$
   $[\![\neg\varphi]\!]_{M,s,g} = 0$ iff $[\![\varphi]\!]_{M,s,g} = 1$
4. $[\![(\varphi \wedge \psi)]\!]_{M,s,g} = 1$ iff $[\![\varphi]\!]_{M,s,g} = 1$ and $[\![\psi]\!]_{M,s,g} = 1$
   $[\![(\varphi \wedge \psi)]\!]_{M,s,g} = 0$ iff $[\![\varphi]\!]_{M,s,g} = 0$ or $[\![\psi]\!]_{M,s,g} = 0$
5. $[\![(\varphi \vee \psi)]\!]_{M,s,g} = 1$ iff $[\![\varphi]\!]_{M,s,g} = 1$ or $[\![\psi]\!]_{M,s,g} = 1$
   $[\![(\varphi \vee \psi)]\!]_{M,s,g} = 0$ iff $[\![\varphi]\!]_{M,s,g} = 0$ and $[\![\psi]\!]_{M,s,g} = 0$
6. $[\![(\varphi \rightarrow \psi)]\!]_{M,s,g} = 1$ iff $[\![\varphi]\!]_{M,s,g} = 0$ or $[\![\psi]\!]_{M,s,g} = 1$
   $[\![(\varphi \rightarrow \psi)]\!]_{M,s,g} = 0$ iff $[\![\varphi]\!]_{M,s,g} = 1$ and $[\![\psi]\!]_{M,s,g} = 0$
7. $[\![\forall x\varphi]\!]_{M,s,g} = 1$ iff for every $d \in D$: $[\![\varphi]\!]_{M,s,g_x^d} = 1$
   $[\![\forall x\varphi]\!]_{M,s,g} = 0$ iff for some $d \in D$: $[\![\varphi]\!]_{M,s,g_x^d} = 0$
8. $[\![\exists x\varphi]\!]_{M,s,g} = 1$ iff for some $d \in D$: $[\![\varphi]\!]_{M,s,g_x^d} = 1$
   $[\![\exists x\varphi]\!]_{M,s,g} = 0$ iff for every $d \in D$: $[\![\varphi]\!]_{M,s,g_x^d} = 0$
9. $[\![\perp\varphi]\!]_{M,s,g} = 1$ iff $[\![\varphi]\!]_{M,s,g} \neq 1$ and $[\![\varphi]\!]_{M,s,g} \neq 0$
   $[\![\perp\varphi]\!]_{M,s,g} = 0$ iff $[\![\varphi]\!]_{M,s,g} = 1$ or $[\![\varphi]\!]_{M,s,g} = 0$

10. $[\![\Box^{\uparrow}\varphi]\!]_{M,s,g} = 1$ iff for every $s' \in S$: if $s \sqsubseteq s'$ then $[\![\varphi]\!]_{M,s',g} = 1$
    $[\![\Box^{\uparrow}\varphi]\!]_{M,s,g} = 0$ iff for some $s' \in S$: $s \sqsubseteq s'$ and $[\![\varphi]\!]_{M,s',g} = 1$

$\Box^{\uparrow}\varphi$ is true in s iff φ is true in every sharpening of s

11. $[\![\blacksquare^{\uparrow}\varphi]\!]_{M,s,g} = 1$ iff for no $s' \in S$: $s \sqsubseteq s'$ and $[\![\varphi]\!]_{M,s',g} = 0$
    $[\![\blacksquare^{\uparrow}\varphi]\!]_{M,s,g} = 0$ iff for some $s' \in S$: $s \sqsubseteq s'$ and $[\![\varphi]\!]_{M,s',g} = 0$

$\blacksquare^{\uparrow}\varphi$ is true if s iff φ is false in no sharpening of s

12. $[\![\Box^{\downarrow}\varphi]\!]_{M,s,g} = 1$ iff for every $s' \in S$: if $s' \sqsubseteq s$ then $[\![\varphi]\!]_{M,s',g} = 1$
    $[\![\Box^{\downarrow}\varphi]\!]_{M,s,g} = 0$ iff for some $s' \in S$: $s' \sqsubseteq s$ and $[\![\varphi]\!]_{M,s',g} = 1$

$\Box^{\downarrow}\varphi$ is true in s iff φ is true in every relaxation of s

13. $[\![\blacksquare^{\downarrow}\varphi]\!]_{M,s,g} = 1$ iff for no $s' \in S$: $s' \sqsubseteq s$ and $[\![\varphi]\!]_{M,s',g} = 0$
    $[\![\blacksquare^{\downarrow}\varphi]\!]_{M,s,g} = 0$ iff for some $s' \in S$: $s' \sqsubseteq s$ and $[\![\varphi]\!]_{M,s',g} = 0$

$\blacksquare^{\downarrow}\varphi$ is true if s iff φ is false in no relaxation of s

$\blacksquare^{\uparrow}\varphi$ expresses, in the strong Kleene three-valued semantics that we set up here, that φ eventually becomes true (φ is true in every world w such that $s \in w$)

Of course, with this we can introduce conditionals like:

$\varphi \Rightarrow^{\uparrow} \psi =_{df} \square^{\uparrow}(\varphi \rightarrow \psi)$ in every sharpening where φ is true, ψ is true as well
$\varphi \Rightarrow^{\uparrow} \psi =_{df} \neg\Diamond^{\uparrow}(\varphi \wedge \neg\psi)$ in no sharpening φ is true and ψ false.

$[\![\varphi \rightarrowtail^{\uparrow} \psi]\!]_{M,s,g} = 1$ iff for every $s_1 \in S$:
if $s \sqsubseteq s_1$ and $[\![\varphi]\!]_{M,s_1,g} = 1$ and for all s_2: if $s \sqsubseteq s_2 \sqsubseteq s_1$ and $[\![\varphi]\!]_{M,s_2,g} = 1$ then $s_2 = s_1$
then $[\![\psi]\!]_{M,s_1,g} = 1$

As soon as you sharpen s to make φ true, ψ becomes true as well
(i.e. ψ is true in every sharpening of s in which φ first becomes true)

And also conditional relaxation relations:

$\varphi \Rightarrow^{\downarrow} \psi =_{df} \square^{\downarrow}(\varphi \rightarrow \psi)$ in every relaxation where φ is true, ψ is true as well
$\varphi \Rightarrow^{\downarrow} \psi =_{df} \neg\Diamond^{\downarrow}(\varphi \wedge \neg\psi)$ in no relaxation φ is true and ψ false.

$[\![\varphi \rightarrowtail^{\downarrow} \psi]\!]_{M,s,g} = 1$ iff for every $s_1 \in S$:
if $s_1 \sqsubseteq s$ and $[\![\varphi]\!]_{M,s_1,g} = 1$ and for all s_2: if $s_1 \sqsubseteq s_2 \sqsubseteq s$ and $[\![\varphi]\!]_{M,s_2,g} = 1$ then $s_2 = s_1$
then $[\![\psi]\!]_{M,s_1,g} = 1$

As soon as you relax s to make φ true , ψ is true as well:
(i.e. ψ is true in every relaxation of s in which φ first becomes true.)

This makes it possible to have formulas of the form:

$$\bot(P(x_1) \wedge \bot(P(x_2) \wedge (P(x_1) \rightarrowtail^{\uparrow} P(x_2)) \wedge (P(x_2) \rightarrowtail^{\uparrow} P(x_1))$$

$P(x_1)$ and $P(x_2)$ are undefined on s, but as soon as you make one of them a P, the other becomes a P as well.

$$P(x_1) \wedge P(x_2) \wedge (\bot(P(x_1)) \rightarrowtail^{\downarrow} \bot(P(x_2))) \wedge (\bot(P(x_2)) \rightarrowtail^{\downarrow} \bot(P(x_1)))$$

Both x_1 and x_2 are P in s, but if you relax s to make one of them undefined, the other becomes undefined as well.

We gave the following definition:

11. $[\![\blacksquare^{\uparrow}\varphi]\!]_{M,s,g} = 1$ iff for no $s' \in S$: $s \sqsubseteq s'$ and $[\![\varphi]\!]_{M,s',g} = 0$
$[\![\blacksquare^{\uparrow}\varphi]\!]_{M,s,g} = 0$ iff for some $s' \in S$: $s \sqsubseteq s'$ and $[\![\varphi]\!]_{M,s',g} = 0$

$\blacksquare^{\uparrow}\varphi$ is true if s iff φ is false in no sharpening of s

An alternative definition is:

11'. $[\![\blacksquare^{\uparrow}\varphi]\!]_{M,s,g} = 1$ iff for every $s' \in S$: $s \sqsubseteq s'$ then there is an $s'' \in S$: $s' \sqsubseteq s''$ and $[\![\varphi]\!]_{M,s',g} = 1$
$[\![\blacksquare^{\uparrow}\varphi]\!]_{M,s,g} = 0$ iff for some $s' \in S$: $s \sqsubseteq s'$ and for every $s'' \in S$: $s' \sqsubseteq s''$ $[\![\varphi]\!]_{M,s',g} = 0$

$\blacksquare^{\uparrow}\varphi$ is true if s iff every sharpening of s has a sharpening where φ is true.

We define:

φ is **monotonic** iff $\forall s \in S$: if $[\![\varphi]\!]_s = 1$ and $s \sqsubseteq s'$ then $[\![\varphi]\!]_{s'} = 1$ and
if $[\![\varphi]\!]_s = 0$ and $s \sqsubseteq s'$ then $[\![\varphi]\!]_{s'} = 0$ and

Claim: if φ is monotonic then:
$[\![\blacksquare^{\uparrow}\varphi]\!]_{M,s,g} =_{[\text{definition 11}]} 1$ iff $[\![\blacksquare^{\uparrow}\varphi]\!]_{M,s,g} =_{[\text{definition 11'}]} 1$

Alternative definitions:

$[\![\blacksquare^{\uparrow 1}\varphi]\!]_{M,s,g} = 1$ iff for no $s' \in S$: $s \sqsubseteq s'$ and $[\![\varphi]\!]_{M,s',g} = 0$
$[\![\blacksquare^{\uparrow 1}\varphi]\!]_{M,s,g} = 0$ iff for some $s' \in S$: $s \sqsubseteq s'$ and $[\![\varphi]\!]_{M,s',g} = 0$

$[\![\blacksquare^{\uparrow 2}\varphi]\!]_{M,s,g} = 1$ iff for every $s' \in S$: $s \sqsubseteq s'$ then there is an $s'' \in S$: $s' \sqsubseteq s''$ and $[\![\varphi]\!]_{M,s',g} = 1$
$[\![\blacksquare^{\uparrow 2}\varphi]\!]_{M,s,g} = 0$ iff for some $s' \in S$: $s \sqsubseteq s'$ and for every $s'' \in S$: $s' \sqsubseteq s''$ $[\![\varphi]\!]_{M,s',g} = 0$

$[\![\blacksquare^{\uparrow 3}\varphi]\!]_{M,s,g} = 1$ iff for every $w \in W$: if $s \sqsubseteq w$ then $[\![\varphi]\!]_{M,w,g} = 1$
$[\![\blacksquare^{\uparrow 3}\varphi]\!]_{M,s,g} = 0$ iff for some $w \in W$: $s \sqsubseteq w$ and $[\![\varphi]\!]_{M,w,g} = 0$

For monotonic sentences, these definitions don't make a difference, but for non-monotonic sentences they do. For instance, φ can be false for a while on for every precisification branch, but always becoming true at some point. In that case $\blacksquare^{\uparrow 1}\varphi$ would be false, but $\blacksquare^{\uparrow 3}\varphi$ would be true.
Discussion in the 1985 dissertation by Frank Veltman and in my own 1986 dissertation.

Truth-value gaps

Let $d \in F_s^{\perp}(P)$
Then $[\![P(x)]\!]_{M,s,g_x^d} \neq 1$ and $[\![P(x)]\!]_{M,s,g_x^d} \neq 0$, hence $[\![\neg P(x)]\!]_{M,s,g_x^d} \neq 1$.
Hence $[\![P(x) \wedge \neg P(x)]\!]_{M,s,g_x^d} \neq 0$ and $[\![P(x) \vee \neg P(x)]\!]_{M,s,g_x^d} \neq 1$

It straightforwardly follows from this that:
Hence $[\![\square^{\uparrow}(P(x) \wedge \neg P(x))]\!]_{M,s,g_x^d} \neq 0$ and $[\![\square^{\uparrow}(P(x) \vee \neg P(x))]\!]_{M,s,g_x^d} \neq 1$

But $[\![\blacksquare^{\uparrow}(P(x) \wedge \neg P(x))]\!]_{M,s,g_x^d} = 0$ and $[\![\blacksquare^{\uparrow}(P(x) \vee \neg P(x))]\!]_{M,s,g_x^d} = 1$

We define:

φ is **supertrue** relative to M,s,g iff $[\![\blacksquare^{\uparrow}(\varphi)]\!]_{M,s,g} = 1$
φ is **superfalse** relative to M,s,g iff $[\![\blacksquare^{\uparrow}(\neg\varphi)]\!]_{M,s,g} = 1$

You can, if you want, maintain classical logic for vagueness by defining entailment in terms of supertruth. See Fine, Kamp, van Fraassen. In fact, the logic of the *worlds* determines the super logic.

Of course, we can also define:

P is **vague relative to** M,s,g iff for some d ∈ D: $[\![\blacklozenge^{\uparrow}(P(x)) \wedge \blacklozenge^{\uparrow}(P(x))]\!]_{M,s,g^{d}_{x}} = 1$
A predicate P is vague in s if there **are** bordeline objects for P in s that can still end up either way, in P or in not-P.

Kamp and Partee 1995 discuss the interaction between vagueness and conditionals.

(9) a. A boy is a male child.
b. A man is a male adult.

The meaning definitions in (9) impose constraints on our modals:

For every M, s, d:
$d \in F_s^{+}(\textit{boy})$ iff $d \in F_s^{+}(\textit{male})$ and $d \in F_s^{+}(\textit{child})$
$d \in F_s^{-}(\textit{boy})$ iff $d \in F_s^{-}(\textit{male})$ or $d \in F_s^{-}(\textit{child})$

$d \in F_s^{+}(\textit{man})$ iff $d \in F_s^{+}(\textit{male})$ and $d \in F_s^{+}(\textit{adult})$
$d \in F_s^{-}(\textit{man})$ iff $d \in F_s^{-}(\textit{male})$ or $d \in F_s^{-}(\textit{adult})$

We are interested in the following sentences:

(10) a. Bob is male.
b. If Bob is a child, then Bob is a boy.
c. If Bob is an adult, then Bob is a man.

Intuitively, given the meaning postulates, the above conditionals are true in any state s where bob $\in F_s^{+}(\textit{male})$:

(11) a. $\textit{child}(\text{bob}) \rightarrow \textit{boy}(\text{bob})$
b. $\textit{adult}(\text{bob}) \rightarrow \textit{man}(\text{bob})$

We do not get that result with material implication:

If bob $\in F_s^{\perp}(\textit{child})$ and bob $\in F_s^{\perp}(\textit{adult})$, then bob $\in F_s^{\perp}(\textit{boy})$ and bob $\in F_s^{\perp}(\textit{man})$
$[\![\neg \textit{child}(\text{bob}) \vee \textit{boy}(\text{bob})]\!]_{M,s,g} \neq 1$ and $[\![\neg \textit{adult}(\text{bob}) \vee \textit{man}(\text{bob})]\!]_{M,s,g} \neq 1$

But the formulas in (12a) **are** true on any model M and state s, where bob $\in F_s^{+}(\textit{male})$, given the meaning postulates in (9):

(12) a. $\square^{\uparrow}(\textit{child}(\text{bob}) \rightarrow \textit{boy}(\text{bob}))$
b. $\square^{\uparrow}(\textit{adult}(\text{bob}) \rightarrow \textit{man}(\text{bob}))$

(12a) is true in s iff for every s’: if $s \sqsubseteq s'$ and bob $\in F_{s'}^{+}(\textit{child})$ then bob $\in F_{s'}^{+}(\textit{man})$.
Let bob $\in F_s^{+}(\textit{male})$, and let $s \sqsubseteq s'$ and bob $\in F_{s'}^{+}(\textit{child})$.
Since bob $\in F_s^{+}(\textit{male})$, and $s \sqsubseteq s'$, bob $\in F_{s'}^{+}(\textit{male})$ (by monotonicity).
Hence, by the meaning postulate, bob $\in F_{s'}^{+}(\textit{boy})$.
(12b) is true in s by a similar argument.

