PART 2: MODALITY

I. MODAL EXPRESSIONS IN DIFFERENT CATEGORIES

Auxiary verbs.

(1) a. You *may* be excused. b. I *must* be hungry.
c. You *can't* do that. d. You *have to* come home at 9.00
e. I *could* have been rich. f. I *had to* be in Amsterdam.
f. I *might* have known that.

Adverbials.

(2) a. *Possibly,* John met Mary in Amsterdam.
b. I don't *necessarily* think that was a good idea.
c. *Maybe* John will come to the party.

Adjectives.

(3) a. This is a *possible* counterexample.
b. John is a *potential* candidate.
c. It is *possible* that John will try to reach you.
d. John was *able* to come.

Suffixes: *-able.*

(4) a. This glass is highly break*able*.
b. This is unthink*able*.
c. The rent is pay*able* at the end of the month. ! *zahlbar* but not *betaalbaar*

Generic present

(5) This car *goes* 200 km/h/ This car *does* 300 km/h/
Fred eats horsemeat (but he hasn't yet)

Progressive

(5a) I *am* draw*ing* a circle
(5b) Professor Lupin *was* creat*ing* a boggard when he was interrupted.

Counterfactual conditionals.

(6) a. *If* she *hadn't* left me, I *wouldn't* be so miserable now.
b. *If* Verdi and Bizet *had been* co-patriots, Bizet *might* have been italian.

...

II. MODALS CREATE INTENSIONAL CONTEXTS

Substitution of extensions (=extensionality) is not valid.

(7) a. Proust could have been the author of Ulysses.
b. The author of Ulyssess is the author of Finnegans Wake.

do not entail

c. Proust could have been the author of Finnegans Wake.

(8) a. If Henrico Granados had written Ulysses, then the author of Ulysses would have died on m.s Essex.
b. The author of Ulysses is James Joyce.

do not entail

c. If Henrico Granados had written Ulysses, James Joyce would have died on m.s. Essex.

De dicto-de re ambiguities.

(9) I could have been married to a Swede.

Situation 1:
Helga and I considered marriage. In the end we decided not to. (7) is true. *de re*.
Situation 2:
There was a time, when I was "into" Sweden. If I had met a Swedish girl then (which I didn't), I might have proposed marriage. (7) is true. *de dicto*.

(10) Every suspect may be innocent.

a. For each suspect, the possibility that that suspect is innocent has to be kept open (though we may know for sure that one of them did it): *de re*.

b. We have to keep open the possibility that the one who did it is not among our suspects. *de dicto*.

Ambiguities with negation:

(1) a. You can not love it, but it is completely unique [γ]	ambiguous
reading a: it is unloveable to anybody	not - can
reading b: there are those that don't love it	can - not
b. You can't love it, but is is completely unique	not ambiguous
reading a: it is unloveable to anybody	not - can

III. INTERACTION OF MODALS WITH ¬, ∧, ∨

I will use the modals *could have* and *had to* as examples.
We assume, for ease of examples, that we restrict ourselves to natural models in which:

I stayed ⇔ I didn't leave   STAY(I) ⇔ ¬LEAVE(I)

In our formal language we use:

□ for *necessity*, *had to*, *must*
◊ for *possibility*, *could have*, *may*

Interaction with negation:

(11) a. I couldn't have stayed. ¬◊¬LEAVE(I)
b. I had to leave. □ LEAVE(I)
(11a) ⇔ (11b)

(12) a. I could have stayed. ◊ ¬LEAVE(I)
b. I didn't have to leave. ¬ □ LEAVE(I)
(12a) ⇔ (12b)

(13) a. I couldn't have left. ¬ ◊ LEAVE(I)
b. I had to stay. □ ¬ LEAVE(I)
(13a) ⇔ (13b)

(14) a. I could have left. ◊ LEAVE(I)
b. I didn't have to stay. ¬ □ ¬LEAVE(I)
(14a) ⇔ (14b)

Interaction with conjunction and disjunction.

(1) I could have sung or danced. (1) $\Diamond$ (SING(I) $\vee$ DANCE(I))
(2) I could have sung or I could have danced. (2) $\Diamond$ SING(I) $\vee$ $\Diamond$ DANCE(I)
(3) I could have sung and I could have danced. (3) $\Diamond$ SING(I) $\wedge$ $\Diamond$ DANCE(I)
(4) I could have sung and danced. (4) $\Diamond$ (SING(I) $\wedge$ DANCE(I))

As always, (3) $\Rightarrow$ (2), and (2) does not entail (3).

-Look at (3) and (4).
Clearly, (4) $\Rightarrow$ (3): If I could have sung and danced, I could have sung, etc.
But (3) does not entail (4).
I am the kind of person who can sing, and who can dance, but, like Gerald Ford, I can do only one thing at a time. (3) is true, but (4) is false.

cf also:
(15) a. I could have stayed and I could have left.
b. I could have stayed and left.

(15a) can easily be true, but (15b) is a contradiction, so (15a) does not entail (15b).

(1) I could have sung or danced. (1) $\Diamond(\text{SING}(I) \vee \text{DANCE}(I))$
(2) I could have sung or I could have danced. (2) $\Diamond \text{SING}(I) \vee \Diamond \text{DANCE}(I)$
(3) I could have sung and I could have danced. (3) $\Diamond \text{SING}(I) \wedge \Diamond \text{DANCE}(I)$
(4) I could have sung and danced. (4) $\Diamond(\text{SING}(I) \wedge \text{DANCE}(I))$

-Look at (1) and (2).
If I could have sung, then I could have sung or danced, etc.
So (2) $\Rightarrow$ (1).

In several kinds of contexts, we may feel that (1) entails (3).

But not in all!

I know that either dancing was allowed and singing forbidden, or singing was allowed and dancing forbidden. But I don't remember which. (1) is true, but (3) is false. Hence (1) does not entail (3).

(1) $\Rightarrow$ (2): If I could have sung or danced, and I couldnt' have sung, then I could have danced.
So: (1) $\Leftrightarrow$ (2).

We get the pattern:

$$\begin{array}{c} (1) \Leftrightarrow (2) \\ \Uparrow \\ (3) \\ \Uparrow \\ (4) \end{array} \qquad \text{SOME}$$

(1) I had to sing or dance.
(2) I had to sing or I had to dance.
(3) I had to sing and I had to dance.
(4) I had to sing and dance.

(1) $\Box$ (SING(I) $\vee$ DANCE(I))
(2) $\Box$ SING(I) $\vee$ $\Box$ DANCE(I)
(3) $\Box$ SING(I) $\wedge$ $\Box$ DANCE(I)
(4) $\Box$ (SING(I) $\wedge$ DANCE(I))

(3) $\Rightarrow$ (2), (2) doesn't entail (3).

Clearly (3) $\Leftrightarrow$ (4):
If I had to sing and dance, I had to sing.
If I had to sing and I had to dance, I had to do both.

(2) $\Rightarrow$ (1)
If I had to sing, I had to sing or dance, etc.
But (1) does not entail (2).
The club membership prescribes that each member chooses to sing a song or dance a dance, but there is no prescription that it has to be a song, nor that it has to be a dance.
(1) is true, (2) is false.

cf.
(16) a. The coin had to come down heads or tails.
b. The coin had to come down heads or the coin had to come down tails.

(16a) expresses that the coin has to land on one of its sides.
(16b) expresses that the coin is tampered with (or being influenced).
Clearly, (16a) does not entail (16b).

We get the pattern:

(1) EVERY
$\Uparrow$
(2)
$\Uparrow$
(3) $\Leftrightarrow$ (4)

We conclude: **Modals are quantifiers**.
□ is a **universal quantifier.**
◊ is an **existential quantifier.**

But if □ and ◊ are quantifiers, there got to be things that they quantify over.
We call them **possibilities**.
Hence there is evidence for **quantification over possibilities** in natural language.

Terminology: possibilities = alternatives = alternative situations =
possible situations = possible worlds.

I will use the latter terminology, althought sometimes we think of worlds, sometimes of world-times, sometimes just of times, sometimes of world-time-contexts.
Montague call them with a neutral term *indices*.
We can also call them *parameters of variation*.

Idea: □φ is true iff φ is true in every possible world.
◊φ is true iff φ is true in some possible world.

Gottfried Wilhelm Leibniz introduced the idea of possible worlds in the context of a philosophic study of the notion of necessity in the 17th century.

Rudolf Carnap revived the idea in Meaning and Necessity 1947, analyzing necessity as truth in all models (treating models as possible situations).

But, C. I. Lewis had started the study of modal logic in 1912, and Lewis introduced different modal systems, in which necessity and possibility had different properties.
This could not be dealt with in Carnap's analysis, it only deals with logical necessity.

By the mid 1950s various logicians were playing with similar ideas to resolve these problems and provide a Tarski style semantics for the modal logics of C. I. Lewis.
I mention Jaakko Hintikka, Richard Montague, Stig Kanger, Evert Beth.

But the person who solved the problems systematically and provided provably correct and complete semantics for the different modal systems, and for intuitionistic logic as well was a teenager: Saul Kripke.
(Classical papers, the first *published* when he was 19:
A Completeness Theorem in Modal Logic 1959
Semantical Considerations on Modal Logic 1963
Semantical analysis of Intuitionistic Logic 1963)

Possible world semantics

Possible world semantics explains intensionality and *de dicto*-*de re* ambiguities.

-Intensionality.

We **evaluate** *John walks* in a situation (= assign a truth value *relative to a world*).

The **extension** (truth value) of *John walks* varies, depending on which possible situation (world) we are looking at.

The **intension** of *John walks* is the **pattern of variation** of the extension of *John walks* across situations (worlds).

The **intension** of *John walks* specifies for each possible situation what the truth value of *John walks* is in that situation. This is a **function from worlds to truth values**.

An **extensional context** is a context which is only sensitive to the **extension** of what fills the context:

Example: negation: $\neg(...)$
The truth value of $\neg(\varphi)$ in possible world w depends only on the truth value of φ in w.

An **intensional context** is a context which is sensitive to the intension of what fills the context, the pattern of variation of the extension of what fills the context across possible worlds.

The modal operators $\Box$ and $\Diamond$ describe properties of the pattern of variation of the extension across possible worlds.
(Just like $\forall x$ and $\exists x$ describe properties of the pattern of variation of the extension across resettings of the value of x in the assignment function)

Thus, to determine the truth value of $\Box\varphi$ and $\Diamond\varphi$ in world w, it is not sufficient to know the truth value of φ in w; we need to know the truth value of φ **in other worlds**.

If α and β have the same extension in world w, that doesn't guarantee that they have the same extension in every other world.

This means that , while $\varphi(\alpha)$ and $\varphi[\beta/\alpha]$ have the same truth value in world w, they may well have different truth values in other worlds.

Consequently, the truth values of $\Box\varphi(\alpha)$ and $\Box\varphi[\beta/\alpha]$ in world w may be different (the same for $\Diamond\varphi(\alpha)$ and $\Diamond\varphi[\beta/\alpha]$).

Consequently, $\Box$ and $\Diamond$ are **intensional contexts**.

***-de dicto-de re* ambiguities**.

$\Box$ and $\Diamond$ are quantifiers over possible worlds. As quantifiers, we expect the same kind of **scope interactions** we find for normal quantifiers.

(10) Every suspect may be innocent.

a. *De dicto*: $\Diamond\forall x[\text{SUSPECT}(x) \rightarrow \text{INNOCENT}(x)]$

There is a possible situation where all of the suspects are innocent.

b. *De re*: $\forall x[\text{SUSPECT}(x) \rightarrow \Diamond\text{INNOCENT}(x)]$

For each suspect, there is a possible situation where that suspect is innocent.

Thus, *de dicto-de re* ambiguities with modals reduce to scope ambiguities.

IV. VARIABILITY AND CONTEXT DEPENDENCY OF MODALS

(1) I could have married you, but now I can't anymore.
(2) Before they changed the law, I had to get a visa, but after, I could come without a visa.

Observation: what is possible **varies with time**.

(3) I am not able to play the saxophone.

(3) is **context dependent**: the **nature** of the modality depends on the context.

(3a) **Skill.**
In view of the fact that I never learned how to play, I am not able to play the saxophone.

(3b) **Opportunity.**
In view of the fact that my instrument was put on a plane to Ipamena, I am not able to play the saxophone.

(3c) **Disablement.**
In view of the fact that my fingers are frozen, I am not able to play the saxophone.

(3d) **Limitation.**
In view of the fact that it is 3.00 am, and my neighbour is a light-sleeping, irritable heavy-weight, I am not able to play the saxophone.
etc.

(3) can have a different truth value dependent on **which** modality is meant.

Consequently, what is possible **varies with the nature of the modality.**

Contradiction test:
(4) a. Can you play the saxophone?
b. I can and I can't.
c. In view of A I can, in view of B I can't.

(4b) need not be a contradiction, because it can be resolved as (4c).

Note:

(5) a. I play, even though I am not able to (play).
b. #I lift the fridge, even though I am not able to (lift the fridge).

(5a) involves different senses of *play*. This is shown by the infelicity of (5b).

Epistemic modality: In view of what we know and don't know.
Deontic modality: In view of what is commanded and allowed.
Ability modality: (also called **dynamic** modality)
In view of our capacities and limitations.

may, must

(6) a. I may have told John this.
b. I must have told John this.

John may be in Amsterdam right now.
As far as I know, he may be anywhere

Epistemic modality: natural

(7) a. You may walk on the grass.
b. You must stay on the sidewalk.

Deontic modality: easily possible

(8) a. You may play the clarinet.
b. You must play the clarinet.

Ability modality: impossible

(8a) does not mean that you have the ability, capacity to play the clarinet

	Epistemic	Deontic	Ability
may, must	✓✓	✓	#

can, has to

(9) a. I can tell you that you're gonna have a problem.
b. I can't tell you who did it (because I don't know)
c. John can’t be the murderer

Epistemic modality: easily possible

(10) a. You can take a cookie.
b. You can't take a chocolate.

Deontic modality: natural

(11) a. I can lift a refrigerator.
b. I can't lift a washing machine.

Ability modality: also possible

	Epistemic	Deontic	Ability
can, must	✓✓	✓	#
can, has to	✓	✓✓	✓

be able to

(12) a. I am not able to tell you who is the murderer, because I don't know.
b. I am not able to play the clarinet, because there is a law against it.
c. He is able to be anywhere.

be able to can only have an epistemic or deontic effect **indirectly*:*** in so far as lack of knowledge or a prescription limits **opportunity**.

(12') a. ✓Because I am not able to get down the stairs, I *have to* stay at home .
Ability
b. ?Because I am not able to get down the stairs, I *must* stay at home.
Not clear that this is ability.

Epistemic modality: impossible
Deontic modality: impossible
Ability modality: natural.

	Epistemic	Deontic	Ability
can, must	✓✓	✓	#
can, has to	✓	✓✓	✓
be able to	**#?**	**#?**	✓✓

To show this in a different way, compare *be able to* with *can*.

(13) Everybody can be the lucky winner.

a. $\forall x[\ \Diamond\ \text{WINNER}(x)\ ]$

b. $\Diamond\ \forall x[\ \text{WINNER}(x)\ ]$

Reading (13a) allows epistemic, deontic or ability interpretations.
Reading (13b) **does not allow** an ability interpretation.
We can paraphrase this reading of (13a) as (13c):

(13) c. It can turn out to be the case that everybody is the lucky winner.

and (13c) does not have an ability interpretation.

(14) Everybody is able to be the lucky winner.

a. $\forall x[\ \Diamond\ \text{WINNER}(x)\ ]$

b. $\Diamond\ \forall x[\ \text{WINNER}(x)\ ]$ Impossible.

Reading (14a) does not allow an epistemic interpretation.
Reading (14b) is impossible, as can be seen in the paraphrase in (14c):

(14) c. #It is able to be the case that everybody is the lucky winner.

Explanation:

Ability modality is subject dependent: $\Diamond_x$
Epistemic modality is not subject dependent: $\Diamond$

be able to expresses subject dependent modality.
can can express subject dependent or subject independent modality.

Consequence:

The modal can scope over the subject iff the modal is not subject dependent.

Reason: variable x in $\Diamond_x$ would be **free**:

\# $\Diamond_{\mathbf{x}}\ \forall x[\text{WINNER}(x)]$

So what we get is:

(13) Everybody can be the lucky winner.

a_1. $\forall x[\ \Diamond\ \text{WINNER}(x)\ ]$

a_2. $\forall x[\ \Diamond_x\ \text{WINNER}(x)\ ]$

b_1. $\Diamond\ \forall x[\ \text{WINNER}(x)\ ]$

b_2. # $\Diamond_x\ \forall x[\text{WINNER}(x)]$

The (a) interpretation allows an episitemic interpretation (by a_1) and an ability interpretation (by a_2).

The (b) interpretation allows an epistemic interpretation, but not an ability interpretation.

(14) Everybody is able to be the lucky winner.

$\forall x[\ \Diamond_x\ \text{WINNER}(x)\ ]$

(14) only allows a narrow scope ability interpretation.

One more type of modality:

Circumstantial modality: in view of the circumstances.

(20) (In view of the laws of physics), the ball that I throw up must come back to earth.

Conclusion: which possible worlds restrict the modal quantifier varies with time and the nature of the modality.

Different interpretations of the modals have **different entailment patterns.**

Ability.

(15) a. (In view of my lack of musical education), I am not able to play the bassoon.
entails
b. I don't play the bassoon.

(15a): $\neg\Diamond$ PLAY(I) = $\Box\neg$PLAY(I)
(15b): $\neg$PLAY(I)
Hence: $\Box\neg$PLAY(I) $\Rightarrow$ $\neg$PLAY(I)

Ability: $\Box\varphi \Rightarrow \varphi$

Deontic.

(16) a. (In view of the law), you can't walk on the grass.
does not entail
b. You don't walk on the grass.

(16a): $\neg\Diamond$ WALK(YOU) = $\Box\neg$WALK(YOU)
(16b): $\neg$WALK(YOU)
Hence: $\Box\neg$WALK(YOU) does not entail $\neg$WALK(YOU)

Deontic: $\Box\varphi$ does not entail φ

Epistemic.

(17) a. (In view of the argument Hercule Poirot made), Bill must be the murderer.
b. Bill is the murderer.
(17a) $\Box$ MURDERER(BILL)
(17b) MURDERER(BILL)
If anything, (17b) entails (17a): (17b) is a **stronger** statement.

Epistemic:(?) $\varphi \Rightarrow \Box\varphi$

Direct and indirect evidence:

I know my digestive system by introspection (**direct**),
yours only by external clues (**indirect**)

(18)	a. I am hungry.	Natural
	b. You are hungry.	Impolite
(19)	a. I must be hungry.	As if I deduce from external clues
	b. You must be hungry.	Natural

The statement without the modal expresses **direct epistemic evidence**, the statement with the modal expresses **indirect epistemic evidence.**
Since direct evidence is stronger than indirect evidence, we get the effect in (17).

V. MODAL BASES AS ACCESSIBILITY RELATIONS (Kripke models)

Conclusions so far:

1. We associate with natural language modals quantifiers over possible worlds.

2. For each natural language modal it is lexically determined what the **force** of that quantifier is:

Modal Force: *can, could, may, be able to, possibly* are
existential quantifiers over possible worlds.

must, has to, had to, neccesarily are
universal quantifiers over possible worlds.

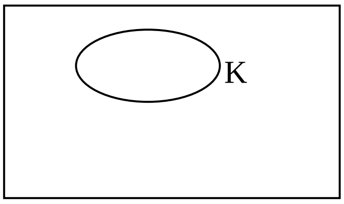
3. The quantification is contextually restricted:
what the relevant alternatives are that we quantify over varies with time and depends on the **nature of the modality**.
The latter we call the **modal base** (Kratzer 1983, the Notional Category of Modality):
The **context** makes available a **modal base** which restricts the quantification.

4. Modals vary in what modal bases are available for them:
i.e. *must, may***:** epistemic, deontic modal base, not ability modal base.
be able to: not epistemic, deontic modal base, subject dependent ability modal base.
etc.

What is a modal base?
A modal base M determines for each world (and moment of time) the set of alternative worlds that are relevant for the modal quantification.

An **epistemic modal base** determines what is known, what isn't known, what is compatible with our knowledge, and what isn't in a situation.

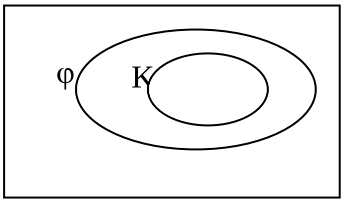
Idea: The **epistemic modal base** associates with a situation **the set of all worlds compatible with what we know.**



W

W: the set of all worlds.
K: the set of worlds compatible with what we know.

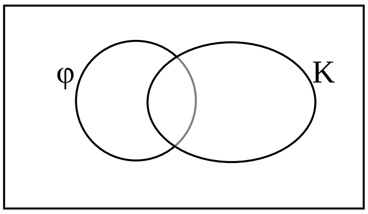
φ **follows from** what we know iff φ is true in all worlds compatible with what we know.



W

W: the set of all worlds.
K: the set of all worlds compatible with what we know.
φ: the set of all worlds where φ is true.

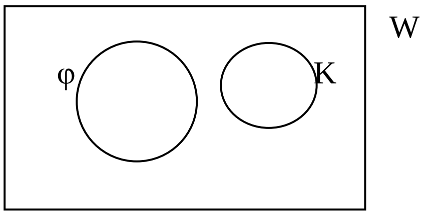
φ is **compatible with** what we know iff φ is true in some world compatible with what we know.



W

W: the set of all worlds.
K: the set of all worlds compatible with what we know.
φ: the set of all worlds where φ is true.

Hence φ is **incompatible** with what we know iff φ is false in all worlds compatible with what we know.



W: the set of all worlds.
K: the set of all worlds compatible with what we know.
φ: the set of all worlds where φ is true.

We don't know whether φ iff both φ and ¬φ are compatible with what we know, and this means that in some of the worlds compatible with what we know φ is true, in the others ¬φ is true.

A **deontic modal base** determines what is commanded, what is allowed in a situation.

Idea: The **deontic modal base** associates with a situation **the set of all worlds compatible with what is commanded, the (contextually given) 'law'.**

φ **follows from** the law iff φ is true in all worlds compatible with the law.
φ is **compatible with** the law iff φ is true in some world compatible with the law.
Hence φ is incompatible with the law iff φ is false in all worlds compatible with the law.

Note: the real world is not necessarily compatible with the law, in fact, in most cases it isn't. This means that the set of worlds compatible with the law will not usually include the real world.

And this means that if φ is true in all worlds compatible with the law, it **doesn't follow** that φ is true in the real world.
(This is going to mean that, on the deontic interpretation, $\Box\varphi$ does not entail φ).

In general:

> **A modal base M associates with every world w a set of worlds, M_w, which is the set of all worlds compatible with the content of M in w.**

Equivalently:

> **A modal base M is a relation between possible worlds.**
>
> **M(w,v)** means: $v \in M_w$
>
> We call such relations between possible worlds **accessibility relations.** (Kripke 1959).
>
> **M(w,v)** means: world v is accessible from world w, according to the content of M.

If M is, say, a deontic modal base, then M(w,v) means:
v is **deontically accessible from** w

v is deontically accessible from w iff v is one of the possible situations that has to be considered when we are asking what is forbidden and what is allowed.

The set of all deontically accessible worlds in w, is the set of all worlds, relevant for w, where no law that is commanded in w by M is broken.

This means, for example, that if it is a law in w that you are not allowed to walk on the grass, then in all worlds, deontically accessible from w, nobody walks on the grass.

Similarly, if M is epistemic accessibility, then M(w,v) means:
in v nothing happens that we know in w is not the case.

If M_d is an ability modal base for d, then $M_d(w,v)$ means:
in v nothing happens that is beyond the ability of d in w.

Note, we can, in context, distinguish different subkinds of modal bases.
Also modal bases may overlap.
cf:

(20) a. (In view of the laws of physics), the ball that I throw up must come back to earth.
Circumstantial modal base.
b.(In view of what we know about the laws of physics), the ball that I throw up must come back to earth.
Circumstantial epistemic modal base.

Also, modal bases can have an ordering relation on them (Kratzer 1983):

(21) a. You must give to the poor.

Modal base: deontic.
In every deontically accessible world you give to the poor.
b. You *ought to* give to the poor.

Modal base: deontic.
Ordering relation: The ideal behaviour of a virtuous person in an ideal situation.
Not: In every deontically accessible world you give to the poor.
(Most religions have that much common sense.)

But: In every deontically accessible world **which is an ethically ideal world** you give to the poor.

In every deontically accessible situation in which the yoke of financial pressures of the real world is lifted and you have high ethical standards, you give to the poor.

In sum:
We associate with a modal a **modal force** and, in context, a **modal base** (accessibility relation) that can be expressed by that modal.
The modal base restricts the quantification of the modal force to quantification over worlds that are accessible, according to the modal base.

VI. L_6, THE LANGUAGE OF MODAL PREDICATE LOGIC (Kripke 1959)

The language of modal predicate logic, L_6 is our language L_4 with **two** new syntactic clauses:

(1) If $\varphi \in$ FORM, then $\Box\varphi \in$ FORM
(2) If $\varphi \in$ FORM, then $\Diamond\varphi \in$ FORM

This gives us formulas of the form:

$\exists x[P(x) \wedge \Diamond R(x,x)]$
$\Box\forall x[P(x) \rightarrow \Diamond R(x,x)]$

To keep things simple, we will only discuss the case where there is **one** modal base available.

We specify the semantics for L_6.

Models.

A **model for** L_6 is a structure: $M = \langle W_M, R_M, D_M, F_M \rangle$ where:

1. W_M is a non-empty set, the set of all **possible worlds**.

2. $R_M \subseteq W_M \times W_M$. R_M, the **accessibility relation**, is a relation between possible worlds. The **modal base**.

3. D_M is a non-empty set, the **domain of possible individuals**.

4. F_M is the interpretation function for the lexical items.

F_M assigns to every lexical item **an extension in every world** (since in this language, extensions of expressions vary from world to world).

This means that F_M is a function **from lexical items and worlds** to extensions.

a. F_M: CON $\times W_M \rightarrow D_M$
for every individual constant $c \in$ CON and every world $w \in W_M$:
$F_M(c,w) \in D_M$.

Condition: Rigidity (discussed later):
for every $c \in$ CON, and every $w,v \in W_M$: $F_M(c,w) = F_M(c,v)$

Names do not vary their extension from world to world (unlike, as we will see definite noun phrases like *the president*, σ(PRESIDENT)).

b. for every $n>0$: F_M: PRED$^n \times W_M \rightarrow pow(D^n)$
for every world $w \in W_M$ and every predicate $P \in$ PRED^n: $F_M(P) \subseteq D^n$.

Thus, a predicate like WALK denotes in each world w a set of individuals: the individuals that walk in w.
Obviously, in different world, different individuals walk, so predicates **do** vary their extension from world to world.

c. $F_M: \{\neg\} \times W_M \to (\{0,1\} \to \{0,1\})$

for every $w \in W_M$: $F_M(\neg,w) = \begin{pmatrix} 0 \to 1 \\ 1 \to 0 \end{pmatrix}$

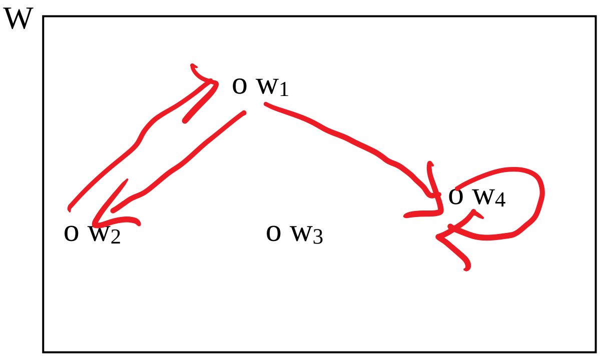
$F_M: \{\wedge,\vee,\to\} \times W_M \to \{0,1\} \times \{0,1\} \to \{0,1\}$

d. for every $w \in W_M$: $F_M(\wedge,w) = \begin{pmatrix} <1,1> \to 1 \\ <1,0> \to 0 \\ <0,1> \to 0 \\ <0,0> \to 0 \end{pmatrix}$

e. for every $w \in W_M$: $F_M(\vee,w) = \begin{pmatrix} <1,1> \to 1 \\ <1,0> \to 1 \\ <0,1> \to 1 \\ <0,0> \to 0 \end{pmatrix}$

f. for every $w \in W_M$: $F_M(\to,w) = \begin{pmatrix} <1,1> \to 1 \\ <1,0> \to 0 \\ <0,1> \to 1 \\ <0,0> \to 1 \end{pmatrix}$

We draw pictures of the accessibility relation between worlds in the same way as usual for two place relations:



This indicates: $R = \{<w_1,w_2>, <w_1,w_4>, <w_2,w_1>, <w_4,w_4>\}$

Thus, the set of worlds accessible from w_1 is $\{w_2,w_4\}$
the set of worlds accessible from w_2 is $\{w_1\}$
the set of worlds accessible from w_3 is $\varnothing$
the set of worlds accessible from w_4 is $\{w_4\}$

Assignment functions:

As before an assignment function for L_6 on M if a function g: VAR $\rightarrow D_M$

Note: assignment functions are **not** sensitive to possible worlds: variables get assigned a value **independent** of possible worlds (this is important).