Note that this, obviously means that also:

(12) a. $\blacksquare^{\uparrow}(\mathit{child}(\text{bob}) \rightarrow \mathit{boy}(\text{bob}))$
b. $\blacksquare^{\uparrow}(\mathit{adult}(\text{bob}) \rightarrow \mathit{man}(\text{bob}))$

because $\square^{\uparrow}\varphi \rightarrow \blacksquare^{\uparrow}\varphi$ holds.

Comparatives.

Let us now define some useful relations.
Let P be a one-place predicate.

$[\![x_1 \prec_P x_2]\!]_{M,s,g} = 1$ iff $g(x_1) \in F_s^{-}(P)$ and $g(x_2) \notin F_s^{-}(P)$; 0 otherwise
x_1 is in not-P, while x_2 is not

$[\![x_1 \succ_P x_2]\!]_{M,s,g} = 1$ iff $g(x_1) \in F_s^{+}(P)$ and $g(x_2) \notin F_s^{+}(P)$; 0 otherwise
x_1 is in P, while x_2 is not

$[\![x_1 -_P x_2]\!]_{M,s,g} = 1$ iff $g(x_1) \in F_s^{-}(P)$ and $g(x_2) \in F_s^{-}(P)$; 0 otherwise
x_1 and x_2 are both in not-P

$[\![x_1 +_P x_2]\!]_{M,s,g} = 1$ iff $g(x_1) \in F_s^{+}(P)$ and $g(x_2) \in F_s^{+}(P)$; 0 otherwise
x_1 and x_2 are both in P

$[\![x_1 \perp_P x_2]\!]_{M,s,g} = 1$ iff $g(x_1) \in F_s^{\perp}(P)$ and $g(x_2) \in F_s^{\perp}(P)$; 0 otherwise
x_1 and x_2 are both in the gap of P

With this we can define:

$x_1 \geq_P x_2$ **x_1 is at least as P as x_2** $=_{df}$
$[x_2 \prec_P x_1] \vee [x_1 \succ_P x_2] \vee [(x_1 \perp_P x_2) \wedge \square^{\uparrow}(P(x_2) \rightarrow P(x_1)) \wedge \square^{\uparrow}(\neg P(x_1) \rightarrow \neg P(x_2))] \vee$
$[(x_1 -_P x_2) \wedge \square^{\downarrow}((\perp(P(x_2) \rightarrow \perp(P(x_1))] \vee [(x_1 +_P x_2) \wedge \square^{\downarrow}((\perp(P(x_1) \rightarrow \perp(P(x_2))]$

x_1 is at least as P as x_2 is true in s iff **one** of the following situations obtains:
1. $[x_2 \prec_P x_1]$
   In s, x_2 is not-P and x_1 isn't
2. $[x_1 \succ_P x_2]$
   In s, x_1 is P and x_2 isn't
3. $[(x_1 \perp_P x_2) \wedge \square^{\uparrow}(P(x_2) \rightarrow P(x_1)) \wedge \square^{\uparrow}(\neg P(x_1) \rightarrow \neg P(x_2))]$
   In s, x_1 and x_2 are in the gap, but in every sharpening where x_2 is added to P, x_1 is already in P and where x_1 is added to not-P, x_2 is already in not-P
4. $[(x_1 -_P x_2) \wedge \square^{\downarrow}((\perp(P(x_2) \rightarrow \perp(P(x_1))]$
   In s, x_1 and x_2 are not-P, but in every relaxation where x_2 is removed from P, x_1 was already removed.
5. $[(x_1 +_P x_2) \wedge \square^{\downarrow}((\perp(P(x_1) \rightarrow \perp(P(x_2))]$
   In s, x_1 and x_2 are P, but in every relaxation where x_1 is removed from P, x_2 was already removed.

$x_1 >_P x_2$ **x_1 is more P than x_2** $=_{df}$

$[x_2 \prec_P x_1] \vee [x_1 \succ_P x_2] \vee$
$[(x_1 \perp_P x_2) \wedge [\Box^{\uparrow}(P(x_2) \rightarrow P(x_1)) \wedge \Box^{\uparrow}(\neg P(x_1) \rightarrow \neg P(x_2)) \wedge$
$(\Diamond^{\uparrow}(x_1 \succ_P x_2)] \vee \Diamond^{\uparrow}(x_2 \prec_P x_1))] \vee$
$[(x_1 -_P x_2) \wedge \Box^{\downarrow}((\perp(P(x_2) \rightarrow \perp(P(x_1)) \wedge \Diamond^{\downarrow}(x_2 \prec_P x_1)] \vee$
$[(x_1 +_P x_2) \wedge \Box^{\downarrow}((\perp(P(x_1) \rightarrow \perp(P(x_2))] \wedge \Diamond^{\downarrow}(x_1 \succ_P x_2)]$

x_1 is more P than x_2 is true in s iff **one** of the following situations obtains:

1. $[x_2 \prec_P x_1]$
   In s, x_2 is not-P and x_1 isn't
2. $[x_1 \succ_P x_2]$
   In s, x_1 is P and x_2 isn't
3. $[(x_1 \perp_P x_2) \wedge [\Box^{\uparrow}(P(x_2) \rightarrow P(x_1)) \wedge \Box^{\uparrow}(\neg P(x_1) \rightarrow \neg P(x_2)) \wedge (\Diamond^{\uparrow}(x_1 \succ_P x_2)] \vee \Diamond^{\uparrow}(x_2 \prec_P x_1))]$
   In s, x_1 and x_2 are in the gap, but in every sharpening where x_2 is added to P, x_1 is already in P and in every sharpening where x_1 is added to not-P, x_2 is already in not-P, **and in some sharpening x_1 is P and x_2 isn't or in some sharpeing x_2 is in not-P and x_1 isn't.**
4. $[(x_1 -_P x_2) \wedge \Box^{\downarrow}((\perp(P(x_2) \rightarrow \perp(P(x_1)) \wedge \Diamond^{\downarrow}(x_2 \prec_P x_1)]$
   In s, x_1 and x_2 are not-P, but in every relaxation where x_2 is removed from P, x_1 was already removed, **and in some relaxation x_2 is not-P and x_1 isn't.**
5. $[(x_1 +_P x_2) \wedge \Box^{\downarrow}((\perp(P(x_1) \rightarrow \perp(P(x_2))] \wedge \Diamond^{\downarrow}(x_1 \succ_P x_2)]$
   In s, x_1 and x_2 are P, but in every relaxation where x_1 is removed from P, x_2 was already removed, **and in some relaxation x_1 is P and x_2 isn't.**

Fact: $\geq_P$ is a pre-order

We define the equivalence relation $\approx_P$, **is equally P as**:

$$x_1 \approx_P x_2 \text{ iff } x_1 \geq_P x_2 \text{ and } x_2 \geq_P x_2$$

We define:

P is a **linear degree predicate** iff $\geq_P$ is $\approx_P$-connected

i.e.: P is a linear degree predicate iff for all x_1, x_2: $(x_1 <_P x_2)$ or $(x_2 <_P x_1)$ or $(x_1 \approx_P x_2)$

We have shown in chapter 1 that for linear degree predicate P the equivalence classes under $\approx_P$ form a linear order, **a scale**.

Gradable adjectives that express degrees along a fixed contextual mixture of dimensions are typically linear degree predicates (if we abstract away from sortal incorrectness):

tall: either x_1 is taller than x_2 or x_2 is taller than x_1 or they are equally tall.

intelligent: either x_1 is more intelligent than x_2 or x_2 is more intelligent than x_1 or x_1 and x_2 are equally intelligent

If we **fix** the mixture of dimensions *intelligent* and keep it constant, arguably *intelligent* is a linear degree predicate. If we **don't** keep it constant, then *is more intelligent than* is not even a partial order, because:

> x_1 is more intelligent than x_2 (wrt criteria $a_1 \ldots a_n$) and
> x_2 is more intelligent than x_1 (wrt criteria $b_1 \ldots b_m$)

The contextual dependency of adjective P on a specified of presupposed *way of being P* is similar to the dependency of modals on specified or presupposed modal bases.

(13) a. Ronya is intelligent and she is not intelligent.
b. Ronya is intelligent in one way and not intelligent in another way.

So linearity is expressed in (14):

(14) For each way of being intelligent, either Minoes is more intelligent than Ronya in a that way or Ronya is more intelligent than Minoes in that way, or they are equally intelligent in that way.

Comparison classes and degree modifiers (following Klein 1980)

We now introduce vagueness models with comparison classes.

A **vagueness model with comparison classes** is a structure $\mathbf{M} = <\mathbf{S}, \mathbf{F}>$ where $\mathbf{S}$ is a vagueness frame and for every n-place predicate P and for every $s \in S$:

$\mathbf{F}_s(P)$ is a triple $\mathbf{F}_s(P) = <F(P), F_{s,F(P)}{}^{-}(P), F_{s,F(P)}{}^{+}(P)>$ where (for $s,s_1,s_2 \in S$, $w \in W$)

1. $F(P) \subseteq D^n$
2. $F_{s,F(P)}{}^{-}(P) \subseteq F(P)$ and $F_{s,F(P)}{}^{+}(P) \subseteq F(P)$
3. $F_{s,F(P)}{}^{-}(P) \cap F_{s,F(P)}{}^{+}(P) = \varnothing$
4. if $s_1 \sqsubseteq s_2$ then $F_{s1,F(P)}{}^{-}(P) \subseteq F_{s2,F(P)}{}^{-}(P)$ and $F_{s1,F(P)}{}^{+}(P) \subseteq F_{s2,F(P)}{}^{+}(P)$
5. $F_{w,F(P)}{}^{-}(P) \cup F_{w,F(P)}{}^{+}(P) = F(P)$

We set:

$[\![P]\!]_{M,s,g} = \mathbf{F}_s(P)$

and:

$[\![P(x)]\!]_{M,s,g} = 1$ iff $g(x) \in {}^{+}[\![P]\!]_{M,s,g}$ iff $\mathbf{F}_s(P)^3$ (which is$F_{s,F(P)}{}^{+}(P)$), etc.

We have now moved away from classical logic even on the set of all worlds:
if $g(x) \notin F(P)$ then $[\![(P(x) \vee \neg P(x))]\!]_{M,w,g} \neq 1$.

We let the interpretation function **F** choose for each predicate P a comparison set F(P). Unlike the structure of standards S, the set of possible comparison sets of P on domain D is only constrained by the meaning of P.
Thus the meaning of *tall* may tell us that interpretation function **F** can only chose comparison sets F(*tall*) which is a set of objects for which it is sortally correct to call them tall, or not tall. But if interpretation function F_M assigns to *tall* comparison set F(*tall*) $\subseteq$ D, then for every non-empty subset Y of F(*tall*), some interpretation function on M: F_M'(*tall*) = Y,

Of course we can assume that predicates P that are not sensitive to comparison sets lexically select interpretation functions F where F(P)= D.

With this we can introduce comparison set restriction in the logical language.
For simplicity we introduce the restriction only for lexical predicates:

If $P, Q \in PRED^1$ then $P{\upharpoonright}Q \in PRED^1$

$[\![P{\upharpoonright}Q]\!]_{m,s,g} = [\![P]\!]_{M,s,g} + [\![Q]\!]_{M,s,g}{}^3$

We define: for one-place predicate P, M,s,g and set $X \subseteq D_M$:

$F_s(P) + X = <F(P) \cap X, F_{s,F(P) \cap X}{}^{-}(P), F_{s,F(P) \cap X}{}^{+}(P)>$.

So if $\mathbf{F}_s(small) = \langle D, F_{s,D}^{-}(small), F_{s,D}^{+}(small) \rangle$
and $\mathbf{F}_s(elephant) = \langle D, F_{s,D}^{-}(elephant), F_{s,D}^{+}(elephant) \rangle$

Then $[\![tall{\upharpoonleft}elephant]\!]_{M,s,g} = \mathbf{F}_{s,D}(small) + F_{s,D}^{+}(elephant)$

Let $F_{s,D}^{+}(elephant) = \text{ELEPHANT}$

Then $[\![small{\upharpoonleft}elephant]\!]_{M,s,g} = \langle \text{ELEPHANT}, F_{s,\text{ELEPHANT}}^{-}(small), F_{s,\text{ELEPHANT}}^{+}(small) \rangle$

This interpretation devides the elephants into the clearly tall ones and the clearly not tall ones.

With this, (11a) entails (11c) but not (11b):

(11)	a. Jumbo is a small elephant	$elephant(\text{jumbo}) \wedge small{\upharpoonleft}elephant(\text{jumbo})$
	b. Jumbo is small	$small{\upharpoonleft}C(\text{jumbo})$
	c. Jumbo is small for an elephant	$small{\upharpoonleft}elephant(\text{jumbo})$

Note the following:

If $\mathbf{F}_s(small) = \langle D, F_{s,D}^{-}(small), F_{s,D}^{+}(small) \rangle$ and $F_{s,D}^{+}(small) = \text{SMALL}$
then:

$[\![small{\upharpoonleft}small]\!]_{M,s,g} = \mathbf{F}_s(small{\upharpoonleft}small)$ where:
$\mathbf{F}_s(small{\upharpoonleft}small) = \langle \text{SMALL}, F_{s,\,\text{SMALL}}^{-}(small), F_{s,\,\text{SMALL}}^{+}(small) \rangle$

If $P \in \text{PRED}^1$ then $very(P) \in \text{PRED}^1$

$[\![very(P)]\!]_{m,s,g} = \mathbf{F}_s(very(P))$ where
$\mathbf{F}_s(very(P)) = \langle F(P), F_{s,F(P)}^{-}(very(P)), F_{s,F(P)}^{+}(very(P)) \rangle$ and:

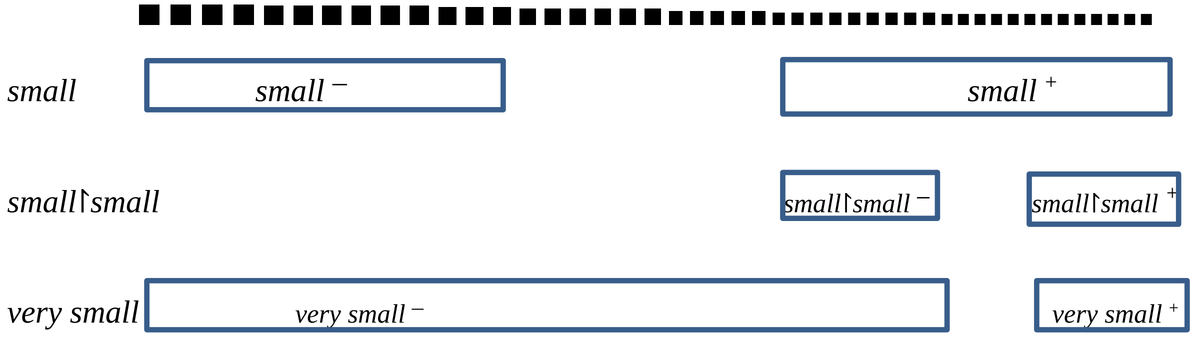
1. $F_{s,F(P)}^{+}(very(P)) = \mathbf{F}_s(P{\upharpoonleft}P)^3$

The positive extension of P↾P)

2. $F_{s,F(P)}^{-}(very(P)) = (\mathbf{F}_s(P)^1 - \mathbf{F}_s(P)^3) \cup \mathbf{F}_s(P{\upharpoonleft}P)^2$

The negative extension and the gap of P together with the negative extension of P↾P

In a picture:



small↾small	'small in the land of the small'
not small↾small	'not-small in the land of the small'
very small	'small in the land of the small'
not very small	'not small, borderline small or not-small in the land of the small'

Of course, for non-gradable predicates like *cat* we will assume that the positive extension of *cat↾cat* is just *cat*, and hence the negative extension is empty. This means that, on this interpretation *very* is not doing anything on *cat*.

Comparison (based on, or inspired by, McConnell-Ginet and Klein)

Instead of defining $[\![(x_1 >_P x_2)]\!]_{M,s,g} = 1$, I will give an 'algorithm' for building $F_{s,F(P)}{}^{+}(>_P)$.

Above I introduced some useful relations in the logical language. Here I introduce them as relations in the models (suppressing the relevant interpretation parameters):

$d_1 \prec_{F_s(P)} d_2$ iff $d_1 \in F_s^{-}(P)$ and $d_2 \in F(P) - F_s^{-}(P)$
$d_1 \succ_{F_s(P)} d_2$ iff $d_1 \in F_s^{+}(P)$ and $d_2 \in F(P) - F_s^{+}(P)$
$d_1 -_{F_s(P)} d_2$ iff $d_1 d_2 \in F_s^{-}(P)$
$d_1 +_{F_s(P)} d_2$ iff $d_1,d_2 \in F_s^{+}(P)$
$d_1 \perp_{F_s(P)} d_2$ iff $d_1,d_2 \in F_s^{\perp}(P)$

Let $\mathbf{M} = \langle \mathbf{S},\mathbf{F} \rangle$ be a model, $s \in S$ and g an assignment, $P \in PRED^1$.

1. $>_P^1 = \{\langle d_1,d_2 \rangle : d_2 \prec_{F_s(P)} d_1 \text{ or } d_1 \succ_{F_s(P)} d_2\}$

1^{+} Let $\boldsymbol{P}^{+} = F_{s,F(P)}{}^{+}$ and $+(\mathbf{F}_s(P)) = < \boldsymbol{P}^{+}, F_{s,\boldsymbol{P}+}{}^{-}(P), F_{s,\boldsymbol{P}+}{}^{+}(P)>$

$>_P{}^{1+} = \{<d_1,d_2>: d_2 \prec_{+(\mathbf{F}s(P))} d_1 \text{ or } d_1 \succ_{+(\mathbf{F}s(P))} d_2\}$

$1^{\perp}$ Let $\boldsymbol{P}^{\perp} = F_{s,F(P)}{}^{\perp}$ and $\perp(\mathbf{F}_s(P)) = < \boldsymbol{P}^{\perp}, F_{s,\boldsymbol{P}}{}^{\perp -}(P), F_{s,\boldsymbol{P}}{}^{\perp +}(P)>$

$>_P{}^{1+} = \{<d_1,d_2>: d_2 \prec_{\perp(\mathbf{F}s(P))} d_1 \text{ or } d_1 \succ_{\perp(\mathbf{F}s(P))} d_2\}$

1^{-} Let $\boldsymbol{P}^{-} = F_{s,F(P)}{}^{-}$ and $-(\mathbf{F}_s(P)) = < \boldsymbol{P}^{-}, F_{s,\boldsymbol{P}}{}^{- -}(P), F_{s,\boldsymbol{P}}{}^{- +}(P)>$

$>_P{}^{1-} = \{<d_1,d_2>: d_2 \prec_{-(\mathbf{F}s(P))} d_1 \text{ or } d_1 \succ_{-(\mathbf{F}s(P))} d_2\}$

Continue the construction for 1^{++}, $1^{+\perp}$, 1^{+-}, $1^{\perp +}$, $1^{\perp\perp}$, $1^{\perp -}$, 1^{-+}, $1^{-\perp}$, 1^{--}, etc.

2. Let $>_P{}^{\cup}$ be the union of all these relations
3. Let $>_P{}^{\cup\, \mathbf{TR}}$ be the transitive closure of that set:

$>_P{}^{\cup\, \mathbf{TR}} = \{<d_1,d_2>: \text{for some } d_3: <d_1,d_3> \in\, >_P{}^{\cup} \text{ and } <d_3,d_2> \in\, >_P{}^{\cup}\}$

4. $F_{s,F(P)}{}^{+}(>_P) = >_P{}^{\cup\, \mathbf{TR}}$

The idea is the following:
1. You start with F_s(*small*) and decide that anybody in the positive extension of *small* according to $\mathbf{F}_s$(*small*) is taller than anybody in F(*small*) not in the positive extension of *small*, and anybody in F(*small*) not in the negative extension of *small* is smaller than anybody in the negative extension of *small*.
2. We are left with pairs of objects that are either both in the positive extension of *small*, or both in the negative extension of *small* or both in F(*small*) but in the gap.
-We look at the interpretation of *small* with the comparison set **reset to each of these three sets**: So we ask:
-Restrict yourself to the people that are small according to s and ask again:
of these people, who is small, who is borderline small and who isn't small.
-Restrict yourself to the people that are borderline small according to s and ask again:
of these people, who is small, who is borderline small and who isn't small.
-Restrict yourself to the people that are not small according to s and ask again:
of these people, who is small, who is borderline small and who isn't small.

The idea is:

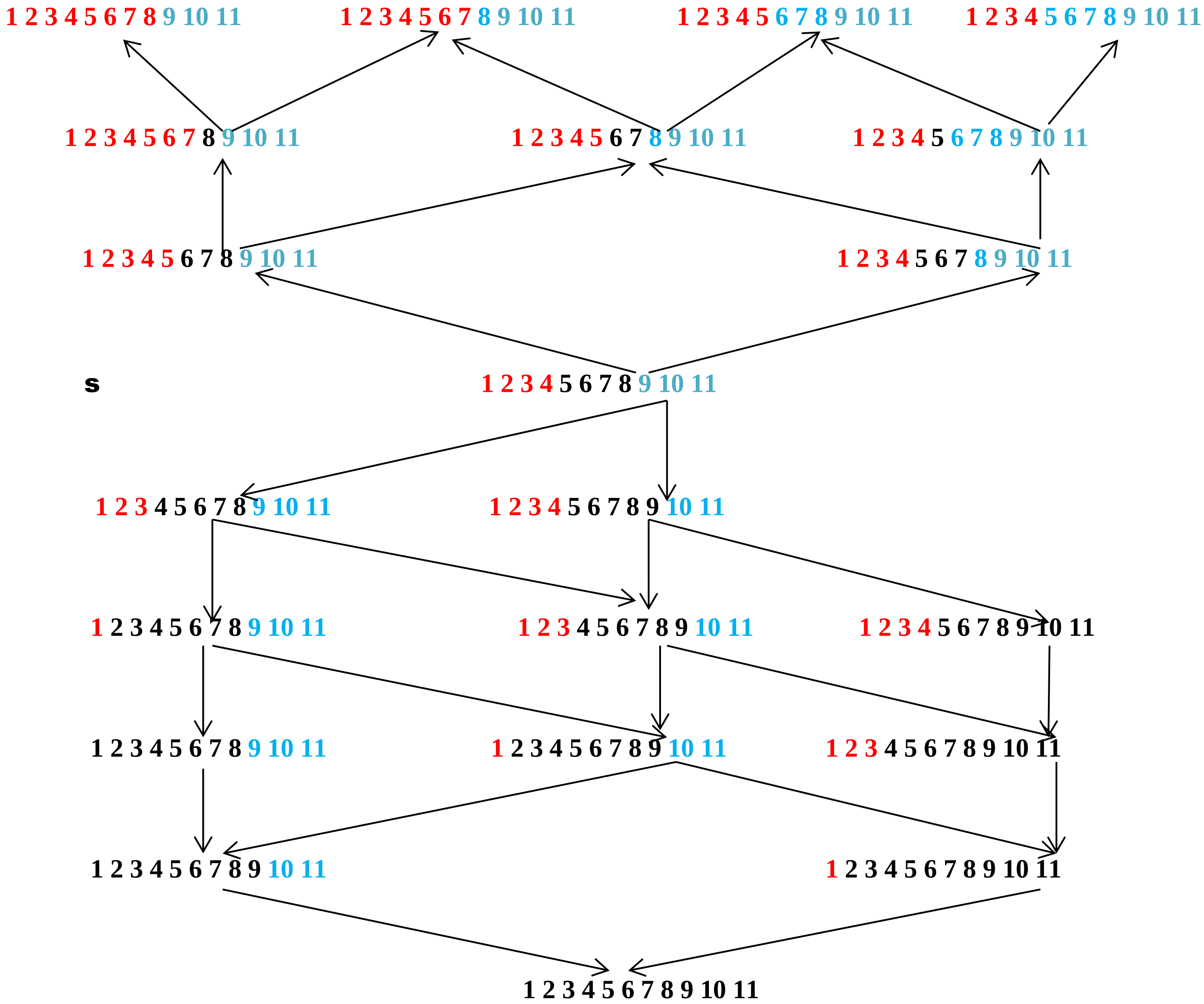
on the postive extension of *small* you reinterpret *small* as: *small for a small person*
on the gap of *small* you reinterpret *small* as: *small for a borderline small person*
on the negative extensions of *small* you reinterpret *small* as: *small for a non-small person*

And you continue on those: on the domain of people that are small for a small person, you reinterpret *small* as *small within the class of people who are small for a small person*, etc.

If we let this procedure define *smaller than in s*, $>_{small,s}$, we can go on to define a less standard dependent definition, by associating with s a set R_s, consisting of s and a chosen set of standards s' such that s' $\sqsubseteq$ s.

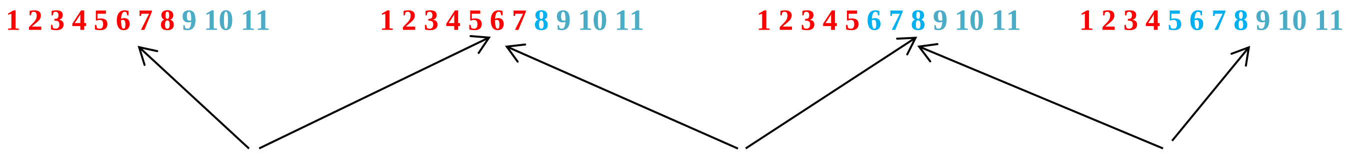
$$>_{P,V_s} = \cup\{ >_{P,s'}: s' \in V_s\}^{\mathbf{TR}}$$

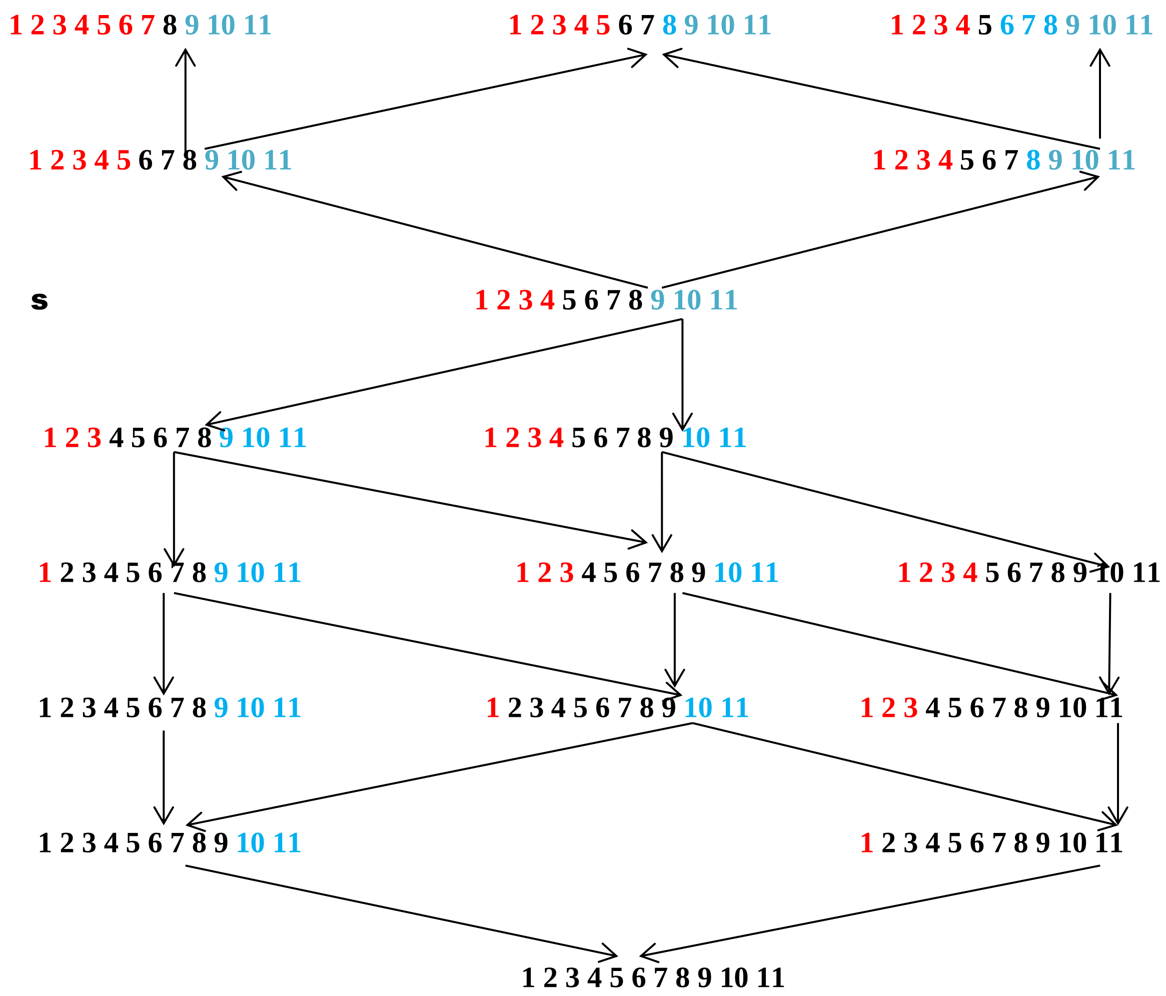
The idea would be that while in standard s two objects d_1 and d_2 may not be distringuishable, even with respect to comparision set $\{d_1,d_2\}$, because the standard s places both without any doubt in the positive extension of P, it may be the case that on a liberalized standard, only one of them is without doubt put in the positive extension of P. In that case, we will want to say that the latter one is more P than the first, even though s doesn't distinguish them.



1. In s: 1234 < 5678 < 9 10 11
2. ↑: 5 < 67 < 8
3. ↓: 1 < 23 < 4 9 < 10 11
hence: 1 < 23 < 4 < 5 < 67 < 8 < 9 < 10 11
2 ~ 3, 6 ~ 7, 10 ~ 11

P





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P is a linear degree predicate iff $\geq_P$ is connected.
Lexical stipulation: scalar adjectives denote lexical degree predicates
(well, relative to a fixed domain, see Kamp 1975)

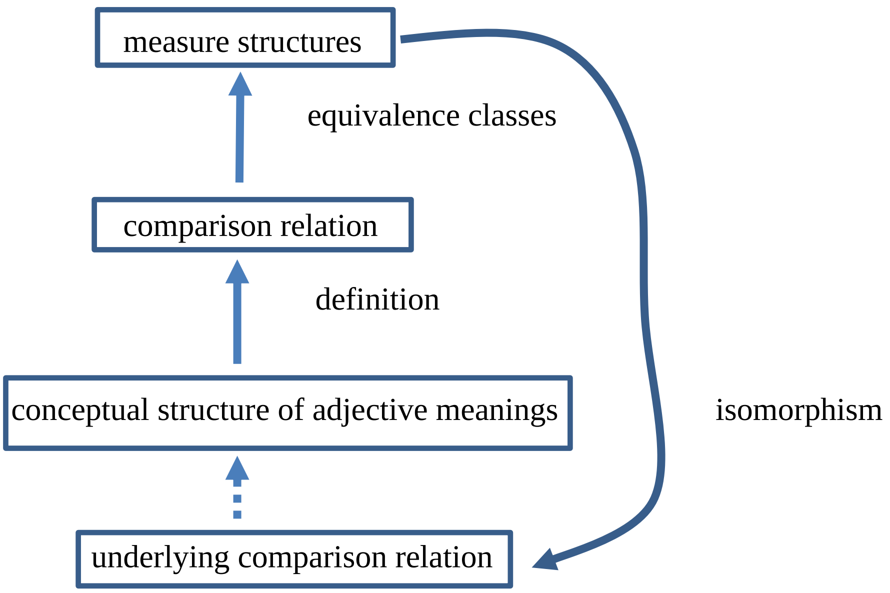
Conceptual Program (initiated by Kamp 1975)

Notions of degrees, scales, and comparison are conceptually derived notions, derived from the basic semantics of the adjectives.