Compositional semantics.

For every L_6 model M = $<W_M,R_M,D_M,F_M>$, every world $w \in W_M$, and every assignment g for L_6 on M,
we define for every L_6 expression α: $[\![\alpha]\!]_{M,g}$

$[\![\alpha]\!]_{M,w,g}$, the extension of expression α in model M in world w relative to assignment g.

0. if $\alpha \in$ LEX, then
$[\![\alpha]\!]_{M,w,g} = F_M(\alpha,w)$

If $x \in$ VAR, then
$[\![x]\!]_{M,w,g} = g(x)$

1. If $\alpha_1,...\alpha_n \in$ TERM and $P \in PRED^n$, then
$[\![P(\alpha_1,...,\alpha_n)]\!]_{M,w,g} = 1$ iff $< [\![\alpha_1]\!]_{M,w,g},..., [\![\alpha_n]\!]_{M,w,g} > \in [\![P]\!]_{M,w,g}$; 0 otherwise.

If $\alpha_1, \alpha_2 \in$ TERM, then
$[\![(\alpha_1=\alpha_2)]\!]_{M,w,g} = 1$ iff $[\![\alpha_1]\!]_{M,w,g} = [\![\alpha_2]\!]_{M,w,g}$; 0 otherwise.

2. If $\varphi,\psi \in$ FORM then:
$[\![\neg\varphi]\!]_{M,w,g} = [\![\neg]\!]_{M,w,g} ([\![\varphi]\!]_{M,w,g})$

$[\![(\varphi \wedge \psi)]\!]_{M,w,g} = [\![\wedge]\!]_{M,w,g} (<[\![\varphi]\!]_{M,w,g}, [\![\psi]\!]_{M,w,g}>)$

$[\![(\varphi \vee \psi)]\!]_{M,w,g} = [\![\vee]\!]_{M,w,g} (<[\![\varphi]\!]_{M,w,g}, [\![\psi]\!]_{M,w,g}>)$

$[\![(\varphi \rightarrow \psi)]\!]_{M,w,g} = [\![\rightarrow]\!]_{M,w,g} (<[\![\varphi]\!]_{M,w,g}, [\![\psi]\!]_{M,w,g}>)$

3. If $x \in$ VAR and $\varphi \in$ FORM then:
$[\![\forall x\varphi]\!]_{M,w,g} = 1$ iff for every $d \in D_M$: $[\![\varphi]\!]_{M,w,g_x^d} =1$; 0 otherwise

$[\![\exists x\varphi]\!]_{M,w,g} = 1$ iff for some $d \in D_M$: $[\![\varphi]\!]_{M,w,g_x^d} =1$; 0 otherwise

4. $[\![\Box\varphi]\!]_{M,w,g} = 1$ iff **for every $v \in W_M$: if R(w,v) then** $[\![\varphi]\!]_{M,v,g} = 1$;
0 otherwise

$[\![\Diamond\varphi]\!]_{M,w,g} = 1$ iff **for some $v \in W_M$: R(w,v) and** $[\![\varphi]\!]_{M,v,g} = 1$;
0 otherwise

Truth in world w.
Let φ be an L_6 sentence.

$[\![\varphi]\!]_{M,w} = 1$ iff for every g: $[\![\varphi]\!]_{M,w,g} = 1$
$[\![\varphi]\!]_{M,w} = 0$ iff for every g: $[\![\varphi]\!]_{M,w,g} = 0$

Entailment for L_6.

φ entails ψ, $\varphi \Rightarrow \psi$ iff for every model M for L_6, for every world $w \in W_M$:
if $[\![\varphi]\!]_{M,w} = 1$ then $[\![\psi]\!]_{M,w} = 1$

$\varphi \Leftrightarrow \psi$ iff $\varphi \Rightarrow \psi$ and $\psi \Rightarrow \varphi$

So φ entails ψ iff for every model and world where φ is true, ψ is true.

Truth in a model:

$[\![\varphi]\!]_M = 1$ iff for every world $w \in W_M$: $[\![\varphi]\!]_{M,w} = 1$

Equivalent definition of entailment:

The **proposition expressed by** φ:

$[\![\varphi]\!]_M = \{w \in W_M: [\![\varphi]\!]_{M,w} = 1\}$

The set of all worlds where φ is true.

φ entails ψ, $\varphi \Rightarrow \psi$ iff for every model M for L_6, $[\![\varphi]\!]_M \subseteq [\![\psi]\!]_M$

Entailment = subset on the set of sets of possible worlds

More connections:

$[\![(\varphi \wedge \psi)]\!]_M = [\![\varphi]\!]_M \cap [\![\psi]\!]_M$

$$\begin{aligned}
[\![(\varphi \wedge \psi)]\!]_M &= \{w \in W_M: [\![(\varphi \wedge \psi)]\!]_{M,w} = 1\} \\
&= \{w \in W_M: [\![\varphi]\!]_{M,w} = 1 \text{ and } [\![\psi]\!]_{M,w} = 1\} \\
&= \{w \in W_M: [\![\varphi]\!]_{M,w} = 1\} \cap \{w \in W_M: [\![\psi]\!]_{M,w} = 1\} \\
&= [\![\varphi]\!]_M \cap [\![\psi]\!]_M
\end{aligned}$$

Conjunction = intersection on the set of of sets of possible worlds

$[\![(\varphi \vee \psi)]\!]_M = [\![\varphi]\!]_M \cup [\![\psi]\!]_M$

$$\begin{aligned}
[\![(\varphi \vee \psi)]\!]_M &= \{w \in W_M: [\![(\varphi \vee \psi)]\!]_{M,w} = 1\} \\
&= \{w \in W_M: [\![\varphi]\!]_{M,w} = 1 \text{ or } [\![\psi]\!]_{M,w} = 1\} \\
&= \{w \in W_M: [\![\varphi]\!]_{M,w} = 1\} \cup \{w \in W_M: [\![\psi]\!]_{M,w} = 1\} \\
&= [\![\varphi]\!]_M \cup [\![\psi]\!]_M
\end{aligned}$$

Disjunction = union on the set of of sets of possible worlds

$[\![\neg\varphi]\!]_M = W_M - [\![\varphi]\!]_M$

$$\begin{aligned}
[\![\neg\varphi]\!]_M &= \{w \in W_M: [\![\neg\varphi]\!]_{M,w} = 1\} \\
&= \{w \in W_M: [\![\varphi]\!]_{M,w} = 0\} \\
&= W_M - \{w \in W_M: [\![\varphi]\!]_{M,w} = 1\} \\
&= W_M - [\![\varphi]\!]_M
\end{aligned}$$

Negation = complementation on the set of sets of possible worlds

The semantics makes the following facts true:

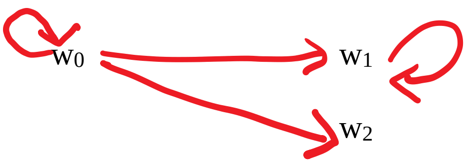
$$\Box\varphi \Leftrightarrow \neg\Diamond\neg\varphi \qquad \Box\neg\varphi \Leftrightarrow \neg\Diamond\varphi$$
$$\neg\Box\varphi \Leftrightarrow \Diamond\neg\varphi \qquad \neg\Box\neg\varphi \Leftrightarrow \Diamond\varphi$$

$$\Diamond(\varphi \vee \psi) \Leftrightarrow \Diamond\varphi \vee \Diamond\psi$$
$$\Uparrow$$
$$\Diamond\varphi \wedge \Diamond\psi$$
$$\Uparrow$$
$$\Diamond(\varphi \wedge \psi)$$

$$\Box(\varphi \vee \psi)$$
$$\Uparrow$$
$$\Box\varphi \vee \Box\psi$$
$$\Uparrow$$
$$\Box(\varphi \wedge \psi) \Leftrightarrow \Box\varphi \wedge \Box\psi$$

The dreamer, the fatalist, and the dogmatic.

Let φ be any formula that is not a contradiction or a tautology and assume that φ is true in some world in W_M and false in some other world in W_M



The dreamer lives in world w_0: in w_0 everything is possible, nothing is necessary.
Since every world is accessible from w_0: $\Diamond\varphi$ is true in w_0 and $\Box\varphi$ is false in w_0.

The fatalist lives in world w_1: in w_1 only what is actual is possible and
what is actual (and only that) is necessary.
Since only w_1 is accessible from w_1: $\Diamond\varphi$ and $\Box\varphi$ are true in w_1 iff φ is true in w_1
Why is the person in w_1 a fatalist? Because for the fatalist there is no hope.

The dogmatic lives in world w_2: in w_2 nothing is possible and everything is necessary.
Since no world is accesible from w_2 $\Diamond\varphi$ is false in w_2 and, trivially, $\Box\varphi$ is true in w_2.

wrt to tautologies and contradictions:
Contradictions are never possible, tautologies are always necessary.
All tautologies are possible in worlds w_0 and w_1, but no tautologogy is possible in world w_2.
No contradiction is necessary in world w_0 and w_1, every contradiction is, trivially, necessary in world w_2.

We add the definite article:

If $P \in PRED^1$, then $\sigma(P) \in TERM$

Semantics:

$$[\![\sigma(P)]\!]_{M,g,w} = \begin{cases} d & \text{if } [\![P]\!]_{M,g,w} = \{d\} \\ \text{undefined otherwise} & \end{cases}$$

Rigidity of names versus non rigidity of definite terms

(1) a. If Kennedy had been a republican, Buck would have been made head of the CIA.
    b. If the president had been a republican, Buck would have been made head of the CIA.

(1b) is ambiguous in a way that (1a) is not.

Reading of (1a): Change the world minimally so as to make Kennedy a republican. In that world Buck is made head of the CIA.

(1b): Reading 1: the same as the reading of (1a): Kennedy is the president, and we change the world minimally to make him republican, Buck becomes head of the CIA.

(1b): Reading 2: Change the world minimally as to make the USA have a republican president (Nixon). In that world, Buck is made head of the CIA.

Crucial fact: (1a) does not have the following reading:

(1a): Non-existent reading 2: Change the world minimally as to make the name *Kennedy* denote a republican (say, Nixon). In that world, Buck is made head of the CIA.

Since the name, individual constant, *Kennedy* denotes the same individual in all possible worlds, you cannot derive the non-existent reading 2 for (1a).
Since the definite *the president* denotes different individuals in different words, you *can* derive reading 2 for (1b).
You can also derive reading 1 for (1b) by given the expression *the president* wide scope over the modal.

VII. EXAMPLES

Example 1.
Let PROUST, JOYCE $\in$ CON.

Let AU stand for 'be author of *Ulysses*'
Let AF stand for 'be author of *Finnegans Wake*'
Let AR stand for 'be author of *A la recherche du temps perdu*'
AU, AF, AR $\in$ $PRED^1$

MODEL M = $\langle W,R,D,F_M \rangle$ where:

$W = \{w_0, w_1, w_2, w_3\}$

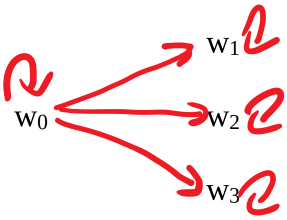
$R = \{\langle w_0,w_0\rangle, \langle w_0,w_1\rangle, \langle w_0,w_2\rangle, \langle w_0,w_3\rangle, \langle w_1,w_1\rangle, \langle w_2,w_2\rangle, \langle w_3,w_3\rangle \}$

$D = \{p, j\}$

F_M is specified in the following table:

F_M	w_0	w_1	w_2	w_3
PROUST	p	p	p	p
JOYCE	j	j	j	j
AR	{p}	{j}	{p}	{j}
AU	{j}	{j}	{p}	{j}
AF	{j}	{j}	{j}	Ø

The accessibility relation is given as follows:



Of course we should add the three novels to the model and add a two place relation A for *author of*. Then we write: σ(λx.A(x,Finnegans wake)) for σ(AF), *the author of Finnegans Wake*.
But we assume that understood and just follow the above specifications of AR, AU and AF

The whole model can be schematically given as:

o w_1
Joyce → À la recherche du temps perdu
Joyce → Ulysses
Joyce → Finnegans wake

o w_0
Proust → À la recherche du temps perdu
Joyce → Ulysses
Joyce → Finnegans wake

o w_2
Proust → À la recherche du temps perdu
Proust → Ulysses
Joyce → Finnegans wake

o w_3
Joyce → À la recherche du temps perdu
Joyce → Ulysses

$AF = \emptyset$

(1) a. Proust is the author of Ulysses.
b. (PROUST = σ(AU))

Truth conditions:

$[\![\text{PROUST} = \sigma(\text{AU})]\!]_{M,w,g} = 1$ iff

$[\![\text{PROUST}]\!]_{M,w,g} = [\![\sigma(\text{AU})]\!]_{M,w,g}$ iff

$F_M(\text{PROUST},w) = d$, where $\{d\} = [\![\text{AU}]\!]_{M,g,w}$ iff

$F_M(\text{AU},w) = \{F_M(\text{PROUST},w)\}$ iff

$F_M(\text{AU},w) = \{p\}$

(since $F_M(\text{PROUST},w) = p$ for every $w \in W$)

We see:

F_M	w_0	w_1	w_2	w_{43}
PROUST	p	p	p	p
JOYCE	j	j	j	j
AU	{j}	{j}	{p}	{j}
AF	{j}	{j}	{j}	Ø
AR	{p}	{j}	{p}	{j}

$F_M(\text{AU},w) = \{p\}$

	w_0	w_1	w_2	w_3
PROUST = σ(AU)	0	0	1	0

(2) a. The author of Ulysses is the author of Finnegans Wake.
b. $(\sigma(AU) = \sigma(AF))$

Truth conditions:

$[\![\sigma(AU) = \sigma(AF)]\!]_{M,w,g} = 1$ iff

$[\![\sigma(AU)]\!]_{M,w,g} = [\![\sigma(AF)]\!]_{M,w,g}$ iff

for some $d \in D$: $[\![AU]\!]_{M,w,g} = [\![AF]\!]_{M,w,g} = \{d\}$ iff

$F_M(AU,w) = F_M(AF,w)$ and $|F_M(AU,w)|=1$

We see:

F_M	w_0	w_1	w_2	w_3
PROUST	p	p	p	p
JOYCE	j	J	j	j
AU	{j}	{j}	{p}	{j}
AF	{j}	{j}	{j}	Ø
AR	{p}	{j}	{p}	{j}

$F_M(AU,w) = F_M(AF,w)$ and $|F_M(AU,w)|=1$

	w_0	w_1	w_2	w_3
$\sigma(AU) = \sigma(AF)$	1	1	0	0

Note: $\sigma(AU) = \sigma(AF)$ is false in w_3, even though $\sigma(AF)$ is not defined there.
This follows from the semantics given for = (i.e. the '0 otherwise').
We could change the semantics for formulas, so that it will come out as undefined instead. But for our purposes here it is just as well that it comes out as false.