$tall_{\text{adjective}}$ + *er* = *taller*

(Against this von Stechow 1984: $tall_{\text{root}}$ + *er* = *taller*; $tall_{\text{root}}$ + Ø = $tall_{\text{adjective}}$)

Constrain the theory of adjective meanings with conceptually plausible axioms, and prove that the comparison structure thus defined is rich enough to be a useful theory of degrees,



Let us assume an assignment g.
Let $V \subseteq S$ be the set of standards compatible with the world as far as we know it.
Let us assume that for every $s \in V$: $[\![x_1 >_{tall} x_2]\!]_{M,s,g} = 1$

Then we know the following things:

1. for no $s \in V$: $g(x_2) \in F_s^{+}(tall)$ and $g(x_1) \notin F_s^{+}(tall)$
2. for no $s \in V$: $g(x_1)\ F_s^{-}(tall)$ and $g(x_2) \notin F_s^{-}(tall)$
3. for no $s \in V$: $g(x_1) \in F_s^{\perp}(tall)$ and $g(x_2) \in F_s^{\perp}(tall)$ and
   for some s': $s \sqsubseteq s'$ and $g(x_2) \in F_{s'}^{+}(tall)$ and $g(x_1) \notin F_{s'}^{+}(tall)$
4. for no $s \in V$: $g(x_1) \in F_s^{\perp}(tall)$ and $g(x_2) \in F_s^{\perp}(tall)$ and
   for some s': $s \sqsubseteq s'$ and $g(x_1) \in F_{s'}^{-}(tall)$ and $g(x_1) \notin F_{s'}^{-}(tall)$
5. for no $s \in V$: $g(x_1) \in F_s^{-}(tall)$ and $g(x_2) \in F_s^{-}(tall)$ and
   for some s': $s' \sqsubseteq s$ and $g(x_1) \in F_{s'}^{-}(tall)$ and $g(x_2) \notin F_{s'}^{-}(tall)$
6. for no $s \in V$: $g(x_1) \in F_s^{+}(tall)$ and $g(x_2) \in F_s^{+}(tall)$ and
   for some s': $s' \sqsubseteq s$ and $g(x_2) \in F_{s'}^{+}(tall)$ and $g(x_2) \notin F_{s'}^{+}(tall)$

Thus, the truth of x_1 *is taller than* x_2 requires not just that certain standards are not in V, but also that in the relation ⊑ as it relates worlds in V to other worlds, there is a *patterns of gaps, absenses* in comparision to a relation that does not omit any logical possibility.

This is **exactly** what we use accessibility relations for: to encode modal patterns which omit **logical** possibilities, so that we can regard the possibilities and connections between possibilities that we **do** find as a different kind of possibility, and as modal connections of a different nature than logical entailment,

This is illuminating and can be philosophically justified in the context of modal logic (see especially the works of David Lewis and of Robert Stalnaker).
But in the case of the conceptual program of reducing the semantics of comparatives to trhe semantics of positive adjectives, there is a conceptual problem.
The truth of x_1 *is taller than* x_2 depends technically on particular constraints on the standards in our set of standards V, and the part of logical space (i.e.their place in ⊑) where they are located. But, while we treat the notions involved in analogy to modal notions, they are *de facto* not modal notions. Even though we do not know which of the worlds extending some standard in V is the real world, we still assume that they are ultimately standards concerning the facts in the real world.
And this means that the particular structure of gaps and absenses around the worlds in V cannot be regarded as encoding modal facts (as they do in modal logic), but must be regarded as encoding facts about the real world.
In other words, we must assume that the structure around the worlds in V is the way it is, because of the facts in the real world. This means that the structure around the worlds in V is ultimately derived from a set of witnessing facts in the real world.

But then, in terms of conceptual reconstruction, it is only fair to ask: which are the facts that witness the structure around the worlds in V?

Kamp 1975 does not answer this question, and in fact, asked in this way, the natural answer is: well, of course, in this particular case, the fact that $g(x_1)$ is, in the real world, has a certain height, and $g(x_2)$ has a certain height, and the first height is smaller than the second.
This, of course, is circular. If this is the answer to the question, then the conceptual program doesn't derive the comparative relation conceptually from the semantics of positive adjectives: it derives **some** comparative relation, which is calibrated (via manipulating the structure of standards V) to fit the natural comparative relation which we find in the real world.

Conceptual approach (Kamp, McConnel-Ginet, Klein, Doetjes)
-Vague adjectives: $<\llbracket tall \rrbracket^{-}, \llbracket tall \rrbracket^{\perp}, \llbracket tall \rrbracket^{+}$
-Primitive comparison relation
-Definiton of comparison and scales in precisification structures

Scalar approach (Cresswell, von Stechow, Bartsch and Vennemann, Kennedy)

-measure function:	$\mathrm{HEIGHT}_{unit}: D \to \mathbb{R}^{+}$
-order:	$>_{\mathrm{HEIGTH}} = \lambda y \lambda x.\mathrm{HEIGHT}_{unit}(x) >_{\mathbb{R}} \mathrm{HEIGHT}_{unit}(y)$
-boundaries	$<\mathrm{HEIGHT}_{unit}^{-}, \mathrm{HEIGHT}_{unit}^{+}>$

$\llbracket taller \rrbracket = >_{\mathrm{HEIGTH}}$
$\llbracket shorter \rrbracket = >_{\mathrm{HEIGTH}}$
$\llbracket tall \rrbracket = \lambda x.\mathrm{HEIGHT}_{unit}(x) >_{\mathrm{HEIGHT}} \mathrm{HEIGHT}_{unit}^{+}$
$\llbracket short \rrbracket = \lambda x.\mathrm{HEIGHT}_{unit}(x) <_{\mathrm{HEIGHT}} \mathrm{HEIGHT}_{unit}^{-}$

Cross linguistic data: quite some languages have Klein-like constructions like:
He was tall among men = He was the tallest man
A was tall among the two (A and B) = A was taller than B

compare Biblical Hebrew: shir ha shirim 1-8
ha yafa be nashim = the beautiful among women = the most beautiful woman

also: yoter-more, hachi-most have no lexical correspondence in Biblical Hebrew.

Stassen 1985: 20 out of 110 languages go like Klein.

Also: comparatives and the scalar system only develop around age 4-5, while the basis of the underlying comparison relation may be innate (and shared with mammals).

Suggestion: a form of both systems may be relevant, and at age 4-5 the two adjectival meanings are identified: $\llbracket tall \rrbracket^{+} = \lambda x.\mathrm{HEIGHT}_{unit}(x) >_{\mathrm{HEIGHT}} \mathrm{HEIGHT}_{unit}^{+}$

Advantages of the conceptual approach

1. Conceptually 'simple' semantics: (1b)
   a. Comparative is an operation on the root *tall*
   b. Adjective meaning = root meaning

2. Standard analysis:

Measure function HEIGHT maps pairs of individuals and worlds onto degrees in a scale of height. (following Lewis, Cresswell):

$d_1 >_{\text{TALL},w} d_2$ iff $\text{HEIGHT}_w(d_1) >_{\text{SCALE(HEIGHT)}} \text{HEIGHT}(d_2)$

or:

Measure relation HEIGHT associates with each individual and world an initial interval of a scale of height (following von Stechow, Heim):

$d_1 >_{\text{TALL},w} d_2$ iff $\text{HEIGHT}_w(d_1) - \text{HEIGHT}_w(d_2) \neq \varnothing$
$>_{\text{SCALE(HEIGHT)}} \text{HEIGHT}(d_2)$

Both require us to associate with gradable predicates as scale of numerical values.

Quantitative analysis, takes measurable adjectives as a model for all gradable predicates. But many gradable predicates do not really have numerical values, but only *tongue-in-cheek values*:

(12) a. Mary is more lovely than Jane
b. Mary is twice as lovely as Jane.

(Although, you **do** need to consider the logic of cases like (12b), as in (13):

(13) a. Ronya is twice as lovely as Minoes
b Poekie is twice as lovely as Ronya
c Poekie is four times as lovely as Minoes)

Also, think about *Ruud is twice as fat as Fred*
Not in terms of weight, but in terms of bulging out of a standard form.
We don't have degrees of bulging, it's just that for Ruud, there's more of it, on more sides, etc.

Cf. Jennie Doetjes' 2011 Amsterdam Colloquium paper.

Problems

Fundamental problem with the analysis (including the modalization):

Vlad the Empaler (the model for Count Dracula) and Gilles de Rais (the companion of Jeanne d'Arc), or for Dante, Judas Iskarioth and Brutus.

On the positive side: Albertine and Odette.

Look at (14):

(14) a. Vlad the Empaler and Gilles de Rais are both quintessentially *despicable*, but Gilles is even more despicable than Vlad.
b. Albertine and Odette are in my world absolutely and totally the most lovely beings. But Albertine is just a tad more lovely than Odette.

In order for the comparative to be true, the above analysis must assume that
either gilles $\in F_{s,\{vlad,gilles\}}^{+}$(*despicable*) and vlad $\notin F_{s,\{vlad,gilles\}}^{+}$(*despicable*) or
vlad $\in F_{s,\{vlad,gilles\}}^{-}$(*despicable*) and gilles $\notin F_{s,\{vlad,gilles\}}^{-}$(*despicable*)
or that this holds for one of the relaxations of s in R_s.

To me this seems incorrect: **any** interpretation of *despicable* that allows Vlad or Gilles to escape from the positive extension of *despicable* misses the point about what *despicable* means relative to our current standard. The same is true with Albertine and Odette. If in order to say that Albertine is more lovely than Odette, I have to recognize a standard relative to which one of them is lovely and the other isn't, or, worse, a standard relative to which one of them is **not**-lovely, misses the point about my standards for *lovely*: such a standard changes the **meaning** of the word *lovely*, and shouldn't be in the ball park to start out with.

At first sight, we may give the following rebuttel:
What Klein is after, besides the conceptual reduction, is a formalization of the basic idea of McConnell-Ginet's analysis:

It's not that we need to assume that there is a standard relative to which Vlad is not despicable. What the analysis tries to capture is that:

Gilles is very very very ***very*** *despicable* and *Vlad is only very very very despicable*.

I grant this, but McConnel-Ginet did not herself give a conceptual reconstruction of this idea, and hence it depends on the analysis of *very despicable* whether or not the criticism is valid or not.

Klein and McConnell-Ginet's analysis of the cmparative in fact quantifies over functions like *very* and *very very* and *very very very*. let K be the class of such functions:
$x_1 >_P x_2$ iff $\exists f \in K[\ f(P)\ (x_1) \wedge \neg\ f(P)\ (x_2)\ ]$

(f could be, for instance, *very very very*)

The crux of the bisquit lies in the definition of what it means to be a function ***like*** *very* and *very very*. What these functions do is easy to explain relative to a scale: They take the extension of the predicate P and map it onto the extension of the predicate f(P) which **right-shifts** the positive extension with respect to P **along the scale associated with P**.

As in the case of Kamp's definition of the comparative given earlier, if you do not define this in terms of conceptual concepts that do not rely on scales, your conceptual reduction has not been succesful, and you have merely defined one scalar concept in terms of another (which may be independently useful, by the way).

When we take another look at Klein's reduction then we see that his analysis is indeed a true conceptual reconstruction of the comparative, but we also see that the problem that I brought up does not go away: I agree that McConnell-Ginet's definition of the comparative is a viable one, but the problem lies hidden in the analysis of *very* P, and with that of functions 'like' *very* P.
The crux of Klein's analysis here is that we do not simply assign *very* P to a subset of $F_{F(P)}^{+}(P)$, but that we assign it $F_{F+(P)}^{+}(P)$, the positive extension of P on the comparison set $F^{+}(P)$. And this is the aspect of the analysis that the scalar order is constructed out of:

> When we look at who is tall within the tall set, that's going to be the taller ones.

So, I don't think that without this aspect, the analysis is going to work. But this ius **exactly** the aspect that I criticized above:

When it comes down to it, I think that the notion *x_1 is more despicable than x_2* cannot be reduced completely to *there is a comparison set and standard relative to which x_1 is despicable and x_2 is not*.

More problems

There are serious questions about the construction of scales in this way.
In essence, if you **derive** scales via equivalence relations, you will never get more degrees, i.e. scalar values than there are individuals compared.

-Thus, on a finite domain you will only get finite scales
-On infinite domains, the above construction will only get you countable scales.
This is not necessarily a disadvantage, for instance, computationally. Some discussion of rationals versus reals in van Hambalgen and Hamm, *The proper treatment of events*.
But it ***is*** a disadvantage if your scalar semantics relies on notions like bounds, supremums, infimums.
-On domains where you do want scalar values, and even on domains where you don't, you do want a notion of **distance** to be part of the scale.

There are various ways in which you can do something about these problems.

Example 1: Height
Equivalence classes will only give you height-classes for the objects there actually **are** and not for heights in between, or above what we have.
What do we do with (15):

(15) Nobody can be smaller than Jane but taller than Emma because there is no height between Jane and Emma.

Introducing intermediate heights is not a real problem if we allow besides the individuals in the domain also **their parts**. If all my parts (and by that I do not just mean my body-parts) are in principle in the domain as well, then a *shaving principle* can introduce enough intermediate height: there is my height, the height of me with my hair shaved, the height of me with the skin of the top of my head shaved, etc…

A second way of introducing intermediate heights is modally, by composing the scale not just from actual objects, but from possible objects as well. But you have to be careful here, a judgement about whether there **could be** someone of intermediate height would have to be reconstructed as a non-scalar statement. You run the risk here of adding possible objects just to fit in the scale.

Example 2: Distances
In my world, everybody (including me) is drab, but Albertine and Odette are not only lovely, they are very very very very much more lovely than anybody else.
On my notion of loveliness, we get only two equivalence classes: everybody else Albertine and Odette, or three, everybody else, Albertine and Odette.

Or compare huggability. My pets are electric eals, jellyfish, and Ronya, my cat.
Again, there are three equivalence classes, hence three degrees of huggability.
How will you express that Ronya is a lot more huggable than any of my other pets.

Direction of resolution.

The standard scalar theory generalizes to the worst case: it equips scales with the topology and arthmetics requires for quatitative measure scales. The conceptual theory (based on Klein) makes the scales clearly not rich enough, and suffers from the Albertine-Odette problem. We do not need to adhere to the conceptual ideology to be attracted to deriving as much as possible of the comparison relation from the positive gradable adjective semantics. (see Doetjes). But we must resolve the Albertine-Odette problems first, because, arguably, if we *do not* accept the last steps in the Albertine-Odette case, we don't get the right comparison relation, because Albertine and Odette come out as equally lovely.

Let us take the order we get with the Klein procedure: $\leq_{P,klein}$. It determines an equivalence relation $\approx_{P,klein}$. Let B be a block of the corresponding partition. We got to B, because at some stage s of the Klein-algorithm, we derived a triple $<X,F_X^{-}(P),F_X^{+}(P)>$ where either $B = F_X^{-}(P)$, or $B = F_X^{\perp}(P)$ or $B = F_X^{+}(P)$, and B was not further refined.

I suggest that at this point we give up the conceptual ideology, and accept that the basis of further divisions may be *comparative facts* that are not reducible any further in the Klein procedure. However, taking stage s to be a standard, we have now a situation $<X_s,F_{s,X}^{-}(P),F_{s,X}^{+}(P)>$ to which we can apply Kamp's semantics:

For Block B and stage s, we establish $\geq_{P,kamp,s}$ relative on the elements of B. Relevant for this is only what B was at stage s: $F_X^{-}(P)$, $F_X^{\perp}(P)$ or $F_X^{+}(P)$. Depending on which it is, the relevant clause of the Kamp definition applies. For instance, Albertine and Odette are clearly, when the refinement process stops in $B = F_{s,X}^{+}$(*lovely*). The relevant Kamp clause tells us that Albertine is more lovely, if every reduction of s that takes Albertine out, takes Odette out first, and some reduction s' of s takes Odette out, but not Albertine.

Waiddaminute! Didn't we just tell Klein that we weren't prepared to do that. We told Klein that we were not prepared to call Albertine lovely and Odette not. That means that if we were put in state s', we would refuse to accept that as a state in which *lovely* means what it means. This is why the process of deciding, on smaller and smaller sets stops before it reaches s'. But we **can** use s' counterfactually in evaluating just the question: don't worry, neither is going to get out, but who would get out *first*?

The last stage, then, is counterfactual (and that is, of course, in essence what Kamp's definition imitates).

So we have:

$>_{P,klein}$

and

$>_{P,kamp,\approx} =$

$\cup$ {$>_{P,kamp,s}$: $B \in \approx_{P.klein}$ and s is the stage of the Klein-procedure where B stabilized}

And we can now define:

$$>_P = [>_{P,klein} \cup >_{P,kamp,\approx}]^{TR} \quad \text{(the transitive closure of the union).}$$

As I indicated under the earlier discussion concerning the Kamp definition, I do not believe that with this the conceptual reduction is succesful: I think the structure of standards around s supporting the counterfactual is indicative of the existence of a witnessing conditional fact (Albertine is more lovely than Odette), rather than producing successfully such a fact.

Also note that, with this modified analysis, I also do not think that Klein is succesful in giving a succesful conceptual reconstruction of the other essential degree notion that he uses, namely: *very*. It is not true that I am ever willing to say that Albertine is lovely, but Odette is not. But, with McConnell-Ginet I *am* willing to say that Albertine is *very*$^{n+1}$ lovely, but Odette is only *very*^{n} lovely, not *veryi*$^{n+1}$ lovely. But that means that the procedure definition Klein propeses for *very* breaks down at exactly the same point where the comparsative breaks down: when it comes to just Albertine and Odette, *very lovely* **does not** mean: *lovely in the land of the lovely*. And again, maybe *there* we can reconstruct it as: if you had to choose one to remove first, it *would* be Odette.