(3) a. Proust could have been the author of Ulysses.
b. $\Diamond$ (PROUST = σ(AU))

Truth conditions:

$[\![\Diamond(\text{PROUST} = \sigma(\text{AU}))]\!]_{M,w,g} = 1$ iff

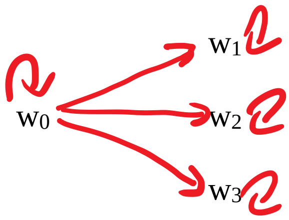
for some $v \in W$: $R(w,v)$ and $[\![\text{PROUST} = \sigma(\text{AU})]\!]_{M,v,g} = 1$ iff

for some $v \in W$: $R(w,v)$ and $F_M(AU,v) = \{p\}$

We see:

F_M	w_0	w_1	w_2	w_3
PROUST	p	p	p	p
JOYCE	j	j	j	j
AU	{j}	{j}	{p}	{j}
AF	{j}	{j}	{j}	Ø
AR	{p}	{j}	{p}	{j}

for some $v \in W$: $R(w,v)$ and $F_M(AU,v) = \{p\}$



	w_0	w_1	w_2	w_3
$\Diamond$(PROUST=σ(AU))	1	0	1	0

true in w_0 because w_2 is accessibe, and there Proust wrote U
true in w_2 for the same reason.
false in the others.

(4) a. Proust could have been the author of Finnegans Wake.
b. $\Diamond(\text{PROUST} = \sigma(\text{AF}))$

Truth conditions:

$[\![\Diamond(\text{PROUST} = \sigma(\text{AF}))]\!]_{M,w,g} = 1$ iff

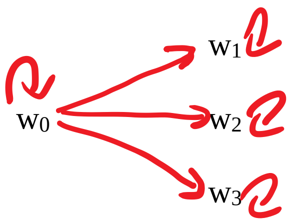
for some $v \in W$: $R(w,v)$ and $[\![\text{PROUST} = \sigma(\text{AF})]\!]_{M,v,g} = 1$ iff

for some $v \in W$: $R(w,v)$ and $F_M(\text{AF},v) = \{p\}$

We see:

F_M	w_0	w_1	w_2	w_{43}
PROUST	p	p	p	p
JOYCE	j	j	j	j
AU	{j}	{j}	{p}	{j}
AF	{j}	{j}	{j}	Ø
AR	{p}	{j}	{p}	{j}

for some $v \in W$: $R(w,v)$ and $F_M(\text{AF},v) = \{p\}$



	w_0	w_1	w_2	w_3
$\Diamond(\text{PROUST}=\sigma(\text{AF}))$	0	0	0	0

Proust did not write F in any world.

This shows that substitution of expre ssions with the same extension is not valid in modal contexts:

(3) b. $\Diamond$ (PROUST = σ(AU))
(2) b. (σ(AU) = σ(AF))
do not entail
(4) b. $\Diamond$(PROUST=σ(AF))

This entailment would hold if for every model M and every world $w \in W_M$ where (3b) and (2) are true, (4b) is true as well.

But model M is a counterexample.
We find a world $w_0 \in W$ where (3b) and (2b) are true, but (4b) is false (check the tables). Hence (3b) and (2b) do not entail (4b).

(5) a. Proust couldn't have been the author of Finnegans Wake.
b. $\neg\Diamond(\text{PROUST} = \sigma(\text{AF}))$

$\neg\Diamond(\text{PROUST} = \sigma(\text{AF})) \Leftrightarrow \Box\neg(\text{PROUST} = \sigma(\text{AF}))$

Truth conditions:
$[\![\Box\neg(\text{PROUST} = \sigma(\text{AF}))]\!]_{M,w,g} = 1$ iff

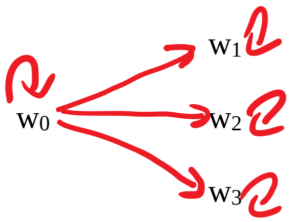
for every $v \in W$: if $R(w,v)$ then $[\![\neg(\text{PROUST} = \sigma(\text{AF}))]\!]_{M,v,g} = 1$ iff

for every $v \in W$: if $R(w,v)$ then $[\![\text{PROUST} = \sigma(\text{AF})]\!]_{M,v,g} = 0$

Obviously:

F_M	w_0	w_1	w_2	w_{43}
PROUST	p	p	p	p
JOYCE	j	j	j	j
AU	{j}	{j}	{p}	{j}
AF	{j}	{j}	{j}	Ø
AR	{p}	{j}	{p}	{j}

for every $v \in W$: if $R(w,v)$ then $[\![\text{PROUST} = \sigma(\text{AF})]\!]_{M,v,g} = 0$



	w_0	w_1	w_2	w_3
$\Box\neg(\text{PROUST}=\sigma(\text{AF}))$	1	1	1	1

(5b) is true in every world in W. Hence (5b) is true on model M.

(6) a. Joyce is necessarily the author of Finnegans Wake.

b. $\Box$(JOYCE = σ(AF))

Truth conditions:

$[\![\Box(\text{JOYCE} = \sigma(\text{AF}))]\!]_{M,w,g} = 1$ iff

for every $v \in W$: if $R(w,v)$ then $[\![\text{JOYCE}{=}\sigma(\text{AF})]\!]_{M,w,g} = 1$ iff

for every $v \in W$: if $R(w,v)$ then $F_M(\text{FW},v) = \{j\}$

We see:

F_M	w_0	w_1	w_2	w_3
PROUST	p	p	p	p
JOYCE	j	J	j	J
AU	{j}	{j}	{p}	{j}
AF	{j}	{j}	{j}	Ø
AR	{p}	{j}	{p}	{j}

	w_0	w_1	w_2	w_3
$\Box$(JOYCE = σ(AF))	0	1	1	0

We see that it is not true on M that Joyce necessarily wrote Finnegans Wake, though it **is** true on M that nobody else could have written Finnegans Wake.

(6) c. Somebody else could have written Finnegans wake instead of Joyce.
$\exists x[\neg(x{=}\text{JOYCE} \wedge \Diamond(x = \sigma(\text{AF})]$
d. Somebody else could have written Ulysses instead of Joyce.
$\exists x[\neg(x{=}\text{JOYCE} \wedge \Diamond(x = \sigma(\text{AU})]$

Example 2.

Let FRED $\in$ CON, SWEDE $\in$ PRED^1, MARRY $\in$ PRED^2.

Model M = $\langle W,R,D,F_M \rangle$, where:

$W = \{w_0,w_1,w_2,v_1,\ldots,v_n\}$

$R = \{\langle w_0,w_0\rangle, \langle w_0,w_1\rangle, \langle w_2,w_2\rangle, \langle w_2,v_1\rangle,\ldots,\langle w_2,v_n\rangle\}$

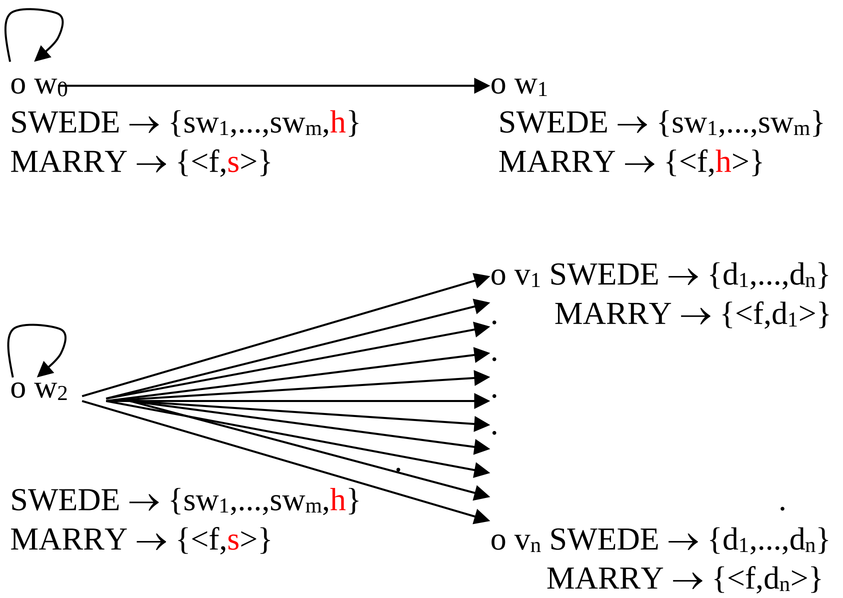
$D = X \cup Y \cup \{f,s,h\}$, where $X = \{sw_1,\ldots,sw_m\}$ and $Y = \{d_1,\ldots,d_n\}$

for every $w \in W$: $F_M(\text{FRED},w) = f$

The interpretations of the predicates SWEDE and MARRY are specified in the following table:

F_M	w_0	w_1	w_2	v_i (for every $i \leq n$)
SWEDE	$X \cup \{h\}$	X	$X \cup \{h\}$	Y
MARRY	$\{\langle f,s\rangle\}$	$\{\langle f,h\rangle\}$	$\{\langle f,s\rangle\}$	$\{\langle f,d_i\rangle\}$

In a picture:



(7) Fred could have been married to a Swede.

a. $\exists x[\text{SWEDE}(x) \wedge \Diamond\text{MARRY}(\text{FRED},x)]$

b. $\Diamond\exists x[\text{SWEDE}(x) \wedge \text{MARRY}(\text{FRED},x)]$

Let me tell you about Helga. I told you we were contemplating marriage. What I didn't tell you (but what the model tells you), is that she wouldn't have married me without changing her nationality first. She felt very strong about that. Or my mother did. I don't remember. (Just try to become Dutch and keep your nationality…)

Truth conditions:

$[\![\exists x[SWEDE(x) \wedge \Diamond MARRY(FRED,x)]]\!]_{M,w,g} = 1$ iff

for some $d \in F_M(SWEDE,w)$ and
for some $v \in W$: $(R(w,v)$ and $[\![MARRY(FRED,x)]\!]_{M,v,g_x^d}=1$ iff

for some $d \in F_M(SWEDE,w)$ for some $v \in W$: $R(w,v)$ and $<f,d> \in F(MARRY,v)$

-since $h \in F_M(SWEDE,w_0)$, $R(w_0,w_1)$ and $<f,h> \in F(MARRY,w_1)$,
(7a) is true in w_0

-since $F_M(SWEDE,w_2) \cap F_M(SWEDE,v_i) = \emptyset$,
for every $i \leq n$, there isn't a $d \in F_M(SWEDE,w_2)$ and a $v \in W$
such that $R(w_2,v)$ and $<f,d> \in F_M(MARRY,v)$.

Hence (7a) is false in w_2.,

Results:

	w_0	w_2
$\exists x[SWEDE(x) \wedge \Diamond MARRY(FRED,x)]$	1	0

$[\![\Diamond\exists x[SWEDE(x) \wedge MARRY(FRED,x)]]\!]_{M,w,g} = 1$

iff for some $v \in W$: $R(w,v)$ and for some $d \in F(SWEDE,v)$: $<f,d> \in F(MARRY,v)$

-since $<f,h> \notin F_M(MARRY,w_0)$ and $h \notin F_M(SWEDE,w_1)$
there isn't a v such that $<w_0,v>$ and
for some d: $d \in F_M(SWEDE,v)$ and $<f,d> \in F_M(MARRY,v)$.

That is,: $R(w_0,w_0)$. In w_0, Helga is a Swede, but Fred is not married to her there.
$R(w_0,w_1)$. In w_1, Fred is married to Helga, but she isn't a Swede there.
Hence (7b) is false in w_0

Any of the worlds v_i is a world such that $<w_2,v_i>$ and
for some $d \in F(SWEDE,v_i)$ $<f,d> \in F(MARRY,v_i)$ (namely, d_i).
Hence, clearly, (7b) is true in w_2.

	w_0	w_2
$\Diamond\exists x[SWEDE(x) \wedge MARRY(FRED,x)]$	0	1

This shows that:

$\exists x[\text{SWEDE}(x) \wedge \Diamond\text{MARRY}(\text{FRED},x)]$ does not entail
$\Diamond\exists x[\text{SWEDE}(x) \wedge \text{MARRY}(\text{FRED},x)]$

(w_0 is a counterexample)

$\Diamond\exists x[\text{SWEDE}(x) \wedge \text{MARRY}(\text{FRED},x)]$ does not entail
$\exists x[\text{SWEDE}(x) \wedge \Diamond\text{MARRY}(\text{FRED},x)]$

(w_2 is a counterexample)

The readings are logically independent.

Note that in w_2 it is not true that I *had to be married to a Swede*:
even though there are many worlds accessible frow w_2 where I am married to a Swede, there is one where I am not, namely w_2 itself.

I chose the set of Swedes in the v_i worlds to be a set different from the Swedes in w_2 (the actual Swedes in w_2).
This was to fit the part of the story that my Swedish fit was not based on an acquaintance with any Swedes
(better would have been to let the set of Swedes in the accessible worlds vary wildly).

Also, I chose a different Swede in each alternative.
This models the **unspecificity** of the modal facts:
My Swedish fit was strong enough, and I was at the time flippant enough that I could have found myself married to anyone of them.

VIII. QUANTIFICATION OVER POSSIBLE INDIVIDUALS

You may have noticed that the domain of individuals was chosen to be a domain of possible individuals, and that quantification is over possible individuals.
This means that $\exists x[\text{BOY}(x) \wedge \text{SING}(x)]$ is true in world w iff some possible individual is a boy in w and sings in w.
But this doesn't actually tell you that that boy should **exist** in w.
Let us make the connection with existence explicit.

> We add a predicate EXIST $\in$ PRED^1.
> We add to the model a function E_M: $W_M \rightarrow \text{pow}D_M$.
> E maps every world w $\in$ W onto $E_M(w)$, which we understand as **the set of possible individuals existing in w**.
>
> We interpret: $F_M(\text{EXIST},w) = E_M(w)$

We could now change the semantics of the quantifiers so that quantification in a world is always over objects existing in that world:

$$[\![\forall x\varphi]\!]_{M,w,g} = 1 \text{ iff for every } d \in E_M(w)\text{: } [\![\varphi]\!]_{M,w,g_x^d} = 1$$

$$[\![\exists x\varphi]\!]_{M,w,g} = 1 \text{ iff for some } d \in E_M(w)\text{: } [\![\varphi]\!]_{M,w,g_x^d} = 1$$

But we are **not** going to do that:
we will continue to assume that quantification is over possible objects.
But then we can formulate our problem:

> Problem:
> (1) Some boy kissed Mary
> (2) Some boy exists.
> (3) Mary exists.
>
> The problem is that (1) should entail (2) and (3), but so far it doesn't.

Instead of building existence into the quantification,
I assume that it follows from the **lexical meaning of the predicates**:

> **Lexical postulates.**
> For every world w $\in$ W: $F_M(\text{BOY},w) \subseteq E_M(w)$
> For every world w $\in$ W: $\text{dom}(F_M(\text{KISS},w) \subseteq E_M(w)$
> $\text{ran}(F_M(\text{KISS},w) \subseteq E_M(w)$

With these postulates, (1) $\Rightarrow$ (2) and (1) $\Rightarrow$ (3).

One advantage of putting the existence claim into the lexical meanings of the predicates is that in this way you can distinguish different predicates:

Lexical postulates.
For every world $w \in W$: $\text{dom}(F_M(\text{RESEMBLE},w) \subseteq E_M(w)$
For every world $w \in W$: $\text{dom}(F_M(\text{WORSHIP},w) \subseteq E_M(w)$

(4) a. Fred resembles Leopold Bloom.
b. Fred worships Anna Livia Plurabella.

Side remark:
Resemble shows temporal asymmetry (Kratzer)

(4') a. ✓I look like my ancester who lived under Napoleon.
b. ?My ancester who lived under Napolean looked like me.
End of side remark

I haven't put an existence requirement on the **range** of the interpretations of *resemble* and *worship*, and that means that if I claim that a resembles b or a worships b, it **does** follow that a exists, but it doesn't follow that b exists.

We see that we need to distinguish different types of predicates, when we're concerned with existence claims. This means that we need these kinds of lexical postulates any way. If so, there is no need to put them as constraints on the quantifiers as well.

But what about (5):

(5) Pegasus is a winged horse.

Is this statement true in world w_0, the real world?
Probably not. But there is a sense in which it is true.

Let's sketch a little analysis of fictional discourse. [More sophisticated analysis: Terrence Parsons – *Non-existent objects*]
I assume that the sense in which (5) is true is the following.
On the 'true' use, (5) contains an **implicit operator**: 'in the story':

(6) (In the story), Pegasus is a winged horse.

How should we analyze this operator?
I assume that in the model **the story** exists in the real world w_0, and I assume that we associate with the real world w_0 a **set**:

S_{w_0} the set of all worlds compatible with the story in w_0.

In fact, I will assume that, we can associate with every world w such a set S_w.

Now we add to the language an operator S (for 'in the story') with the following syntax and semantics:

If $\varphi \in \text{FORM}$, then $S(\varphi) \in \text{FORM}$
$[\![S(\varphi)]\!]_{M,w,g} = 1$ iff for every $v \in S_w$: $[\![\varphi]\!]_{M,v,g} = 1$

Let WH be the predicate *winged horse*, and assume that WH satisfies the same postulate as BOY:

Lexical postulate.
For every world $w \in W$: $F_M(WH,w) \subseteq E_M(w)$

In the real world w_0, $F_M(WH,w_0) = \emptyset$

Consequently:

$[\![WH(PEGASUS)]\!]_{M,w_0,g} = 0$, because $F_M(PEGASUS) \notin F_M(WH,w_0)$

But:
$[\![S(WH(PEGASUS))]\!]_{M,w_0,g} = 1$ iff
for every $v \in S_{w_0}$: $F_M(PEGASUS,v) \in F_M(WH,v)$

Since the story distinctly specifies the winged-horsedness of Pegasus, we assume that indeed, only worlds where Pegasus is in the extension of WH are in S_{w_0}.
Consequently, $[\![S(WH(PEGASUS))]\!]_{M,w_0,g} = 1$.

Note that it is important to realize that, while we talk about 'the world according to the story', there is no such thing: there are only the **worlds** compatible with the story. This is because the story leaves many things open that a world does not leave open. When the Duke de Guermantes makes a scene about the color of his wife's shoes, the story doesn't tell you, for instance, **what size those shoes were**:
This means that in some worlds compatible with the story, the Dutchess wore size 36, in others 37, etc.

We can show now that the following entailment relations hold:

(7) a. Pegasus is a winged horse.
WH(PEGASUS)
entails
b. Pegasus exists.
EXIST(PEGASUS)

(8) a. (In the story) Pegasus is a winged horse.
S(WH(PEGASUS))
does not entail
b. Pegasus exists.
EXIST(PEGASUS)

(9) a. (In the story) Pegasus is a winged horse.
S(WH(PEGASUS))
entails
b.(In the story) Pegasus exists.
S(EXIST(PEGASUS))

IX. PROPOSITIONS AND PROPOSITIONAL ATTITUDE VERBS

For the use of the examples below, we add a new syntactic category of propositions, $TERM_{prop}$ And we add a category of relations between individuals and propositions : $PRED^2_{<ind,prop>}$:

$PRED^2_{<ind,prop>}$= {BELIEVE, CLAIM}
If $t \in TERM$ and $p \in TERM_{prop}$ and $R \in$: ,$PRED^2_{<ind,prop>}$ then $R(t,p) \in FORM$
$[\![R(t,p)]\!]_{M,w,g} = 1$ iff $<[\![t]\!]_{M,w,g}, [\![p]\!]_{M,w,g}> \in F_M(R)$

We form propositions from formulas by a proposition forming operation ^ ("up")

If $\varphi \in FORM$, then $^\wedge\varphi \in TERM_{prop}$
$[\![^\wedge\varphi]\!]_{M,w,g} = \{v \in W: [\![\varphi]\!]_{M,v,g} = 1\}$

Thus, in world w, ^SMART(SASHA) denotes
$\{v \in W: F_M(SASHA,v) \in F_M(SMART,v)\}$ the set of worlds v such that Sasha is smart in v. Interpreting *that* as the operator ^, we can have expressions like:

(1) a. Fred believes that sasha is smart
b. BELIEVE(FRED, ^SMART(SASHA))

(1b) is true in M in world w iff
$<F_M(FRED,w), \{v \in W:F_M(SASHA,v) \in F_M(SMART,v)\}> \in F_M(BELIEVE,w)$

The pair consisting of Fred and the set of worlds where Sasha is smart stand in the believe relation, i.e. Fred stands in the believe relation to the set of worlds where Sasha is smart.
^(…) creates an intensional context.

We can go on and constrain the meaning of $F_M(BELIEVE,w)$ further.
For instance, we can assume in the model for each individual $d \in D$ and world w a set $B_{d,w}$, the set of worlds compatible with what d believes in w. And we can impose a meaning constraint on $F_M(BELIEVE,w)$ that:

$<d,p> \in F_M(BELIEVE,w)$ iff $B_{d,w} \subseteq p$

This gives a possible world semantics for *believe* (Hintikka 1962). With this semantic constraint: (1b) is true in world w if in every world v compatible with what Fred believes in in w, Sasha is smart.
See Stalnaker's book *Inquiry* for extensive discussion of this analysis.

X. MODALS AS GENERALIZED QUANTIFIERS

We add a term MB $\in$ CON_{prop}
$F_M(MB,w) = \{v \in W: R(w,v)\}$

MB stands for the *modal base* in w, the set of worlds accessible from w.
We now add generalized quanfiers relating sets of possible worlds:

$EVERY_{prop}$, $SOME_{prop} \in DET_{prop}$
If $\alpha \in DET_{prop}$ then $F_M(\alpha) \subseteq pow(W) \times pow(W)$
a relation between propositions, sets of possible worlds.

If $\alpha \in DET_{prop}$ and p,q $\in$ $TERM_{prop}$ then $\alpha[p, q] \in FORM$
$[\![\alpha[p, q]]\!]_{M,w,g} = 1$ iff $<[\![p]\!]_{M,w,g}, [\![q]\!]_{M,w,g}> \in [\![\alpha]\!]_{M,w,g}$

$F_M(EVERY_{prop}) = \{<p,q>: p, q \subseteq W \text{ and } p \subseteq q\}$
$F_M(SOME_{prop}) = \{<p,q>: p, q \subseteq W \text{ and } p \cap q \neq \emptyset\}$

Fact: $\Box\varphi = EVERY_{prop}[MB, {}^\wedge\varphi]$
$\Diamond\varphi = SOME_{prop}[MB, {}^\wedge\varphi]$

i.e.

$[\![EVERY_{prop}[MB, {}^\wedge\varphi]]\!]_{M,w,g} = 1$ iff
$[\![MB]\!]_{M,w,g} \subseteq [\![{}^\wedge\varphi]\!]_{M,w,g}$ iff
$\{v \in W: R(w,v)\} \subseteq \{v \in W: [\![\varphi]\!]_{M,v,g} = 1\}$ iff
for every $v \in W$: if $R(w,v)$ then $[\![\varphi]\!]_{M,v,g} = 1$ iff
$[\![\Box\varphi]\!]_{M,w,v} = 1$

The generalized quantifier perspective is useful to represent other modals than $\Box$ and $\Diamond$, for instance, *probably*.

We have dealt in the nominal domain with cardinality *most*:
$F_M(MOST_{|\,|}) = \{<X,Y>: X, Y \subseteq D_M \text{ and } |X \cap Y| > |X - Y|\}$

Most cats are smart:
$MOST_{||}[CAT, SMART]$ $|CAT \cap SMART| > |CAT - SMART|$
More cats are smart than not smart

If we extend the theory with mass nouns, we will need to deal with *most* that doesn't compare in terms of cardinality but in terms of other measures, like volume or weight:

Let $\mathbb{R}^+$ be the set of non-negative real numbers.
$P,Q \subseteq D_M$

An *additive measure* is a function μ: $pow(D_M) \rightarrow \mathbb{R}^+$
such that: $\mu(0) = 0$ and $\mu(P \cup Q) = \mu(P - Q) + \mu(Q - P) + \mu(P \cap Q)$

So the weight of the union of the books that A and B own is the weight of the books that A owns alone plus the weight of the books that B owns alone, plus the weight of the books that A and B jointly own.

Let μ be an additive measure:

$F_M(MOST_\mu) = \{<X,Y>: X, Y \subseteq D_M \text{ and } \mu(X \cap Y) > \mu(X - Y)\}$

Most Marc de Bourgogne is drunk in France
$MOST_\mu[$ MARC, DIF$]$ $\mu(MARC \cap DIF) > \mu(MARC - DIF)$
More Marc is drunk in France than is drunk abroad

Standard probability theory defined a probability measure on pow(W), the set of all sets of possible worlds:

Let W be the set of worlds, $w \in W$ and $P, Q \subseteq W$, and let $[0,1]_{\mathbb{R}}$ be the set of real numbers between 0 and 1.

A *probability measure* is an additive measure π^w: $pow(W) \rightarrow [0,1]_{\mathbb{R}}$ such that $\pi^w(W) = 1$

Additivity says that the probability that P or Q holds is the probability that P holds but not Q plus the probability that Q holds but not Q plus the probability that P and Q both hold.

It follows, for instance, from this that $\pi^w(^\wedge\neg\varphi) = 1 - \pi^w(\varphi)$
(or in other words: the more probable $\neg\varphi$ is the less probable φ is)

We index the probability measure here with w to let w function as a background context.

With this, we can now propose:
Let π^w be a probability measure.

$MOST_{\pi,w} \in$ DETprop
$F_M(MOST_{\pi,w}) = \{<p,q>: p, q \subseteq W \text{ and } \pi^w(P \cap Q) > \pi^w(P — Q)\}$

and we represent *probably* as:

$[\![$*probably* $\varphi]\!]_{Mw,g} = 1$ iff $[\![MOST_{\pi,w}[MB, ^\wedge\varphi]]\!]_{Mw,g} = 1$ iff
$\pi^w(MB \cap {}^\wedge\varphi) > \pi^w(MB \cap {}^\wedge\neg\varphi)$

φ is probably in w iff the set of accessible worlds where φ is true is more probable in w than the set of accessible worlds where φ is false.

Accessible worlds where φ is true here may be in context w be thought of as futures of the present in w where φ gets realized within a given time framl; and accessible worlds where φ is false would then be in context w futures of the present in w where φ doesn't get realized within a given time frame.

On that interpretation φ is probable means that the claim that φ is gonna happen in the given period is more likely than that φ is not gonna happen in that period.
That seems a reasonable interpretation.

XI. INDIVIDUAL CONCEPTS

Individual concepts are functions from possible worlds to individuals.

Possible world is short for index, parameter of variation of extensions. So often when we say world, we mean world-time, and more specifically world-times where the world parameter is kept constant, i.e. times.
In several of the examples below, the individual concepts we use are functions from moments of time to individuals (including degrees on a scale in the first example).

1. THE TEMPERATURE PARADOX (Partee)

The need for individual concepts is motivated by an analysis of Partee's temperature puzzle:

(1) The temperature is 90. [90 F = 32.222 C]
(2) The temperature is rising
hence: (3) Ninity is rising

This pattern is intuitively **invalid**, but its representation in predicate logic is valid:

Let TEMP, RISE $\in$ PRED^1, 90 $\in$ CON

(4) σ(TEMP) = 90
(5) RISE(σ(TEMP))
Hence: (6) RISE(90)

This is valid by extensionality.

The same problem can be formulated with normal individuals as well:

(7) The trainer is Michels.
(8) The trainer changes.
Hence: (9) Michels changes.

Let TRAINER, CHANGE $\in$ PRED^1, MICHELS $\in$ CON

(10) σ(TRAINER) = MICHELS
(11) CHANGE(σ(TRAINER))
Hence: (12) CHANGE(MICHELS)

There is a reading of the pattern in (7)-(9) which is valid, but there is another reading, the more prominent one, which is not valid. It is the latter we are concerned with.
We find the same ambiguity in (13): (said, say, in 1961).

(13) The president is a democrat, but he could have been a republican.
(14) a. Kennedy could have been a republican
b. There could have been a republican president.

It is the (14b) reading that we are interested in.

2. ADDING INDIVIDUAL CONCEPTS

We analyze the puzzles with individual concepts.

Individual concepts are functions from world to individuals.