Now suppose we have the corrected comparative relation $>_P$. We take equivalence classes with our new equivalence relation $\approx_P$.

If the adjective we started out with was a mono-dimensional gradable adjective, the resulting structure is a linear order.

Now instead of associating with the scalar adjective directly a full-blown real-valued scale, we can start *piecemeal* from the other side. We get, of course, the linear order $<P_{\approx}, < >$ of the equivalence classes. Instead of assuming that these form (part of) one single scale, we can assume that the induced scale is a **set of** homomorphisms into **R**. The semantics, then, puts constraints on this set, but there is no assumption that there is necessarily something which is *the scale* associated with P (cf. Robert van Rooy's paper on vagueness).

One thing that would be expressed at this level, is a constraint that every homomorphism maps my electric eals and jelly fish on a degree of huggability that is considerably smaller than that of Ronya. In other words, constraints on *distance* between equivalence classes are expressed here.

As far as I am concerned, *this* is also the natural place to express the meaning of the adjective modifier *very*, in terms of distance between degrees. But the distance constraints may be approximative: *twice as lovely* need not be literally *twice as lovely*, it tells you that the homomorphisms map two degrees at some approximate distance.

Semantic scalar constraints can be incorporated too, like the constraint that only scales with a maximum are appropriate for certain adjectives (like *clean*), but not for others (like *dirty*).

But the scalar values themselves are not necessarily numbers, and do not necessarily have a quantitative topology defined on them.

In other words: you determine *qualitatively* who is more lovely than whom, and you determine *qualitatively* how much more lovely. After that, you map this onto a set of scales: the scales compatible with the qualitative scale, using equivalence relations and constraints.

This means that there are initially no degrees of loveliness: these are derived.

Thus, we determine 'qualitatively' what it means that Ruud is 'twice as fat' as Fred by comparing 'on the eye' for each the material that bulges outside a Standard model:

Fred Ruud

We do not start out for *fat* with a measure *lipels* such that Fred is n *lipels fat* and Ruud is 2n *lipels fat*.

Note about the counterfactual way out

Klein determines all cases extensionally:
x is in P^+ relative to {x,y} and y is not in P^+ relative to {x,y}
x is in P^- relative to {x,y} and y is not in P^- relative to {x,y}

This leaves the cases of Odette and Albertine and of Vlad and Gilles.

Assume that here we access the modal theory and determine counterfactually who gets in first/who goes out first. Let this determine the ultimate comparative relation.

Technically this could work, of course, but I still think that this is fundamentally changing the meaning of what counts as *lovely*, even if counterfactually, i.e. resetting the meaning of what counts as lovely.

2.3. QUESTIONS AS PARTITIONS

Groenendijk and Stokhof 1985, *Studies in the Semantics of Questions and the Pragmatics of Answers*, PhD Diss. University of Amsterdam
Groenendijk and Stokhof 1990, 'Partitioning logical space.' (both on their webpages).

(1) a. Fred believed that Ronya was asleep.
b. $believe_w(Fred, \{v \in W: asleep_v(Ronya)=1\})$

Fred stands in world w in the believe-relation to a propostion:
$\{v \in W: asleep_v(Ronya)=1\}$
The set of worlds v in which *asleep* maps Ronya onto truthvalue 1.

Fact 1: *that*-complements and *wh*-complements can be conjoined**:**

(2) Fred knows that there is going to be a party, whether it is come as you are, and who is going to come.

Fact 2: Entailment relations with *know* and with *tell* between *that*-complements and *wh*-complements.

(3) a. Fred knew whether Buck was angry.
b_1. Buck was angry
c_1. Fred knew that Buck was asleep.

a. Fred knew whether Buck was angry.
b_2 Buck wasn't angry
c_2 Fred knew that Buck wasn't angry.

Not to do with the factiveness of *know*:

(4) a. Fred told me at some point whether Buck was angry.
b_1. Buck was angry
c_1. Fred told me at some point that Buck was angry.

a. Fred told me at some point whether Buck was angry.
b_1. Buck wasn't angry
c_1. Fred told me at some point that Buck wasn't angry.

Cf. (4')

(4') a. Fred told me that Anna Sophie won the Songfestival, but he lied, she didn't win.
✓But he *did* tell me that Anna Sophie won.
b. Fred told me that who won the Songfestival, Anna Sophie, but he lied, she didn't win.
#But he *did* tell me who won.

Fact 3: Some verbs only take *that*-complements, some verbs only take *wh*-complements.

(5) a. ✓Fred believed that Buck was angry.
b. #Fred believed whether Buck was angry

c. #Fred wondered that Buck was angry.
d. ✓Fred wondered whether Buck was angry.

Also inferences with other *wh*-complements:

(6) a. At that point Fred knew who was the murderer.
b. Jack was the murderer
c. At that point Fred knew that Jack was the murderer.

On the strongest theory:

(6) a. At that point Fred knew who was the murderer.
b. Jack wasn't the murderer
c. At that point Fred knew that Jack wasn't the murderer.

Exhaustiveness:

(7) a. Fred knows who has passed the test.
b Jane, Mary and Joanna are the ones who passed the test.
c. Fred knows that Jane, Mary and Joanna have passed the test.

Problemetic case:

(8) a. Fred knows which prime numbers are even
b. 2 is the only even primenumber
c. Fred knows that 2 is the only even primenumber

a. Fred knows which prime numbers are even
b. 7919 is a prime number, but not an even primenumber
c. Fred knows that 7919 is a prime number, but not an even primenumber

Groenendijk and Stokhof would argue that you don't 'really' know which prime numbers are even if you don't know what the prime numbers are (i.e. if *for you* the set of prime numbers can vary from world to world). I don't agree with that, but once we have defined the stronger semantics, we can see various ways of weakening it.

Groenendijk and Stokhof:
Semantic answer: complete exhaustive direct answer.
Pragmatic answer: less complete, less exhaustive, less direct….

Fact 1 suggests that *wh*-complements have the same type as *that*-complements.
Since *that*-complements denote propositions, *wh*-complements denote propositions.
But which propositions?
Answer: propositions that make the above entailments come out valid.

Basic intuition:
whether Buck is angry contemplates two answers:
Buck is angry versus *Buck is not angry*
The answers are propositions:
$A = \{v \in W: angry_v(b)=1\}$ versus $\neg A = \{v \in W: angry_v(b)=0\}$
Knowing in world w_0 whether Buck is angry is knowing whether $w_0 \in A$ or $w_0 \in \neg A$.

Which proposition does *whether Buck is angry* denote depends on where w_0, the world of evaluation, is:

if $w_0 \in A$, *whether Buck is angry* denotes A
if $w_0 \in \neg A$, *whether Buck is angry* denotes $\neg A$

Let w_0 be the world of evaluation:

whether Buck is angry denotes:
$\{v \in W: angry_v(b) = angry_{w0}(b)\}$

The set of worlds v in which *angry* maps *b* onto
the same truthvalue as it maps b onto in w_0.

In general, let φ_w be the truthvalue of φ in w.

that φ denotes $\{v \in W: \varphi_v = 1\}$
The set of worlds v where the truthvalue of φ is 1

that $\neg\varphi$ denotes $\{v \in W: \varphi_v = 0\}$
The set of worlds v where the truthvalue of φ is 0

whether φ denotes: $\{v \in W: \varphi_v = \varphi_{w0}\}$
The set of worlds v where the truthvalue of φ is the same as it is in w_0.

With this interpretation of *whether* φ the patterns in (9) and (10) become valid:

(9) a. Fred tells whether φ	$tell_{w0}(f, \{v \in W: \varphi_v = \varphi_{w0}\})$
b. φ is true in w_0	$\varphi_{w0} = 1$
c. Fred tells that φ	$tell_{w0}(f, \{v \in W: \varphi_v = 1\})$
(10) a. Fred tells whether φ	$tell_{w0}(f, \{v \in W: \varphi_v = \varphi_{w0}\})$
b. φ is false in w_0	$\varphi_{w0} = 0$
c. Fred tells that $\neg\varphi$	$tell_{w0}(f, \{v \in W: \varphi_v = 0\})$

Wh-complements:

(11) a. Fred knows who passed.
b Ruth, Mary and Joanna are the ones who passed.
c. Fred knows that Ruth, Mary and Joanna passed.

Relative clause: *who - passed*

$passed_w$ The set of individuals that passed in w

Question:

who P denotes: $\{v \in W: P_v = P_{w0}\}$

The set of worlds v where the denotation of P is the same as it is in w_0.

(7) a. $know_{w0}(f, \{v \in W: passed_v = passed_{w0}\})$

b $passed_{w0} = \{r, m, j\}$

c. $know_{w0}(f, \{v \in W: passed_v = \{r, m, j\})$

Multiple relative: *who loves whom* denotes $love_{w0}$

Question:

who R denotes $\{v \in W: R_v = R_{w0}\}$

So far we have been concerned with the denotation, extension of the question, the proposition expressed.
The intension of a propositional type expression α is the function that maps every world w_0 onto the extension of α in w_0.
If the extension of α is a proposition, a set of worlds, the intension of α is a function from worlds into sets of worlds, which is (up to isomorphism) nothing but **a relation between worlds**.

that Buck is angry: extension in w_0: $\{v \in W: angry_v(b)\}$
intension in w_0: $\{\langle w,v\rangle \in W^2: angry_v(b)\}$

(abstract over world w_0, rename as w for clarity):
The intension is a constrant function from worlds to sets of worlds.

whether Buck is angry: extension in w_0: $\{v \in W: angry_v(b) = angry_{w0}(b)\}$
intension in w_0: $\{\langle w,v\rangle \in W^2: angry_v(b) = angry_w(b)\}$

The relation that holds between w and v iff *Buck is angry* has the same truthvalue in w and in v.

who is angry: extension in w_0: $\{v \in W: angry_v = angry_{w0}\}$
intension in w_0: $\{\langle w,v\rangle \in W^2: angry_v = angry_w\}$

The relation that holds between w and v iff the people that are angry in w are the same as the people who are angry in v.

who loves whom: extension in w_0: $\{v \in W: love_v = love_{w0}\}$
intension in w_0: $\{\langle w,v\rangle \in W^2: love_v = love_w\}$
The relation that holds between w and v iff the loving couples in w are the same as the loving couples in v.

Typal concerns for propositional complements.

believe, know and *tell* operate on the **extension** of the complement.
wonder operates on the **intension** of the complement.

Lexical postulates:

believe requires a complement whose intension is constant
wonder requires a complement whose intension is not constant
know, *tell* allow both kinds

Extension of the wh-complement in w_0 = The answer to the question in w_0
Intension of the wh-complement in w_0 = The question itself

(11) Fred knows whether Buck is angry
$know_{w0}(f, \{v: angry_v(b) = angry_{w0}(b)\})$
Fred wonders whether Buck is angry
$wonder_{w0}(f, \{\langle w,v\rangle: angry_v(b) = angry_w(b)\})$

know and *tell* relate Fred to the answer in w_0 to the question *Is Buck angry*.
wonder relates Fred in w_0 not to the answer, but to the question *Is Buck angry* itself.
A natural lexical postulate would be:

$$\forall z(wonder_z(f, \{\langle w,v\rangle: \varphi_v = \varphi_w \}) \rightarrow \neg know_z(f, \{v: \varphi_v\}) \wedge \neg know_z f, \{v: \neg\varphi_v\})$$

Now we relate this to partitions:

Fact: Let R be an n-place relation (including formulas as 0-place relations).
The relation $\{\langle w,v\rangle \in W^2: R_v = R_w\}$ is an equivalence relation on W.
Proof: This is obvious from the definition in terms of identity.

Questions are partitionings of the space of possible worlds.
For instance, the following two questions: *Is Buck angry?* and *Is Sara angry*?

	W
$\{v \in W: angry_v(b) = 1 \}$	*angry*(*b*)?
$\{v \in W: angry_v(b) = 0 \}$	

		W
$\{v \in W: angry_v(s) = 1\}$	$\{v \in W: angry_v(s) = 0\}$	*angry*(*s*)?

As we see, depending on where w_0 is, the extension of *angry*(*b*)? is the extension of *that Buck is angry* or the extension of *that Buck is not angry*, and the the extension of *angry*(*j*)? is the extension of *that Sara is angry* or *that Sara is not angry*.

Let us assume D = {buck, jane, sara}. Then the set of worlds W partitions according to what the extension of angry_v is, for $v \in W$. There are 8 logical possibilities:

	W
$\{v \in W: \mathit{angry}_v = \{\text{ buck, jane, sara }\}$	B_1
$\{v \in W: \mathit{angry}_v = \{\text{ buck, jane }\}$	B_2
$\{v \in W: \mathit{angry}_v = \{\text{ buck, sara }\}$	B_3
$\{v \in W: \mathit{angry}_v = \{\text{ jane, sara }\}$	B_4
$\{v \in W: \mathit{angry}_v = \{\text{ buck }\}$	B_5
$\{v \in W: \mathit{angry}_v = \{\text{ jane }\}$	B_6
$\{v \in W: \mathit{angry}_v = \{\text{ sara }\}$	B_7
$\{v \in W: \mathit{angry}_v = \emptyset$	B_8

Who is angry?

Again, if $w_0 \in B_4$ then the true complete semantic answer to the question is:
Jane and Sara are the ones that are angry

In the next picture we show the two partitions, *Who is angry*? and *Is Buck angry*?

	W
$\{v \in W: \mathit{angry}_v = \{\text{ buck, jane, sara }\}$	B_1
$\{v \in W: \mathit{angry}_v = \{\text{ buck, jane }\}$	B_2
$\{v \in W: \mathit{angry}_v = \{\text{ buck, sara }\}$	B_3
$\{v \in W: \mathit{angry}_v = \{\text{ buck }\}$	B_5
$\{v \in W: \mathit{angry}_v = \{\text{ jane, sara }\}$	B_4
$\{v \in W: \mathit{angry}_v = \{\text{ jane }\}$	B_6
$\{v \in W: \mathit{angry}_v = \{\text{ sara }\}$	B_7
$\{v \in W: \mathit{angry}_v = \emptyset$	B_8

Now, clearly, there is a relation between these questions, namely:
Every possible true complete answer to the question: *Who is angry* entails a true complete answer to the question: *Is Buck angry*?

That is, if the real world is w_0, then, whichever block of partition *Who is angry* w_0 is in, that block is a subset of a block of partition *Is Buck angry*, and hence that answer to *Is Buck angry* is also a true answer in w_0.
The subset relation on the set of possible worlds is the entailment relation between propositions, hence, we can define entailment between questions in terms of the refinement relation on partitions:

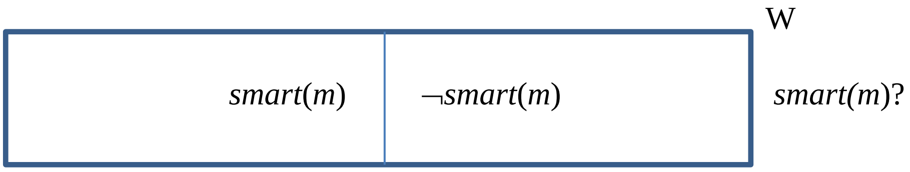
> Q_1 **entails** Q_2 iff $Q_1 \sqsubseteq Q_2$, iff
> For every world w: the true complete answer to Q_1 in w entails the true complete answer to Q_2 in w

Suppose you are interested in getting a cat, and someone seeks a new house for her cat Minoes. You ask:

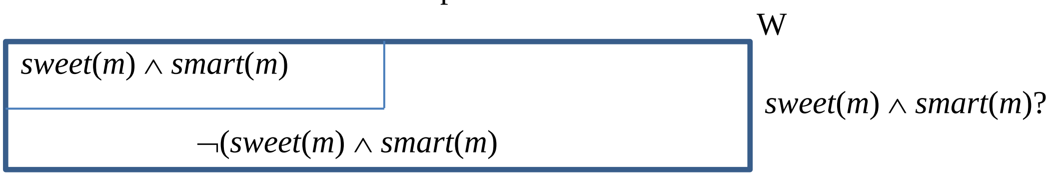
(12) a. Is she sweet? And is she smart?
b. Is she sweet and smart?