In this section we will add individual concepts as a special kind of individual to the predicate logical modal semantics that we have. That is, we are going to treat individual concepts in the same way as we have treated individuals. That is, we are going to have names for individual concepts, variables over individual concepts, predicates of individual concepts, quantifiers over individual concepts, and abstraction over individual concepts, and all these clauses are in analogy to the clauses for individuals we have given before.

This is done systematically in the *intensional type logic* that Montague developed in the sixties and that I teach in Advanced Semantics. Here I will only introduce what I need for the dealing with the examples to come.

Enriching the logical language with individual concepts.

1. We start with the language of predicate logic enriched with the definite article and generalized quantifiers, i.e. the language we ended up with in the first part.
We call the relevant basic sets: CON_{ind}, VAR_{ind}, $TERM_{ind}$, $PRED^n_{ind}$,

2. We add the modal operators $\Box$ and $\Diamond$.
We give the by now standard modal interpretation for this language in terms of models M that contain $<D_M, W, R_W, F_M>$.

We will use in what follows domains based on these sets and extend the interpretation functions and assignment functions where necessary.

3. We add propositions, relations between individuals and propositions and the proposition forming operation ^, I repeat this from the previous section:

We add set of terms $TERM_{prop}$, and the clause and interpretation:

If $\varphi \in FORM$ then $^\wedge\varphi \in TERM_{prop}$
$[\![^\wedge\varphi]\!]_{M,w,g} = \{v \in W: [\![\varphi]\!]_{M,v,g} = 1\}$
The proposition expressed by φ is the set of all worlds where φ is true.

We add a new set of relations to the language: $PRED^2_{<ind,prop>}$ and the rules:

If $P \in PRED^2_{<ind,prop>}$ then $F_M(P) \subseteq (D_M \times \textbf{pow}(W))$
A relation between individuals and propositions, like BELIEVE, CLAIM

If $P \in PRED^2_{<ind,prop>}$ and $t \in TERM_{ind}$ and $p \in TERM_{prop}$ then $P(t,p) \in FORM$
$[\![P(t,p)]\!]_{M,w,g} = 1$ iff $<[\![t]\!]_{M,g}, [\![p]\!]_{M,g}> \in [\![P]\!]_{M,g}$

4. We now add individual concepts to the theory.

For clarity I will write individual concept terms, predicates of individual concepts, and relations between sets of individual concepts in the colour green.

4_a We add a set of individual concept constants, variables and terms:

$CON_{ic} = \{c_1, c_2, c_3\}$ The set of individual concept constants (names of individual concepts)
$VAR_{ic} = \{x_1, x_2, x_3\}$ The set of individual concept variables
$TERM_{ic} = CON_{ic} \cup VAR_{ic}$

These expressions are interpreted in the domain of individual concepts:

> $(W \to D_M)$, the set of all functions from worlds to individuals, is the domain of individual concepts.

So:

> For $c \in CON_{ic}$: $F_M(c) \in (W \to D_M)$
> For $x \in VAR_{ic}$: $g(x) \in (W \to D_M)$

This means that we let assignment functions g be functions from VAR_{ind} into D_M and from VAR_{ic} into $(W \to D_M)$.

As I said, I will not try to be systematic here but only add enought to the model so that I can deal with the examples below: we do need predicates of individual concepts:

4b. $PRED^1_{ic}$ is the set of one place predicates of individual concepts.
If $P \in PRED^1_{ic}$ then $F_M(P) \subseteq (W \to D_M)$
Predicates of individual concepts like CHANGE denote sets of individual concepts.

It $t \in TERM_{ic}$ and $P \in PRED^1_{ic}$ then $P(t) \in FORM$
$[\![P(t)]\!]_{M,g} = 1$ iff $[\![t]\!]_{M,g} \in [\![P]\!]_{M,g}$

4c. We will not introduce Frege-Tarski quantification over individual concepts, but generalization quantifiers for them.
Just as individual determiners denote relations between sets of individuals, individual concept determiners denote relations between sets of individual concepts.

$DET_{ic} = \{EVERY, SOME, ...\}$
If $\alpha \in DET_{ic}$ then:
$F_M(\alpha) = \{<\mathbf{X}, \mathbf{Y}>: \mathbf{X} \subseteq (W \to D_M)$ and $\mathbf{Y} \subseteq (W \to D_M)$ and $r_\alpha(|\mathbf{X} \cap \mathbf{Y}|, |\mathbf{X} - \mathbf{Y}|)$

Note that, since $\mathbf{r}_\alpha$ is a relation between numbers, we do not have to separately define $\mathbf{r}_\alpha$, we use the same relation $\mathbf{r}_\alpha$ as before for $\mathbf{r}_\alpha$.

If $P, Q \in PRED^1_{ic}$ and $\alpha \in DET_{ic}$ then $\alpha[P, Q] \in FORM$
$[\![\alpha[P, Q]]\!]_{M,w,g} = 1$ iff $<[\![P]\!]_{M,w,g}, [\![Q]\!]_{M,w,g}> \in [\![\alpha]\!]_{M,w,g}$

4_d. We add abstraction over individual concepts to the language:

If $x \in VAR_{ic}$ and $\varphi \in FORM$ then $\lambda x.\varphi \in PRED^1_{ic}$
$[\![\lambda x.\varphi]\!]_{M,w,g} = \{f \in (W \to D_M)\colon\ [\![\varphi]\!]_{M,w,g^f_x} = 1\ \}$

Just as abstracting over an individual variable gives a predicate of individuals, a set of individuals,
over an individual concept variable gives a predicate of individual concepts, a set of individual concepts:
$\lambda x.\varphi$ denotes the set of all individual concepts f for which φ is true relative to g^f_x

5. The operations DOWN($^\vee$) and UP ($^\wedge$) .

The last thing that we add to the logical language are two operations that relate the categories $TERM_{ind}$ and $TERM_{ic}$:

DOWN: From $TERM_{ic}$ to $TERM_{ind}$ **Extension of α at w**
If $\alpha \in TERM_{ic}$, then $^\vee\alpha \in TERM_{ind}$
$[\![^\vee\alpha]\!]_{M,w,g} = [\![\alpha]\!]_{M,w,g}(w)$

α denotes in world w an individual concept, a function f from worlds into individuals.
$^\vee\alpha$ denotes in world w **the value of that individual concept** f **for world w**: f(w), the individual that is the value of f for world w.

So if MISTER UNIVERSE denotes at time t_0 the function f from times to individuals which maps each time t onto the individual who holds at that time t the title *Mister Universe*, then $^\vee$MISTER UNIVERSE denotes at the present time t_0 the value of that function for time t_0, $f(t_0)$, which is the individual who currently holds the title (i.e. at t_0).

UP: From $TERM_{ind}$ to $TERM_{ic}$ **Intension of α**
If $\alpha \in TERM$, then $^\wedge\alpha \in$ i-TERM
$[\![^\wedge\alpha]\!]_{M,w,g}$ is the function in $(W \to D_M)$
which maps every world $v \in W$ onto $[\![\alpha]\!]_{M,v,g}$, the extension of α in w.

$^\wedge\alpha$ denotes in world w the individual concept which maps every world v onto the denotation of α in v. (this is the intension of α)

So, if PRESIDENT $\in PRED^1_{ind}$,
σ(PRESIDENT) denotes in world w the person who is in w the president.
$^\wedge\sigma$(PRESIDENT) denotes in world w the function that maps every world v onto the person who is in v the president.

We will be more interested in finding the correct readings than in systematically argue about how these readings come about in the grammar.

I will assume that *rise*, *change* denote properties of functions, and hence are interpreted as $PRED^1_{ic}$ predicates RISE, CHANGE $\in PRED^1_{ic}$

3. THE TRAINER PUZZLE

For the sake of the examples here we think of possible worlds as world-times, and the variation in the present section involves the time parameter. We allow ourselves here to be a bit imprecise, and just talk about world-times.

(7) The trainer is Michels.

We give this the same interpretation as before:

(7a) σ(TRAINER) = MICHELS
the trainer in world w is Michels

(8) The trainer changes.

Here we change the analysis: *change* is a predicate of functions, expressing that the function is different at later world-times than it is at earlier world-times.
This can mean various things.

-For instance, if the function f is **constant**, i.e. assigns the same individual to all relevant worlds-times, the natural interpretation of $f \in F_M$(CHANGE,w) is that the **individual** which is the value of f **has very different propertie**s at earlier world-times than at later world times (for instance, a different world-view, nationality,…)

-On the other hand, when function f is not a constant function, a very natural interpretation of $f \in F_M$(CHANGE,w) is than the **value of f at earlier world times is not the same as the value of f at later world-times**: the club changed its trainer, as when Rinus Michels resigned in 1973 as trainer of Ajax and was replaced by Stefan Kovács.
It is the latter interpretation that we are interested in here.

Note that we cannot write:

CHANGE(σ(TRAINER)) #

because that is not well-formed: σ(TRAINER) is not in $\mathrm{TERM_{ic}}$.

We need an expression in $\mathrm{TERM_{ic}}$ as the argument of CHANGE.

Systematic meaning shift: [type shifting – Advanced Semantics]
if $\alpha \in \mathrm{TERM_{ind}}$ and $\beta \in \mathrm{PRED}^1_{\mathrm{ic}}$ then:
You can shift from α to $^\wedge\alpha$ to resolve type mismatch.

if $\alpha \in \mathrm{TERM_{ic}}$ and $\beta \in \mathrm{PRED}^1_{\mathrm{ind}}$ then:
You can shift from α to $^\vee\alpha$ to resolve type mismatch.

So we can resolve the mismatch as in (8) by shifting from σ(TRAINER) to ^σ(TRAINER):

(8) The trainer changes.
(8a) CHANGE(^σ(TRAINER))

We will assume that in world w (8a) is true precisely because first the trainer was Michels and later it was Kovácz: the interpretation of ^σ(TRAINER) in world w is a function that maps all relevant earlier world-times onto Michels and all relevant later ones onto Kovácz.

(9) Michels changes.

Again, CHANGE(MICHELS) is unwellformed. We have to shift MICHELS to ^MICHELS:

(9) Michels changes.
(9a) CHANGE(^MICHELS)

We get as pattern:

(7) The trainer is Michels.
(8) The trainer changes.
(9) Michels changes.

(7a) σ(TRAINER) = MICHELS
(8a) CHANGE(^σ(TRAINER))
(9a) CHANGE(^MICHELS)

We note two things:
First, ^σ(TRAINER) and ^MICHELS denote **different** functions in world w.

^σ(TRAINER) denotes the function f that maps every world v onto the trainer in world v, earlier worlds onto Michels, later worlds onto Kovácz.

^MICHELS denotes the function g that maps every world onto Michels.

(7a) only says that in our world w these two functions have the same value:
f(w) = g(w) = Michels.

Let us assume that z is a world such that f(z)= Kovácz. Then f(z) ≠ g(z), since f(z)=Michels. hence f≠g.

This means that (7a) and (8a) do not entail (9a).
The pattern is invalid, as it should be.

Second, in (8a) *change* can mean that the function g takes different values at different times; in (9a) *change* can only mean that the individual value of f changes his properties, world-view, etc. from earlier world-times to later world-times.

4. THE VALID PATTERN: *DE RE* READINGS

What about the interpretation on which this pattern is valid?

(7) The trainer is Michels.
(8) The trainer changes.
(9) Michels changes.

On the individual concept analysis, *change* creates an intensional context. We can get the other reading by assuming that *the trainer* in (8) can have an interpretation which is *de re* and takes scope **over** the intensional context. We can make fruitful use of λ-abstraction to represent this case, as in (8c):

(7a) $\sigma(\text{TRAINER}) = \text{MICHELS}$
(8b) $\lambda x.\text{CHANGE}(^\wedge x)\ (\sigma(\text{TRAINER}))$
(8c) $\lambda x.\text{CHANGE}(^\wedge x)\ (\text{MICHELS})$

Now, MICHELS is rigid, so (8c) is equivalent to (9a)

(9a) $\text{CHANGE}(^\wedge\text{MICHELS})$

If MICHELS has the property that you have if the function that maps every world onto you is a changing function, then the function that maps every word onto Michels is a changing function.

Now (8b) is true in world w if whoever is the trainer in world w (i.e. Michels) has in world w the property $\lambda x.\text{CHANGE}(^\wedge x)$.

$\lambda x.\text{CHANGE}(^\wedge x)$ is the property that **you** have in w if the function that maps every world onto **you** has the *change* property, which can only mean that its value, **you**, has different properties, world views etc at earlier world-times than it has at later world-times.

Since the trainer in world w, according to (7a), is Michels, (8a) expresses that Michels has the property $\lambda x.\text{CHANGE}(^\wedge x)$, which means that the function f that maps every world onto Michels has the *change* property.
But function f is the denotation of $^\wedge$MICHELS, hence, on this reading, (7a) and (8b) entail (9b), because, on the assumption that (7a) is true, (8b) and (9b) express the same thing.

5. THE PRESIDENT

With this instrumentarium we can attack the other problems as well:

(13) The president is a democrat, but (s)he could have been a republican.

The two readings can be represented as in (14) and (15):

(14) $\lambda x.\text{DEMOCRAT}({}^{\vee}x) \wedge \Diamond\text{REPUBLICAN}({}^{\vee}x)\ ({}^{\wedge}\sigma(\text{PRESIDENT}))$

The president-function ${}^{\wedge}\sigma(\text{PRESIDENT})$ has in w the property that an individual concept has if its value in w is a democrat in w, while its value in some other world v is a republican in v.

This is equivalent to (14a):

(14a) $\text{DEMOCRAT}({}^{\vee\wedge}\sigma(\text{PRESIDENT})) \wedge \Diamond\text{REP}({}^{\vee\wedge}\sigma(\text{PRESIDENT}))$

The value of the president-function in w is a democrat in w and the value of the president function in some other world v is a republican in v.

And this is equivalent to (14b):

(14b) $\text{DEMOCRAT}(\sigma(\text{PRESIDENT})) \wedge \Diamond\text{REPUBLICAN}(\sigma(\text{PRESIDENT}))$

The president in w is a democrat in w and in some other world v, the president in v is a republican in v.

On this reading (14) is true in the real world w (at time t), where, we assume Kennedy is the president (at t), if in some world v accessible from w a republican, for instance, Nixon, is president (at time t).

The *de re* reading we get by giving the **individual reading** wide scope, as in (15):

(15) $\lambda x.\text{DEMOCRAT}(x) \wedge \Diamond\text{REPUBLICAN}(x)\ (\sigma(\text{PRESIDENT}))$

The president (at t) in w has the property that an individual has if he/she is a democrat in world w (at t) and a republican in some other world v (at t).

(15) is equivalent to (15a):

(15) $\text{DEMOCRAT}(\sigma(\text{PRESIDENT})) \wedge \lambda x.\Diamond\text{REPUBLICAN}(x)\ (\sigma(\text{PRESIDENT}))$

The president in w is a democrat in w and has the property $\lambda x.\Diamond\text{REPUBLICAN}(x)$, which is the property that you have if in some other world you are a republican.

This cannot be reduced any further.
(14) is true on this reading in the real world w if in some world v accessible from w Kennedy is a republican.

6. NEW PLANETS

Here is a little story. It concerns two astronomical observatories A and B that spot the sky in search of new planets. These observatories are in a fierce competition (on is the Sylvanian astronomical centre, and the other the corresponding Bordurian one). In fact, the astronomers at observatory B are so competitive that it is a bit unpleasant. I tell you (16):

(16) Every new planet that observatory A claims to have discovered, observatory B claims to have discovered first.

But, in fact, there is an added complication, that both you and I know: these observatories are no good; they are always wrong, no new planet has ever been discovered by either of them.

The crucial observation is that **I** can assert sentence (16), **without committing myself to the existence of new planets**.
(It doesn't have to do with the fact that the example has *every*, the story works just as well for other quantifiers.)

Analysis

-We have, inside the relative clause, an intensional context (*claim*).

-We have, inside the relative clause a gap, which we interpret as a variable that we λ-abstract over at the level of the head of the relative.

-We make the assumption that if a gap is in an intensional context inside the relative clause, we have a **choice** of interpreting the gap as an **individual variable** (in VAR_{ind}) or as an **individual concept variable** (in VAR_{ic}).
This choice is part of the relativization mechanism and is triggered by the intensional context inside the relative clause.

-This gives two interpretations (17a) and (17b):

(17a) $\mathrm{EVERY}[\ \lambda x.\mathrm{NEWPLANET}(x) \wedge \mathrm{CLAIM}(A,{}^\wedge\mathrm{DISCOVERED}(A,x))\ ,$
$\lambda x.\mathrm{CLAIM}(B,{}^\wedge\mathrm{DISCOVERED\text{-}FIRST}(B,x))]$

(17a) is true in w if for every new planet **existing** in w of which A has claimed that they discovered it, B has claimed they discovered it first.

(17b) $\mathrm{EVERY}[\ \lambda x.\mathrm{NEWPLANET}(x) \wedge \mathrm{CLAIM}(A,{}^\wedge\mathrm{DISCOVERED}(A,{}^\vee x))\ ,$
$\lambda x.\mathrm{CLAIM}(B,{}^\wedge\mathrm{DISCOVERED\text{-}FIRST}(B,{}^\vee x))]$

(17b) is true in w if for every *new planet individual concept* of which A has claimed that they discovered **an existing instance of it**, B has claimed they discovered **an existing instance of it** first.

What are *new planet concepts*? We think of those as concepts introduced in the discourse context [here the story] and understood as contextually linked to the story given: the story introduces individual concepts **new planet**$_1$ the concept that was an issue in, say, 1956, and **new planet**$_2$, which was an issue in 1957,…. These are **tentatively** existing ‘objects’, both of which turned out not to be instantiated in the real world w. The predicate NEWPLANET takes these functions in its extension, and EVERY hence quantifies over new-planet individual concepts that have been made relevant in the discourse.
A journalist interested in the controversy, will go to the archives and pull out all cases of ***putative*** new planets. The quantification is over those, and not over individual planets.

This means that (17b) does not commit the speaker to the existence of new planets, it just requires new planet individual concepts to be contextually relevant, as in the story given.

In a model, we could have four worlds: w_0 the real world, and w_1, w_2 w_3 the worlds compatible with what the Observatories claim
three relevant individual concepts:

f_1: $w_1 \rightarrow d_1$	f_2: $w_1 \rightarrow d_2$	f_3: $w_1 \rightarrow d_3$
$w_2 \rightarrow d_1$	$w_1 \rightarrow d_2$	$w_1 \rightarrow d_3$
$w_3 \rightarrow d_1$	$w_1 \rightarrow d_2$	$w_1 \rightarrow d_3$
$w_o \rightarrow$ venus	$w_0 \rightarrow$ mars	$w_0 \rightarrow$ x, where x is an airplane

d_1 may be the non-existent planet Vulcanus (which supposedly circles the earth shielded from vision by Mercurius.
d_2 is the non-existent planet Ursa Minor Beta (the planet where it is always Saturday afternoon, just before the beach bars close).
d_3 is the planet Homoterrae which is postulated by the obscure Israeli astronomer Pered Am-ha-aretz.

We assume that:
$\{f_1, f_2, f_3\} \subseteq \lambda x.\mathrm{NEWPLANET}(x) \wedge \mathrm{CLAIM}(A,{}^\wedge\mathrm{DISCOVERED}(A,{}^\vee x)$

Say: in 1956 A observed Venus and claimed: we have found Vulcanus, ….

(17b) is true if:

$\{f_1, f_2, f_3\} \subseteq \lambda x.\mathrm{CLAIM}(B,{}^\wedge\mathrm{DISCOVERED\text{-}FIRST}(B,{}^\vee x)$

We check the records and find indeed: in 1956, the same week, the Bordurians say: we already made that observation, their spies stole the information from us….

7. HOB-NOB SENTENCES

Similar to this are Hob-nob sentences introduced by Peter Geach in *Reference and Generality* 1962 which involve quantification over different people's belief, **again without the speaker being committed to the existence of witches**. Geach discusses the following case: ((18) is Geach's example)

> A rumour goes around that there is a witch in the village.
> Hob says in the pub: "That explains! That's why my horse got sick."
> Nob says in church: "And my sow died suddenly! remember?"
>
> (18) Hob believes that a witch blighted his mare, and Nob believes that *she* killed his sow.

Again, just giving **wide scope**, but abstracting over **individual variables** commits the speaker to the existence of witches, as in (19a), but **wide scope** and abstraction over **individual concept variables** doesn't, as in (19b):

(19) a SOME[WITCH,
$\lambda x.\text{BELIEVE}(\text{hob}, {}^\wedge\exists y[\text{MARE}(y,\text{hob}) \wedge \text{BLIGHT}(x,y)]) \wedge \text{BELIEVE}(\text{nob}, {}^\wedge\exists z[\text{SOW}(z,\text{nob}) \wedge \text{KILL}(x,z)])$]

(19) a SOME[WITCH,
$\lambda x.\text{BELIEVE}(\text{hob}, {}^\wedge\exists y[\text{MARE}(y,\text{hob}) \wedge \text{BLIGHT}({}^\vee x,y)]) \wedge \text{BELIEVE}(\text{nob}, {}^\wedge\exists z[\text{SOW}(z,\text{nob}) \wedge \text{KILL}({}^\vee x,z)])$]

There is a witch-concept made relevant in the discourse to which both Hob and Nob are linked, for instance, via a rumour that both Hob and Nob heared, and in all Hob's belief-worlds, someone instantiates that concept and blighted Hob's mare, and in all Nob's belief-worlds, someone instantiates that concept and killed Nob's cow.

(There is a lot of philosophical literature about exactly what this causal attachment condition involves.)

With individual concepts we avoid the conclusion that (18) involves a *de re* belief of Hob and Nob about and individual.

XII. MODAL FORMULAS EXPRESSING PROPERTIES OF THE ACCESSIBILITY RELATION

Let M = $<W_M,R_M,D_M,F_M>$ be a model for L_6.

The **frame of** M: $\mathit{F}_M = <W_M,R_M,D_M>$

The frame of M is the model **minus** the interpretation function F_M.

In general, a frame is a triple $\mathit{F} = <W,R,D>$, with W a non-empty set of possible worlds, R an accessibility relation on W and D a non-empty set of possible individuals.

A model, then is a pair M = $<\mathit{F},F>$, with F a frame and F an interpretation function for the lexical items.

(A set-theoretic subtelty: we don't distinguish between $<\mathit{F},F>$ (= $<<W_{\mathit{F}},R_{\mathit{F}},D_{\mathit{F}}>,F>$) and $<W_{\mathit{F}},R_{\mathit{F}},D_{\mathit{F}},F>$.)

Let φ be an L_6 sentence.
We define φ is true on frame $\mathit{F} = <W_{\mathit{F}},R_{\mathit{F}},D_{\mathit{F}}>$

$[\![\varphi]\!]_{\mathit{F}} = 1$, φ is true on frame F iff
for every interpretation function F for L_6 such that $<\mathit{F},F>$ is a model for L_6:
$[\![\varphi]\!]_{<\mathit{F},F>} = 1$; otherwise $[\![\varphi]\!]_{\mathit{F}} = 0$

φ is true on frame F iff for every interpretation function F for F,
φ is true on model $<\mathit{F},F>$, false on F otherwise.

Intuitively, φ is true on a frame F iff φ is true **in virtue of** the structure of the frame, independent of the interpretation of the lexcal items.

Let $\mathcal{F}_P$ be the the class of all frames F in which the accessibility relation R_{F} has property P.

Property P of accessibility relations is ***modally definable***, ***definable in L_6*** iff there is an L_6 formula φ which is true on all the frames in class $\mathcal{F}_P$ and false on every frame not in $\mathcal{F}_P$.

This means, *vive versa*, that you can check what property of accessibility relations, if any, is defined by a formula φ:

-φ *modally defines* P iff
1. For every frame $\mathit{F} \in \mathcal{F}_P$ and every interpretation function F:$[\![\varphi]\!]_{<\mathit{F},F>} = 1$
2. For every frame $\mathit{F} \notin \mathcal{F}_P$ there is an interpretation function F such that:$[\![\varphi]\!]_{<\mathit{F},F>} = 0$

As above, let us use φ for a contingent non-modal sentence, a formula (without free variables) that can be made true in some worlds and false in others (like SMART(RONYA)).

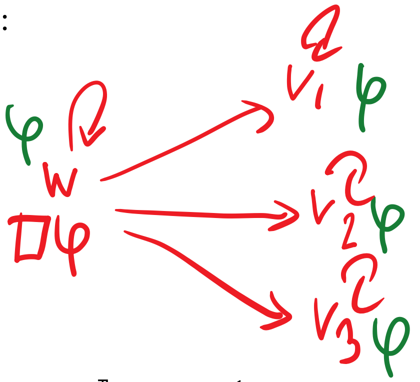
Here are some of the basic facts (going back to Kripke's work):

FACT 1: $\Box\varphi \rightarrow \varphi$ defines reflexivity of the accessibility relation.

1. If F is a reflexive frame, a frame where R_F is reflexive then $\Box\varphi \rightarrow \varphi$ is true on F

Proof:
Let F be a reflexive frame, and $w \in W_F$ and let F be any interpretation function such that $[\![\Box\varphi]\!]_{<F,\mathrm{F}>,w} = 1$.
Then for every $v \in W_F$: if $R_F(w,v)\}$ then $[\![\varphi]\!]_{<F,\mathrm{F}>,v} = 1$.
Since R_F is reflexive, $R_F(w,w)$ and hence $[\![\varphi]\!]_{<F,\mathrm{F}>,w} = 1$.

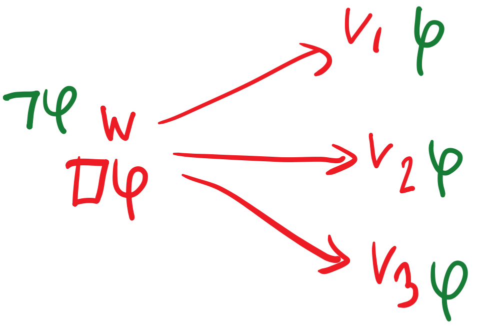
a picture:



If $\Box\varphi$ is true in w, φ is true in all the accessible worlds, one of which is w, by reflexivity

Hence $[\![\Box\varphi \rightarrow \varphi]\!]_{<F,\mathrm{F}>,w} = 1$
So $\Box\varphi \rightarrow \varphi$ is true in every world in W_F if R_F is reflexive.

2. If F is not a reflexive frame then, we can make a counterexampe, we can choose an interpretation function F and a world w where $[\![\Box\varphi \rightarrow \varphi]\!]_{<F,\mathrm{F}>,w} = 0$, which is a world w where $[\![\Box\varphi]\!]_{<F,\mathrm{F}>,w} = 1$ but $[\![\varphi]\!]_{<F,\mathrm{F}>,w} = 0$
We choose a world w such that $<w,w> \notin R_F$. Then we choose an interpretation function F that makes φ true in every world $v \in W_F$ such that $R_F(w,v)$, but false in w:



If the frame is not reflexive you can make φ true in all worlds accessible from w but false in w.
That situation is the required counterexample.

By the assumption about the contingency of φ we can do that.
Hence $[\![\Box\varphi \rightarrow \varphi]\!]_{<F,\mathrm{F}>,w} = 0$

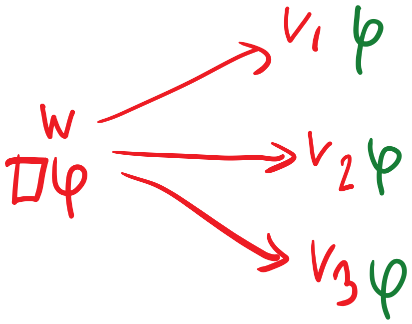
So, indeed $\Box\varphi \rightarrow \varphi$ is true on every reflexive frame (true in every world), and false on non-reflexive frames (meaning, not true in every world).

Hence indeed, $(\Box\varphi \rightarrow \varphi)$ defined (or characterizes) the class of frames with a reflexive accessibility relation and $(\Box\varphi \rightarrow \varphi)$ expresses that the accessibility relation is reflexive.

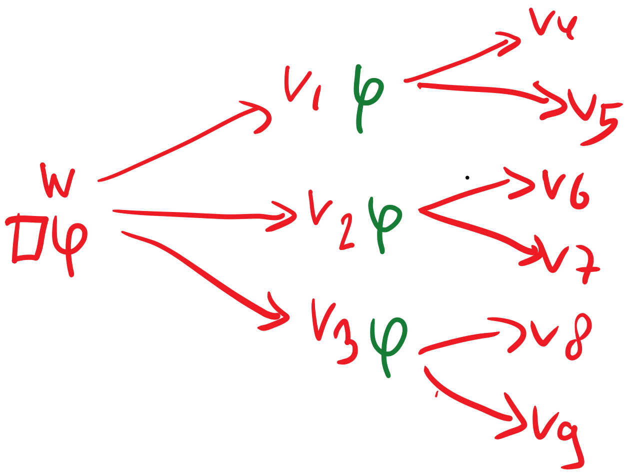
FACT 2: $\Box\varphi \rightarrow \Box\Box\varphi$ defines transitivity of the accessibility relation.