Is Minoes sweet? is the following bipartition:

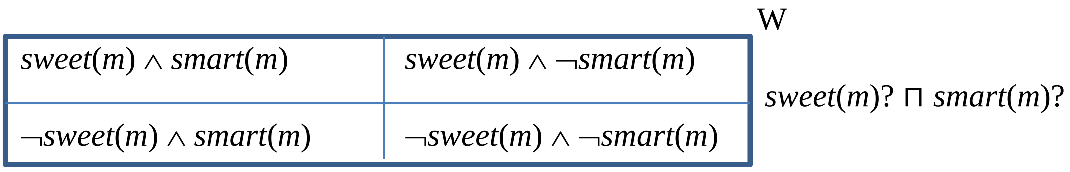
Is Minoes smart? is the following bipartition:



Is Minoes sweet and smart? is also a bipartition:



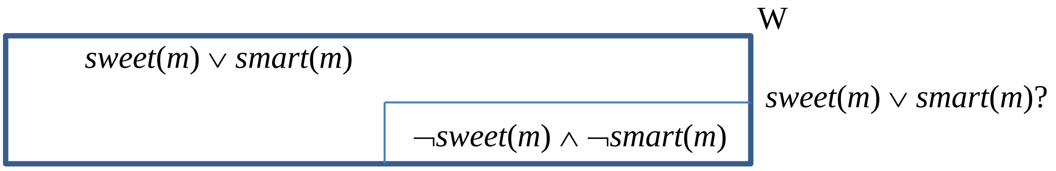
But the conjunction of the two questions *Is Minoes sweet? And is she smart?* is a partition with four blocks:



And, again, every answer to the the conjunction of the two questions entails an answer to each of the questions that are conjoined.

It is not so clear that disjunctions of questions are actually interpreted as joins in the partition lattice. While in the case of conjunction, I think there is a clear reading where the conjunction of questions is a partition with four blocks, rather than two, it is not clear to me that there is a sense in which the following disjunction is trivial rather than a bipartition:

(13) a. Is Minoes sweet? Or is Minoes smart?
b. Is Minoes one of sweet and smart?



But then, trivial readings are rarely preferred when non-trivial readings are available.

Look at the questions:
Is more than one person angry?
This partition has blocks: $\{B_1 \cup B_2 \cup B_3 \cup B_4\}$ and $\{B_5 \cup B_6 \cup B_7 \cup B_8\}$

	W
$\{v \in W$: $angry_v = \{$ buck, jane, sara $\}$	B_1
$\{v \in W$: $angry_v = \{$ buck, jane $\}$	B_2
$\{v \in W$: $angry_v = \{$ buck, sara $\}$	B_3
$\{v \in W$: $angry_v = \{$ jane, sara $\}$	B_4
$\{v \in W$: $angry_v = \{$ buck $\}$	B_5
$\{v \in W$: $angry_v = \{$ jane $\}$	B_6
$\{v \in W$: $angry_v = \{$ sara $\}$	B_7
$\{v \in W$: $angry_v = \emptyset$	B_8

Is nobody angry?
This partition has blocks: $\{B_8\}$ and $\{B_1 \cup B_2 \cup B_3 \cup B_4 \cup B_5 \cup B_6 \cup B_7\}$

	W
$\{v \in W: angry_v = \{$ buck, jane, sara $\}$	B_1
$\{v \in W: angry_v = \{$ buck, jane $\}$	B_2
$\{v \in W: angry_v = \{$ buck, sara $\}$	B_3
$\{v \in W: angry_v = \{$ sara, jane $\}$	B_4
$\{v \in W: angry_v = \{$ buck $\}$	B_5
$\{v \in W: angry_v = \{$ jane $\}$	B_6
$\{v \in W: angry_v = \{$ sara $\}$	B_7
$\{v \in W: angry_v = \emptyset$	B_8

Is more than one person angry? $\sqcup$ *Is nobody angry*?
has blocks: $\{B_1 \cup B_2 \cup B_3 \cup B_4 \cup B_8\}$ and $\{B_5 \cup B_6 \cup B_7\}$

The questions is whether there is a notion of disjunction on which the following two questions are equivalent:

(14) a. Is more than one person angry? Or is nobody angry?
b. Is exactly one person angry?

I am not sure (meaning, I think not).
Notice that the definition of join of two partitions is dramatically more complex than meet of two partitions. Could this be reason for the lack of the join of partition reading for disjunction?...

With this bit of theory in place, Groenendijk and Stokhof define a variety of relations between questions and answers. I give variants of their notions here.

Proposition p is **semantically non-trivial** iff $p \neq \emptyset$ and $p \neq W$
The **tautological question** is $\{W\}$.

Let p be a proposition and Q a question.

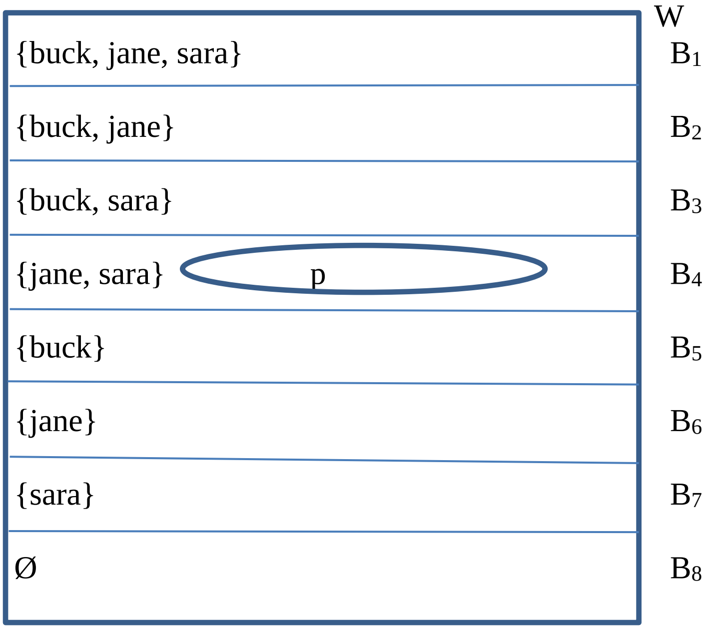
p is a **semantic answer** to Q iff $p \in Q$
p is a **partial answer** to Q if p is non-trivial and for some $X \subseteq Q$: $p = \cup X$

Any of the blocks in Q is a semantic answer to Q.
Any union of blocks in Q which is non-trivial is a partial answer to Q.

Fact 1: If p is a semantic answer to Q and Q is not the tautological question, then p is a partial answer to Q
Fact 2: Every partial answer to a yes-no question is a semantic answer.

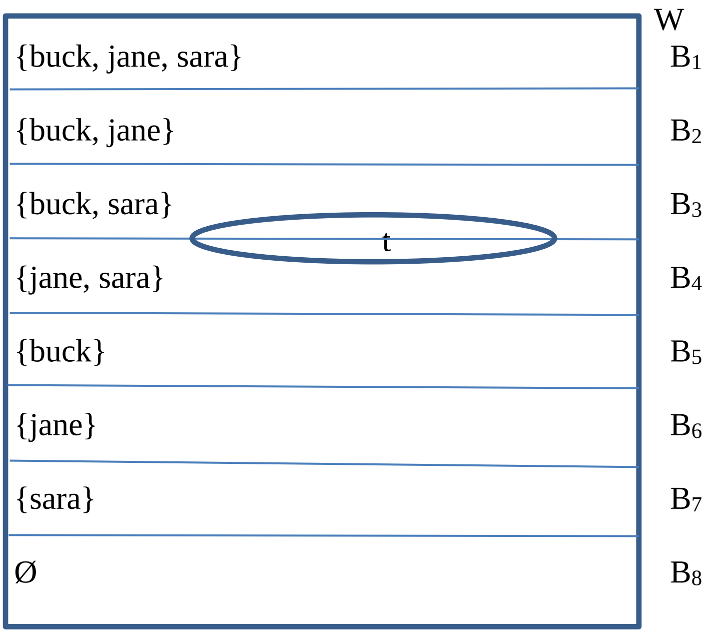
Let $p \neq \varnothing$
p **entails a semantic answer to** Q iff for some p_1: $p \subseteq p_1$ and p_1 is a semantic answer to Q
p **entails a partial answer to** Q iff for some p_1: $p \subseteq p_1$ and p_1 is a partial answer to Q

Who is angry?



Let p be proposition: *Jane and Sara lost at the horses, and everybody else won.*
p entails a semantic answer to the question *who is angry*?

Who is angry?



Let t be the proposition:
Sara lost at the horses and quarelled with Buck or with Jane, but not with both.
t entails a partial answer to the question *Who is angry*?

Let $w \in W$:

p is a **semantic answer** to Q **true in w** iff $p \in Q$ and $w \in p$

p is a **partial answer** to Q **true in w** if p is non-trivial and for some $X \subseteq Q$: $p = \cup X$ and $w \in X$

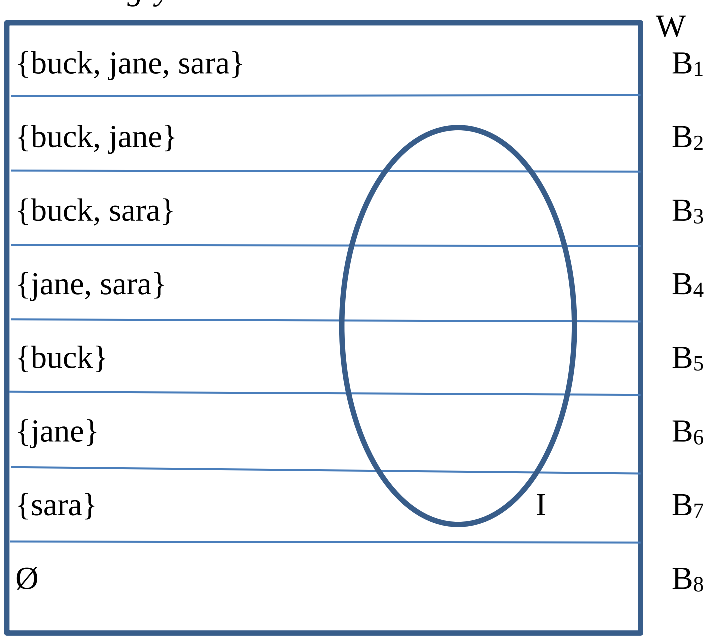
p **entails a semantic answer to** Q **true in w** iff for some p_1: $p \subseteq p_1$ and p_1 is a semantic answer to Q true in w

p **entails a partial answer to** Q **true in w** iff for some p_1: $p \subseteq p_1$ and p_1 is a partial answer to Q true in w

Note that if p entails a true semantic or partial answer, that doesn't mean that p is itself true.

For pragmatic notions of questions and answers, we relativize the questions and answers to a set $I \subseteq W$ which represents the information relative to which the question is evaluated.

Who is angry?



I is the set of worlds compatible with the information.
We will assume that I is the information against the background of which the question Q is asked. As you can see in the picture, and as we proved in chapter 1, we have a lemma:

Lemma: If Q is a partition on W and $I \subseteq W$, $I \neq Ø$ then:

$Q_I = \{p \cap I: p \cap I \neq Ø \text{ and } p \in Q\}$ is a partition on I

Proof:
-Clearly, by the definition of Q_I: for every $b \in Q_I$: $b \subseteq I$ and $b \neq \emptyset$.
-If $b_1, b_2 \in Q_I$ then for some $p_1, p_2 \in Q$: $b_1 = p_1 \cap I$ and $b_2 = p_2 \cap I$.
Since Q is itself a partition, $p_1 \cap p_2 = \emptyset$. Hence $b_1 \cap b_2 = \emptyset$
-Let $w \in \cup Q_I$. Then for some $b \in Q_I$; $w \in b$, and since for some $p \in Q$, $b = p \cap I$, $w \in I$
-Let $w \in I$, then $w \in W$ and, since Q is a partition of W, for some $p \in Q$: $w \in Q$.
Then $w \in p \cap I$. Hence for some $b \in Q_I$: $w \in b$. Then $w \in \cup Q_I$.
So indeed Q_I is a partition on I.

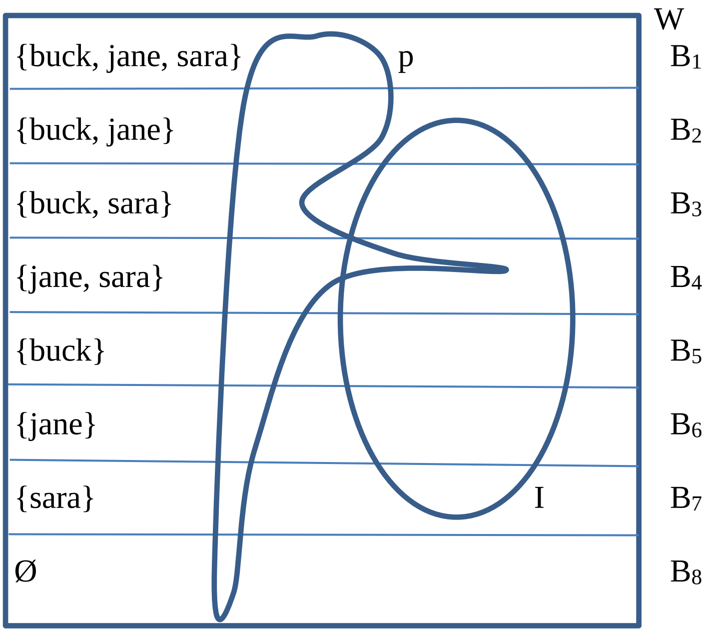
Let Q be a question and I a non-empty information set.

Q is **an open question on** I iff $|Q_I| > 1$.

A question Q is open on I iff Q_I is at least a bipartition on I.

Let Q be a question open on I, $w \in W$:
p is a **pragmatic answer** to Q, rel I iff $p \cap I$ is a semantic answer to Q_I
p is a **partial pragmatic answer** to Q rel I if $p \cap I$ is a partial answer to Q_I
p **entails a pragmatic answer to** Q rel I iff $p \cap I$ entails a semantic answer to Q_I
p **entails a partial pragmatic answer to** Q rel I iff $p \cap I$ entails a partial answer to Q_I
p is a **pragmatic answer** to Q **true in w** rel I iff p is a pragmatic answer to Q_I true in w
p is a **partial pragmatic answer** to Q **true in w** rel I iff p is partial answer to Q_I true in w
p **entails a pragmatic answer to** Q **true in w** rel I iff p entails an answer to Q_I true in w
p **entails a partial pragmatic answer to** Q **true in w** rel I iff p entails a partial answer to Q_I true in w

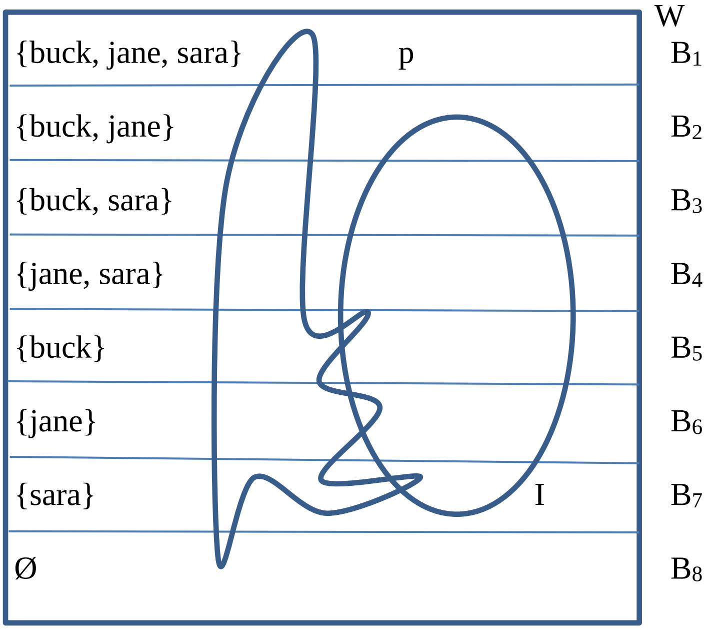
Who is angry?



In this example p does not entail a semantic answer to question Q at all:
no block of Q gets eliminated if we intersect Q with p.
But, given our information I, it turns out that p entails a pragmatic answer to Q relative to I.
So, if p is the proposition: *Jane and Sara lost at the horses*.

And the background information is:
Buck won at horses, and Buck, Sara, and Jane are notoriously bad losers. Buck, Sara and jane are the occupants of the appartment next door.
Then, if you ask me: I hear screaming in the appartment next door. *Who is angry*?
I can answer: *Sara and Jane lost at the horses*. And you say: *oops*.

Who is angry?



This is the next day, and my answer is you: *One of them lost at the horses*.
This time, you may know that only one of them was at the race track, without it being known which of them. In this case p entails a partial pragmatic answer.

Further topics:
-Different theories of questions: Hamblin, Karttunen, Heim, GS,…
-Exhaustiveness and answerhood (see the paper, and also Aldo Sevi's dissertation)
-Mention all vs mention some readings; choice readings, *de dicto-de re* in questions, functional readings in questions,…
-Pragmatics of questions and pragmatics with questions:
formalize Gricean theory in question theory (see the section in the paper mentioned)
GS work on questions started as a way of formalizing the pragmatic notion of *discourse topic*. See also work by Craige Roberts, Gerhard Jager, and many others.
-Inquisitive semantics. See the next section.

2.4 INQUISITIVE SEMANTICS

Ciardelli, Groenendijk and Roelofson 2012, 'Inquisitive semantics,'

2.4.1. Issues

We start with the possible world view of propositions as sets of possible worls, Stalnaker's view of the common ground or an information state as a set of worlds, and accepting a proposition as intersecting it with the common ground, i.e. information growth as kicking out alternative.

W is the set of possible worlds.
S = **pow**(W) is the set of all information states.

An information state $s \in S$ as the set of worlds compatible with the common information is taken to be the set of worlds that are still candidates for being the real world.
There is, of course, the actual real world w_0, but, as far as our information goes, we don't know which world that it.
If t is a proper subset of our information state s, then t kicks out some candidates for being the real world as no longer in the running. From the perspective of t, you have come closer to the real world than you were when your information was only s.
CGR call t an enhancement of s. I am not happy with that terminology and will call t a **sharpening** of s if $t \subseteq s$.

> W is the set of possible worlds.
> S = **pow**(W) is the set of all **information states**.
> t is a **sharpening of** s iff $t \subseteq s$
>
> W is the state of **ignorance**
> Ø is the **inconsistent state (overly sharp)**
> $\{\{w\}: w \in W\}$ is the set of the **precise states**.

Precise states are maximally (but not overly) sharp: they fix one world.

Let s be an information state.
We are interested in getting better informed, which means, sharpening the information.
An **issue over** s is, in essence, a specific set of **alternatives for sharpening** to be contemplated in s: should we go on and sharpen s in direction s_0, in direction s_1, or in direction s_2? Thus it is a set of **choices** of sharpening paths to be contemplated.

From this we see, as a first constraint, that an **issue I over** s should be a non-empty set of sharpenings of s: I is a non-empty set of subsets of s.

Now let us think of the issue as a choice between sharpening in the direction of s_1 or of s_2, and for clarity, let us think of the issue as a clear choice between non-overlapping sharpenings s_1 and s_2. Then it should be clear that sharpening $t_1 \subseteq s_1$ settles the issue in favour of s_1, and similarly, $t_2 \subseteq s_2$ settles the issue in favour of s_2.
This means that we want an **issue I over** s to be closed under sharpening: if $u \in I$ and $t \subseteq u$ then $t \in I$.

Finally, the worlds that are in s are there for a good reason: each of them could turn out to be the real world. If we raise an issue, to be resolved by sharpening, we do not want to **define** the issue in such a way, that some of the worlds in s are not engaged in the choices at all. If such a world turns out to be the real world after all (and s says that it may well!) then the choices in the issue were clearly ill-conceived. We do not want issues to be ill-conceived. We will assume the choices that the issue presents, form a cover of $\cup I = s$

With this we define:

I is an **issue over** s iff 1. $I \subseteq$ **pow**(s) and $I \neq \emptyset$
2. If $t \in I$ and $u \subseteq t$ then $u \subseteq I$
2. $\cup I = s$

If I and J are both issues over s then I **is at least as inquisitive as** J iff $I \subseteq J$
The least inquisitive issue over s is the trivial issue **pow**(s).
The most inquisitive issue over s is $\{\{w\}: w \in s\} \cup \{\emptyset\}$
A proper issue over s is an issue I over s such that $s \notin I$.

2.4.2. Propositions

Inquisitive semantics uses the notion of issue to define the basic semantic notion of **proposition**. A proposition in inquisitive semantics is **not** a set of possible worlds, **but** an issue, a set of information states:

A **proposition** P over state s is an issue over a sharpening of s.
Π_s is the set of propositions over s

W is the set of all worlds, so Π_W is the set of all propositions over W, which, by the definition given, clearly includes for every $s \in S$: Π_s.

We write Π for Π_W:
Π is the set of all propositions.

Of course, there is a natural mapping from propositions to states:

if $P \in \Pi$ then $\cup P$ **is the information state covered by** P

In the past, a statement expressed a proposition, and the only informative function that the proposition had was to function as an assertion and sharpen the common ground, if accepted. In inquisitive semantics, the statement that expresses proposition P can have two informative functions:

1. As before, the statement function as an assertion of proposition P and we can let the information state covered by P sharpen the common ground, if accepted.
2. the statement can **raise the issue of P**. The statement can be regarded as a **request** to the speech partitipant to sharpen the information by adressing the choices raised by P.

These two functions correspond to different relations between proposition P and information state s:

P is **informative on** s iff $\cup(P) \subset s$
P is **inquisitive on s** iff $\cup(P) \notin P$

P is informative if $\cup(P)$ properly sharpens s.
P is inquisitive if it offers a real choice. if $\cup(P) \in P$, then P = **pow**($\cup$P) and $\cup$P is the maximum. That means that P doesn't really offer true alternative paths.

As we will see, there are purely informative propositions,
there are purely inquisitive propositions,
and, most interestingly, there are also hybrid propositions which are informative and inquisitive at the same time.

Some technical notions:

Let $P, Q \in \Pi$
P **is at least as informative as** Q iff $\cup(P) \subseteq \cup(Q)$
P is **at least as inquisitive as** Q iff $\cup(P) = \cup(Q)$ and $P \subseteq Q$
P **entails** Q iff $P \subseteq Q$

Let $P \in \Pi_s$ and $t \subseteq s$
The **restriction of** P **to** t is: $P\restriction t = \{t' \subseteq t: t' \in P\}$

The proposition $P\restriction t$ sharpens the issue P on s to an issue on sharpening t of s.

2.4.3. Meaning functions

The interpretation function maps every sentence in the language onto a **meaning function**:

A **meaning function** is a function $m: S \rightarrow \Pi$ such that:
1. for every $s \in S$: $m(s) \in \Pi_s$
2. if $t \subseteq s$ then $m(t) = m(s)\restriction t$

M is the set of all meanings functions.

Let $m \in \mathbf{M}$
m is **informative** iff there is a state $s \in S$ such that m(s) is informative in s
m is **inquisitive** iff there is a state $s \in S$ such that m(s) is inquisitive in s

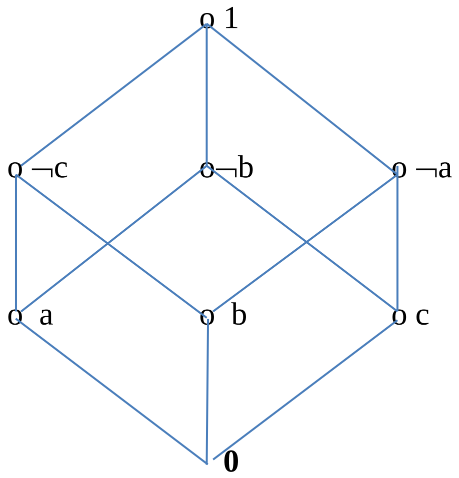
m_1 **entails** m_2 iff for every $s \in S$: $m_1(s)$ entails $m_2(s)$

Fact 1: There is a one-one correspondence between meanings and propositions
Fact 2: Let $m \in M$: m is informative iff m(W) is informative
m is inquisitive iff m(W) is inquisitive
Fact 3: m_1 entails m_2 iff $m_1(W) \subseteq m_2(W)$

Fact 4: The structure of propositions is a Heyting algebra.

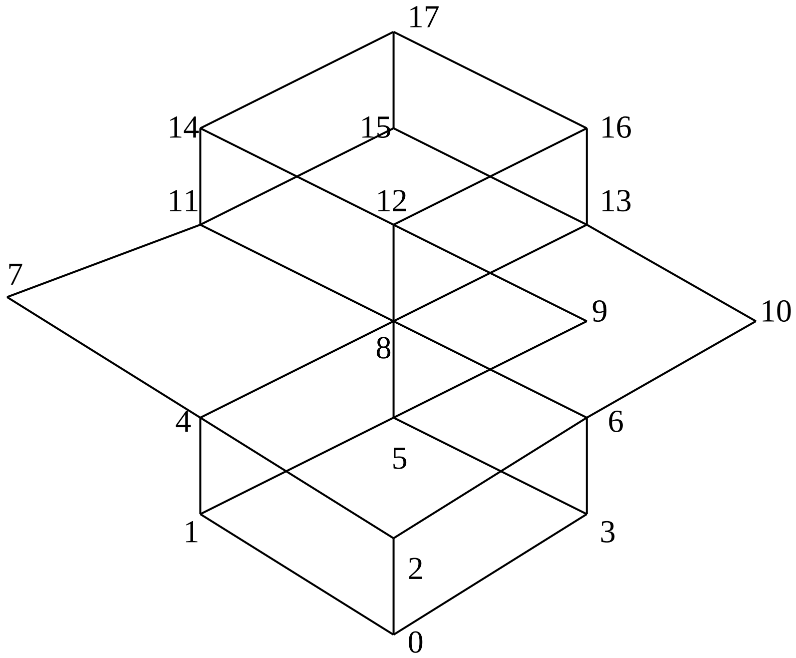
Preview of some aspects of Heyting Algebras.

Boolean algebra with Boolean complementation:

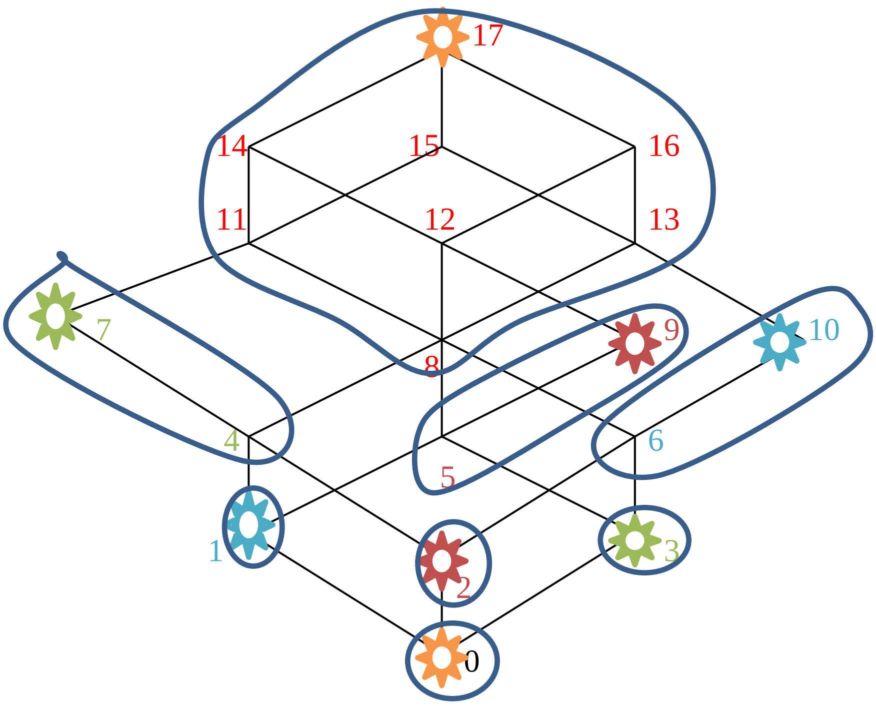


Fact: Complete Heyting algebras = complete distributive lattices = complete pseudo complemented lattices.

The following structure is an example (the free distributive lattice on three generators):



a*, the pseudo complement of a is the maximal element such that a ⊓ a* = 0
Pseudocomplements are indicated in compatible colors in the following picture:



In a Boolean algebra a* = ¬a.
In general, it is not the case that a** = a, what is the case is: a ⊑ a**.
i.e. look at 4: 4** = 7. (i.e. a is a **stronger** proposition than a**). * corresponds to *intuitionistic negation*.

The operation →, called *relative pseudo complement*, corresponds to *intuitionistic implication*.
(a → b), the relative pseudo complement of a rel b, is the maximal element such that
a ⊓ (a → b) = b

For instance, (4 → 2) = 10, because 10 is the maximal element such that 4 ⊓ (4 → 2) = 2.

Intuitionistic logic rejects the classical inference principle:

From	$\neg\neg\varphi$	infer	φ

(In fact, the inference system for intuitionistic logic can be formulated as that of classical logic minus only the above rule)

With that, it rejects other well known classical principles like:

From	Ø	infer	$\varphi \vee \neg\varphi$	(Law of excluded middle)
From	$\neg(\varphi \wedge \psi)$	infer	$\neg\varphi \vee \neg\psi$	(One de Morgan law)
From	$\neg\forall x\varphi$	infer	$\exists x\neg\varphi$	(One quantifier law)

For conditionals, intuitionistic logic satisfies classical principles like:

From	$(\varphi \vee \psi) \rightarrow \chi$	infer	$(\varphi \rightarrow \chi) \wedge (\psi \rightarrow \chi)$
From	$\varphi \rightarrow (\psi \wedge \chi)$	infer	$(\varphi \rightarrow \psi) \wedge (\varphi \rightarrow \chi)$
From	$\varphi \rightarrow \psi$	infer	$\neg(\varphi \wedge \neg\psi)$

But rejects classical troublemakers like:

From	Ø	infer	$(\varphi \rightarrow \psi) \vee (\psi \rightarrow \varphi)$
From	$\neg(\varphi \rightarrow \psi)$	infer	$\varphi \wedge \neg\psi$
From	$(\varphi \wedge \psi) \rightarrow \chi$	infer	$(\varphi \rightarrow \chi) \vee (\psi \rightarrow \chi)$
From	$\varphi \rightarrow (\psi \vee \chi)$	infer	$(\varphi \rightarrow \psi) \vee (\varphi \rightarrow \chi)$

Intuitionistic logic models a constructive interpretation for mathematics:
-A proof of $(\varphi \rightarrow \psi)$ is a method for turning any proof for φ into a proof for ψ
-A proof of $\neg\varphi$ is a method for deriving a contradiction for any proof for φ
-A proof of $\varphi \wedge \psi$ is a proof of φ and a proof of ψ
-A proof of $\varphi \vee \psi$ is a proof of one of them.
-A proof for a universal statement is a method for proving all instances
-A proof for an existental statement is a proof of an instance.

Intuitionistic mathematics does not accept non-constructive proof (Like the famous Bolzano-Weissenstrass ‘proof’ that every continuous function the values of which run from negative values to positive values on the Y-axis, crosses the X-axis in some point: the B-W proof accepts the existence of this point even when we cannot construct it.) The intuitionists will not accept the theorem *in that form*, and intuitionistic mathematics will tell you under what circumstances instances of the theorem *are* acceptable.

2.4.4. Inquisitive semantics

We have a first order language with $\bot, \wedge, \vee, \rightarrow, \exists, \forall$
We define:

$$\neg\varphi \quad = \quad (\varphi \rightarrow \bot)$$
$$!\varphi \quad = \quad \neg\neg\varphi$$
$$?\varphi \quad = \quad (\varphi \vee \neg\varphi)$$

We fix a domain D of possible objects.
A **D**-world is a pair $w = \langle D, F_w \rangle$, where for every $R \in PRED^n$: $F_w(R) \subseteq D^n$
W, the set of possible worlds is the set of all D-worlds.

CGR give a substitutional semantics for predicate logic:
instead of using assignment functions, they assume you can always extend the language for every object d in the domain with a constant *d* which rigidly denotes d.
This way you can avoid using variable assignments
(although it has the disadvantage of making the language as large as its interpretation domains) .

We define a classical semantics: $[\varphi]_w = 1/0$:

$[R(t_1,\ldots,t_n)]_w = 1$ iff $\langle [t_1]_w,\ldots,[t_n]_w \rangle \in F_w(R)$; 0 otherwise
$[\bot]_w = 0$, for all $w \in W$
$[\varphi \wedge \psi]_w = 1$ iff $[\varphi]_w = 1$ and $[\psi]_w = 1$; 0 otherwise
$[\varphi \vee \psi]_w = 1$ iff $[\varphi]_w = 1$ or $[\psi]_w = 1$; 0 otherwise
$[\varphi \rightarrow \psi]_w = 1$ iff $[\varphi]_w = 0$ or $[\psi]_w = 1$; 0 otherwise
$[\forall x\varphi]_w = 1$ iff for every $d \in D$: $[\varphi[d/x]\ ]_w = 1$; 0 otherwise
$[\exists x\varphi]_w = 1$ iff for some $d \in D$: $[\varphi[d/x]\ ]_w = 1$; 0 otherwise

$[\varphi] = \{w \in W: [\varphi]_w = 1\}$
pow($[\varphi]$) is the set of all states s such that for all $w \in s$: $[\varphi]_w = 1$

With the help of this, we define the inquisitive semantics:
$[\![\varphi]\!]$ is the proposition expressed by φ

for every sentence φ: $[\![\varphi]\!]$ is a set of information states defined by:

$[\![\ R(t_1,\ldots t_n)\ ]\!] = \mathbf{pow}([R(t_1,\ldots t_n)])$
$[\![\bot]\!] = \{\emptyset\}$
$[\![\varphi \wedge \psi]\!] = [\![\varphi]\!] \cap [\![\psi]\!]$
$[\![\varphi \vee \psi]\!] = [\![\varphi]\!] \cup [\![\psi]\!]$
$[\![\varphi \rightarrow \psi]\!] = [\![\varphi]\!] \rightarrow [\![\psi]\!]$
$[\![\forall x\varphi]\!] = \cap\{[\![\varphi[d/x]\ ]\!]: d \in D\}$
$[\![\exists x\varphi]\!]_g = \cup\{[\![\varphi[d/x]\ ]\!]: d \in D\}$

Fact: for every sentence φ: $[\![\varphi]\!] \in \Pi$

$\rightarrow$ is the operation of relative pseudocomplement (see chapter 3)
* is the operation of pseudo complement (see chapter 3)

$P^* = \{s \in S$: for all $\alpha \in P$: $s \cap \alpha = \emptyset\}$
$P \rightarrow Q = \{s \in S: \forall t \subseteq s$: if $t \in P$ then $t \in Q\}$

Fact: $P^* = P \rightarrow \{\emptyset\}$

Negation is pseudocomplement:

$[\![\neg\varphi]\!] \quad = \quad [\![\varphi \rightarrow \bot]\!] \quad = \quad [\![\varphi]\!] \rightarrow \{\emptyset\} \quad = \quad [\![\varphi]\!]^*$

With this, the notions defined for propositions above can be extended to sentences:

φ **entails** ψ iff $[\![\varphi]\!] \subseteq [\![\psi]\!]$

Sentence φ is **informative** iff $\cup([\![\varphi]\!]) \neq W$
Sentence φ is **inquisitive** iff $\cup([\![\varphi]\!]) \notin [\![\varphi]\!]$

With this, CGR distinguish four categories of sentences:

φ is an **assertion** iff φ is non-inquisitive
φ is a **question** iff φ is non-informative
φ is a **tautology** iff φ is neither inquisitive nor informative
φ is a **hybrid** iff φ is both inquisitive and informative

φ is an **assertion** iff $\cup([\![\varphi]\!]) \in [\![\varphi]\!]$
φ is a **question** iff $\cup([\![\varphi]\!]) = W$
φ is a **tautology** iff $\cup([\![\varphi]\!]) = W$ and $\cup([\![\varphi]\!]) \in [\![\varphi]\!]$
φ is a **hybrid** iff $\cup([\![\varphi]\!]) \neq W$ and $\cup([\![\varphi]\!]) \notin [\![\varphi]\!]$

Some more facts:

Fact: 1. Information content is classical:
for every φ: $\cup([\![\varphi]\!]) = [\varphi]$

Fact 2: φ is a question iff $[\varphi] = W$

Questions are statements that are classical tautologies.
The question φ? is interpreted as *is φ true or isφ true?*
As a **statement** this is the tautology: $(\varphi \vee \neg\varphi)$

Fact 3: φ is an assertion iff $[\varphi] \in [\![\varphi]\!]$ iff $[\![\varphi]\!] \in$ **pow**$([\varphi])$
φ is an assertion iff $[\![\varphi]\!] =$ **pow**$(\cup([\![\varphi]\!]))$
φ is an assertion iff $[\![\varphi]\!]$ has a maximal element $(\cup([\![\varphi]\!]))$

2.4.5. Examples

Suppose we have two objects r (Ronya) and m (Minoes) and one predicate SMART
we have four worlds w_{11}, w_{10}, w_{01}, w_{00}
and SMART(r) is true in world w_{10} and SMART(m) is false in world w_{10}, etc.

Atomic sentences:

⟦SMART(r)⟧ is the set of all states s such that all the worlds in s make SMART(r) true:

$⟦\text{SMART}(r)⟧ = \{ \emptyset, \{w_{10}\}, \{w_{11}\}, \{w_{10}, w_{11}\} \}$
$[\text{SMART}(r)] = \{w_{10}, w_{11}\}$

We observe that $[\text{SMART}(r)] \in ⟦\text{SMART}(r)⟧$, hence SMART(r) is an assertion.

Fact: atomic sentences are assertions

Disjunctions:
$⟦\text{SMART}(r)⟧ = \{ \emptyset, \{w_{10}\}, \{w_{11}\}, \{w_{10}, w_{11}\} \}$
$⟦\text{SMART}(m)⟧ = \{ \emptyset, \{w_{01}\}, \{w_{11}\}, \{w_{01}, w_{11}\} \}$

$⟦\text{SMART}(r) \vee \text{SMART}(m)⟧ = ⟦\text{SMART}(r)⟧ \cup ⟦\text{SMART}(m)⟧ =$
$\{ \emptyset, \{w_{01}\}, \{w_{10}\}, \{w_{11}\}, \{w_{10}, w_{11}\} \}$

$\cup(⟦\text{SMART}(r) \vee \text{SMART}(m)⟧) = \{w_{10}, w_{01}, w_{11}\} = W - \{w_{00}\}$
Hence, $\cup(⟦\text{SMART}(r) \vee \text{SMART}(m)⟧) \neq W$, and SMART(r) ∨ SMART(m) is informative.

$\cup⟦\text{SMART}(r) \vee \text{SMART}(m)⟧ = \{w_{10}, w_{01}, w_{11}\} \notin ⟦\text{SMART}(r) \vee \text{SMART}(m)⟧$,
hence SMART(r) ∨ SMART(m) is inquisitive

Hence, SMART(r) ∨ SMART(m) is inquisitive **and** informative, and hence a **hybrid**:
it gives the information that one of the disjuncts is true, and requests information as to which of them.

Negation:
$⟦\neg\text{SMART}(r)⟧ = \{ \emptyset, \{w_{00}\}, \{w_{01}\}, \{w_{00}, w_{01}\} \}$
the set of all states that have no world in common with any state in ⟦SMART(r)⟧
Like ⟦SMART(r)⟧, this set has a maximum, and hence ⟦¬SMART(r)⟧ is an assertion.

Fact: for every φ: $\neg\varphi$ is an assertion.

Interrogatives:
?SMART(r) = SMART(r) ∨ ¬SMART(r)

$⟦\text{SMART}(r)⟧ = \{ \emptyset, \{w_{10}\}, \{w_{11}\}, \{w_{10}, w_{11}\} \}$
$⟦\neg\text{SMART}(r)⟧ = \{ \emptyset, \{w_{00}\}, \{w_{01}\}, \{w_{00}, w_{01}\} \}$
$⟦\text{SMART}(r) \vee \neg\text{SMART}(r)⟧ = \{ \emptyset, \{w_{00}\}, \{w_{10}\}, \{w_{01}\}, \{w_{11}\}, \{w_{10}, w_{11}\}, \{w_{00}, w_{01}\} \}$

$\cup$([[SMART(r) $\vee$ $\neg$SMART(r)]]) = W, hence SMART(r) $\vee$ $\neg$SMART(r) is not informative.
W $\notin$ [[SMART(r) $\vee$ $\neg$SMART(r)]], so SMART(r) $\vee$ $\neg$ SMART(r) is inquisitive.
Hence SMART(r) $\vee$ $\neg$SMART(r) is a question.

Conjunction:

Fact 1: The conjunction of two assertions is an assertion
Fact 2: The conjunction of two questions is a question

Quantified statements:

Fact 1: $\exists$xSMART(x) behaves like SMART(r) $\vee$ SMART(m)
Fact 2: $\forall$x?SMART(x) is an exhaustive question

[[SMART(r) $\vee$ $\neg$SMART(r)]] = { Ø, {w_{00}}, {w_{10}}, {w_{01}}, {w_{11}}, {w_{10}, w_{11}}, {w_{00}, w_{01}} }

[[SMART(m) $\vee$ $\neg$SMART(m)]] = { Ø, {w_{00}}, {w_{10}}, {w_{01}}, {w_{11}}, {w_{01}, w_{11}}, {w_{00}, w_{10}} }

[[$\forall$x?SMART(x)]] = [[SMART(r) $\vee$ $\neg$SMART(r)]] $\cap$ [[SMART(m) $\vee$ $\neg$ SMART(m)]] =
{Ø, {w_{00}}, {w_{01}}, {w_{10}}, {w_{11}}}

This is a question.
But it is (the closure under subset of) a **partition**.
This means that the question $\forall$x?SMART(x) is not necessarily the proper representation for *Is everybody smart* (which would be ?$\forall$xSMART(x)) but, it is an **exhaustive** question, like Groenendijk and Stokhof's wh-question: *who is smart*?

2.4.6. The connection with Hamblin semantic for questions.

GS's partition semantics for questions starts out with n-place relation α of type <s,<e^n,t>>, mapping a world and n-individuals onto a truth value.
The question forming schema is: {<w,v>: $\alpha_w = \alpha$ v}
The n-place relations form the interpretation of the interrogatives.

(1)	a.	Does Fred come?	0-place relation
	b.	Who comes?	1-place relation
	c.	Who loves whom?	2-place relation
	d.	Who introduced whom to whom?	3-place relation
	….		

The question is: how does inquisitive semantics combine with a compositional theory of the interpretation of interrogatives?
Inquisitive semantics encodes more in the notion of proposition than classical possible world semantics does. In possible world semantics you form the proposition expressed by φ by inspecting the extension of φ in different worlds. The relation between α of type <s,<e^n,t>> and associated propositions at type <s,t> is rather straightforward.

Already in Cresswell 1973 it was argued that possible world semantics allows for an alternative treatment of n-place relations, namely as relations of type <e^n,<s,t>>, propositional functions. Thomason 1980 argues that if you replace type <s,t> here by a primitive type p of propositions, structured according to what you think is appropriate, you get a finegrained theory of n-place properties as propositional functions of type $<e^n,p>$, mapping n-arguments onto a proposition. If we assume that p is the type of inquisitive propositions, we can straightforwardly lift the inquisitive semantics to n-place relations:

(1)	a.	Does Fred come?	*come*(*fred*)	of type p
	b.	Who comes?	*come*	of type $<e,p>$
	c.	Who loves whom?	*love*	of type $<e^2,p>$
	d.	Who introduced whom to whom?	*introduce*	of type $<e^3,p>$
	….			

What does this mean about the denotations of the interrogative:

(1) a. Does Fred come? *come*(*fred*) of type $<e^0,p>$

come(*fred*)

b. Who comes? *come* of type $<e^1,p>$

$\{<d_1,come(d_1)>, <d_2,come(d_2)>, <d_3,come(d_3)>\ldots\}$

c. Who loves whom? *love* of type $<e^2,p>$

$\{<<d_1,d_2>, love(d_1,d_2)>, <<d_3,d_4>, love(d_3,d_4)>, <<d_5,d_6>, love(d_5,d_6)>\ldots\}$

…

Now, apart from the n-tuples of arguments, these propositions are just the propositions that you find in question denotations according to Hamblin.

But this means that the discussion on how to derive which interpretation from which meaning (see in particular Heim *versus* Groenendijk and Stokhof) can be done from propositional functions.

Consider, for instance, the following operations on n-place properties of type $<e^n,p>$

Let α be op type $<e^n,p>$.

$$\alpha_\forall \quad = \quad \forall x_1\ldots\forall x_n.?\alpha(x_1,\ldots,x_n)$$

$$\alpha_\exists \quad = \quad \exists x_1\ldots\exists x_n.?\alpha(x_1,\ldots,x_n)$$

Consider the question (2):

(2) a. Who knows the answer to question 3?
b. $\forall x[know(x,\textit{the answer to q3}) \vee \neg\, know(x,\textit{the answer to q3}))$
c. $\exists x[know(x,\textit{the answer to q3}) \vee \neg\, know(x,\textit{the answer to q3}))$

(2b) is an exhaustive interpretation of the question, (2c) a *mention-some* interpretation. The first is appropriate in context (3a), the second in context (3b):

(3) a. After I have corrected this exam I will have found out who knows the answer to question 3.
b. Look at the screen, finger on the buzzer. Who knows the answer to question 3?

In (3b) only the person who knows and is fastest counts. I definitely do not want to know who ***also*** knew the answer.

2.4.7. The category of inquisitive semantic theories.

Classical semantics:
-A proposition is a set of worlds, a subset of W
-The set of all propositions is pow(W)
-pow(W) has the structure of a complete atomic Boolean algebra.
The Boolean structure is induced by the semantics:
$[\![\neg\varphi]\!] = W - [\![\varphi]\!]$, $[\![\varphi \wedge \psi]\!] = [\![\varphi]\!] \cap [\![\psi]\!]$, etc.
However, arguably, the situation works both ways: the powerset is most naturally endowed with a Boolean structure and directly leads us to the semantics.

Inquisitive semantics:
-The set of all propositions is a Heyting algebra.
The Heyting structure is induced by the semantics.

However, the set of propositions is actually a **complete** Heyting algebra (just like the set of all propositions in classical semantics is a **complete** Boolean algebra).

But this changes matters considerably, because there is a theorem which says:

Theorem: The class of **complete** Heyting algebras coincides with the class of **complete** distributive lattices.

Thus, while Heyting algebras are a special kind of distributive lattices, complete Heyting algebras are not are not a special kind: all complete distributive lattices are Heyting algebras.

The inquisitive semantics (and in particular the negation) is tailored to Heyting algebras, but this is not defended or motivated independently.
Within distributive lattices, Heyting algebras are one kind of generalization of Boolean algebras (weakening the negation in one way), other types of distributive lattices wiuth other kinds of negation have been studied in the literature as well. For instance, de Morgan lattices, structures that keep the law of double negation ($\varphi = \neg\neg\varphi$), but weaken the link with the laws of 0 and 1 ($\varphi \vee \neg\varphi = 1$, $\varphi \wedge \neg\varphi = 0$) arise naturally in the context of partial semantics (like the vagueness semantics discussed above).

Hence the structure of propositions given does not determine the semantics as obviously as it does in the case of classical semantics. In fact, given this general notion of proposition determining complete distributive lattices, it becomes possible and interesting to develop and compare different types of inquisitive semantics, in particular with different notions of negation. Thus, what they present is one instance of a category of inquisitive semantic theories.

2.4.8 Further topics:

-Relation with finegrained alternative semantics.

-Adding finegrainedness for highlighting and attention.

The notion of proposition is now enriched with a ‘choice of alternative paths for sharpening’. One can add think of adding a weight function for paths, making certain paths prominent and others not. (see the paper for an alternative). This can be used to distinguish, for instance, explicitly mentioned alternatives from implicit alternatives, i.e. distinguishing, for instance, $(\varphi \vee \psi \vee (\varphi \wedge \psi))$ from $(\varphi \vee \psi))$ and dealing with polarity in questions.

The latter concerns, for instance, the difference between *is the door open*? and *isn’t the door open*?

Puzzle: if both are semantically the same question, why are they answered differently (i.e. *yes* selects a different answers).

$\text{Open}(d)? = \text{Open}(d) \vee \neg\text{Open}(d)$

$\neg\text{Open}(d)? = \neg\text{Open}(d) \vee \neg\neg\text{Open}(d)$

While it is true that $\neg\neg\text{Open}(d)$ is not equivalent to Open(d) in this logic, it is not clear that *that* is very useful here, because we may well assume, contextually, that we are in a situation where also intuitionists would conclude φ, even though they have only derived $\neg\neg\varphi$ (say, where the predicate involved is decidable and the domain is tractable).

-The approach to inquisitive semantics makes one wonder about *imperative semantics*. abstracts relate to imperatives in the way that abstracts relate to questions (one place: *go away*, two-place: *one, two, three, kiss*). Can the approach of inquisitive semantics contribute to imperitive semantics?

In fact, on the ILLC paper there is a link to the Festschrift for Jeroen Groenendijk, Martin Stokhof and Frank Veltman, with a link to a paper by Maria Aloni and Ivano Ciardelli which addresses just this issue.