1. If F is a transitive frame, a frame where R_F is transitive then $\Box\varphi \rightarrow \Box\Box\varphi$ is true on F

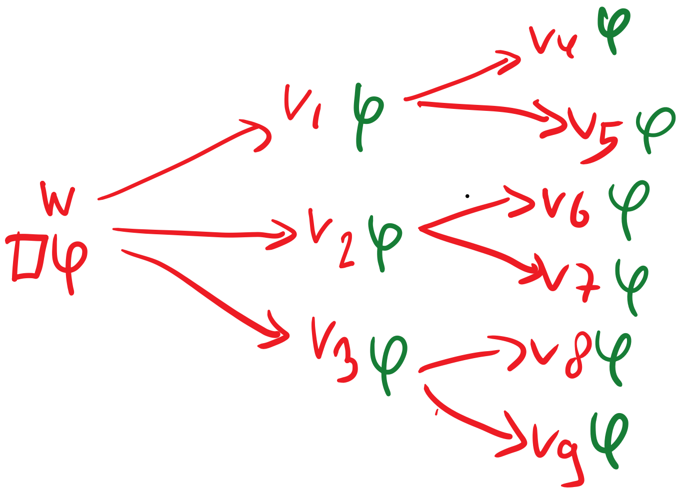
Step 1: Assume $\Box\varphi$ is true in w. Then φ is true in all accessible worlds:



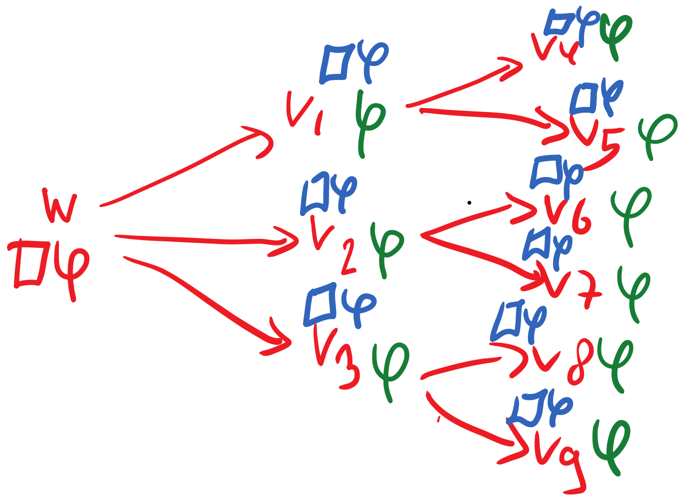
Look at all the worlds accessible from v_1, v_2, v_3, say:



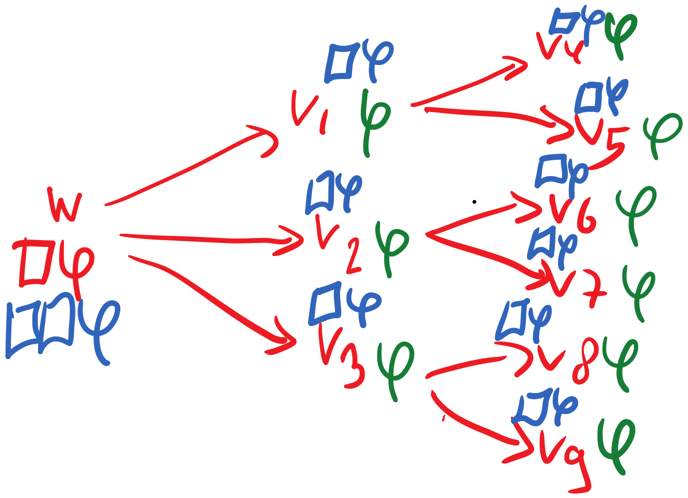
By transitivity these worlds are accessible from w, and hence φ is true in them as well:



But that means that for each world v which is accessible from w, φ is true in all worlds accessible from v, and hence $\Box\varphi$ is true in v.
This shows that $\Box\varphi$ is true in all worlds accessible from w:

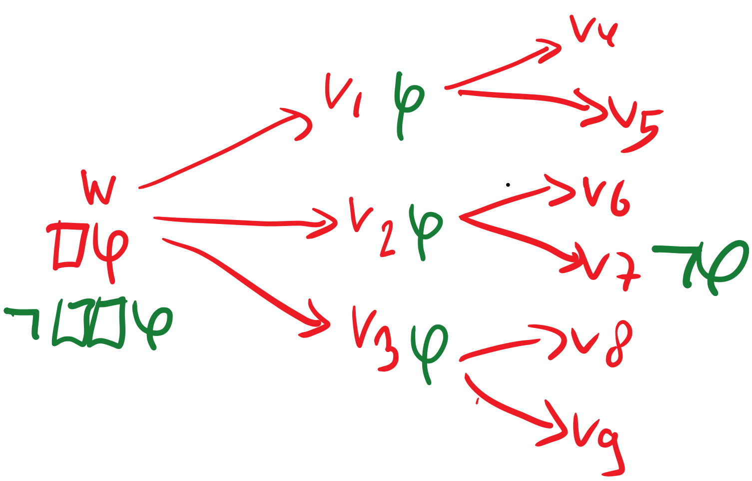


But if □φ is true in all worlds accessible from w, then □□φ is true in w:



And that means that □φ → □□φ is true in w.

If R_F is not transitive you easily make a counterexample:

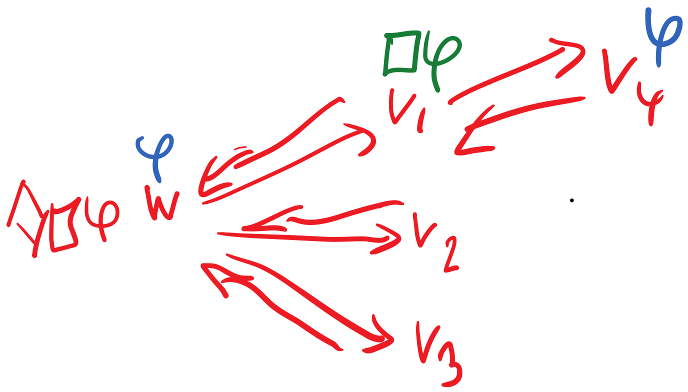


You make φ true in all worlds accessible from w, including v_2, and false in some worlds accessible from v_2, but not from w. This is perfectly possible in R_F is not transitive, and is a counterexample.

FACT 2: $\Diamond\Box\varphi \rightarrow \varphi$ defines symmetry of the accessibility relation.

1. If *F* is a symmetric frame then $\Diamond\Box\varphi \rightarrow \varphi$ is true on *F*

Assume $\Diamond\Box\varphi$ is true in w.
Then for some accessible world $\Box\varphi$ is true, say v_1. Then in all worlds accessible from there φ is true. One of those is w, by symmetry:



Hence $\Diamond\Box\varphi \rightarrow \varphi$ is true in w.

Priorian Tense logic

In tense logic, the set of worlds W_F is renamed T_F, the set of moments of time, and the accessibility relation R_F is renamed $<_F$ and is assumed to be a strict partial order of *earlier than*.
In Priorian tense logic we introduce four tense operators, two futurate and two past:

P	at some time in the past
H	at every time in the past
F	at some time in the future
G	at every time in the future

with the obvious semantics:

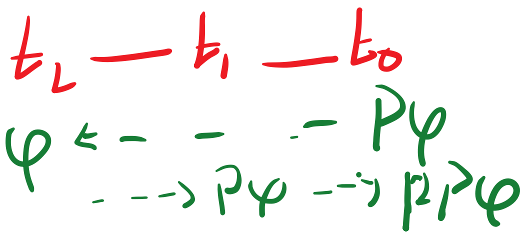
$[\![P\varphi]\!]_{M,t,g} = 1$ iff for some $t' \in T_M$: $t' < t$ and $[\![\varphi]\!]_{M,t',g} = 1$
$[\![H\varphi]\!]_{M,t,g} = 1$ iff for every $t' \in T_M$: if $t' < t$ then $[\![\varphi]\!]_{M,t',g} = 1$

$[\![F\varphi]\!]_{M,t,g} = 1$ iff for some $t' \in T_M$: $t < t'$ and $[\![\varphi]\!]_{M,t',g} = 1$
$[\![G\varphi]\!]_{M,t,g} = 1$ iff for some $t' \in T_M$: if $t < t$ then $[\![\varphi]\!]_{M,t',g} = 1$

If you find modal definability interesting, you will find tense logical definability even more interesting.
In tense logic $H\varphi \rightarrow HH\varphi$ defines transitivity (and so does and $G\varphi \rightarrow GG\varphi$).
$H\varphi \rightarrow \neg HH\varphi$ is logically equivalent to $PP\neg\varphi \rightarrow \neg\varphi$

$H\varphi \rightarrow HH\varphi \quad \Leftrightarrow \quad \neg HH\varphi \rightarrow \neg H\varphi \quad \Leftrightarrow \quad P\neg H\varphi \rightarrow P\neg\varphi \ \Leftrightarrow$
$PP\neg\varphi \rightarrow P\neg\varphi$

That means that the formula $PP\varphi \rightarrow P\varphi$ also defined transitivity.
What about $P\varphi \rightarrow PP\varphi$?
As it turns out $P\varphi \rightarrow PP\varphi$ expresses that the temporal order is **dense**: between any two points of time there is a third.
Intuition:



Assume that $P\varphi$ is true at t_0. Then at some past moment, say, t_2 φ is true.
By density there is a point between t_2 and t_0, say, t_1, and since φ is true at t_2, $P\varphi$ is trua at t_1, because t_2 is in the past of t_1. But then $PP\varphi$ is true at t_0, because t_1 is in the past of t_0.
Again, giving a counterexample on a non-dense frame is simple.

XIII. SEMANTICS AND PRAGMATICS OF CONDITIONALS

(1) a. It is not the case that if it rains it is cold.
b. It rains and it isn't cold.
c. It may rain and not be cold.

Problem: $\neg(\varphi \to \psi) \Leftrightarrow (\varphi \wedge \neg\psi)$ material implication $\to$

Intuitively: $\neg(\varphi \blacktriangleright\!\!\blacktriangleright \psi) \Leftrightarrow \Diamond(\varphi \wedge \neg\psi)$ natural language implication $\blacktriangleright\!\!\blacktriangleright$

That means: $(\varphi \blacktriangleright\!\!\blacktriangleright \psi) \Leftrightarrow \neg\Diamond(\varphi \wedge \neg\psi)$

$\Leftrightarrow \Box\neg(\varphi \wedge \neg\psi)$

$\Leftrightarrow \Box(\varphi \to \psi)$

Thus: **conditionals are modals**.

In the following discussion we will assume an **informational interpretation** of the modals $\Box$, $\Diamond$ and $\blacktriangleright\!\!\blacktriangleright$.
By this, I mean the following.
We will assume in our models an accessibility relation I.

The modal base I represents: what follows from or is compatible with the *conversional information*.

This means:

For every world $w \in W$: $\{v \in W: I(w,v)\}$ is the set of all worlds compatible with conversational information in w.
Notation: $I_w = \{v \in W: I(w,v)\}$

We introduce the set of all worlds where φ is true:

$[\![\varphi]\!]_M = \{w \in W: [\![\varphi]\!]_{M,w} = 1\}$

As usual:

$[\![\Box\varphi]\!]_{M,w} = 1$ iff for every $v \in I_w$: $[\![\varphi]\!]_{M,v} = 1$
$\Box\varphi$ is true in w iff φ **follows from the information** in w.

Equivalently:

$[\![\Box\varphi]\!]_{M,w} = 1$ iff $I_w \subseteq [\![\varphi]\!]_M$

$[\![\Diamond\varphi]\!]_{M,w} = 1$ iff for some $v \in I_w$: $[\![\varphi]\!]_{M,v} = 1$
$\Diamond\varphi$ is true in w iff φ **is compatible with the information** in w.

Equivalently:
$[\![\Diamond\varphi]\!]_{M,w} = 1$ iff $I_w \cap [\![\varphi]\!]_M \neq \emptyset$

$[\![(\varphi \blacktriangleright\!\blacktriangleright \psi)]\!]_{M,w} = 1$ iff for every $v \in I_w$: if $[\![\varphi]\!]_{M,v} = 1$ then $[\![\psi]\!]_{M,v} = 1$

Equivalently:

$[\![(\varphi \blacktriangleright\!\blacktriangleright \psi)]\!]_{M,w} = 1$ iff $I_w \cap]\!]\varphi]\!]_M \subseteq [\![\psi]\!]_M$

$I_w \cap [\![\varphi]\!]_M$ is **the result of adding φ to your information**
$(\varphi \blacktriangleright\!\blacktriangleright \psi)$ is true in w iff ψ follows from the result of adding φ to I_w.

GRICE'S MAXIM OF QUALITY.

"Do not say what you know not to be true."

CASE A: Non-modal statements.

Only say φ in w if φ follows from your information.

Quality for non-modal statements φ:
Say φ in w only if $\Box\varphi$ is true in w

CASE B: Modal statements.
Modal statements are **already** themselves **about** I:

Quality for modal statements φ:
Say φ in w only if φ is true in w

So:

Quality for (φ ⏩ ψ):
Say (φ ⏩ ψ) in w only if (φ ⏩ ψ) is true in w

IRRELLEVANT ENTAILMENTS

FACT 1: $\Box\psi \Rightarrow$ (φ ⏩ ψ)

Reason: If ψ is true in every world compatible with the information, then ψ is also true in every world compatible with the information where φ is true.

FACT 2: $\Box\neg\varphi \Rightarrow$ (φ ⏩ ψ)

Reason: If φ is false in every world compatible with the information, then ψ is true in every world compatible with the information where φ is true.

Hence:

both $\Box\psi$ and $\Box\neg\varphi$ are semantically stronger statements than (φ ⏩ ψ).

In all the following discussion, we assume that φ and ψ themselves are **non-modal** statements.

For non-modal statements, Quality says:
only say ψ if $\Box\psi$ is true
only say $\neg\varphi$ if $\Box\neg\varphi$ is true

We conclude:

With the maxim of quality:
ψ is pragmatically a stronger statement than (φ ⏩ ψ)
$\neg\varphi$ is pragmatically a stronger statement than (φ ⏩ ψ)

GRICE'S MAXIM OF QUANTITY.

"Give as much information as you can (but not more than is necessary)"

Quantity:
If φ is pragmatically stronger than ψ, and both are relevant, etc., then you should say φ rather than ψ.

Consequently:

If (φ ⏩ ψ) is true in w and $\Box\psi$ is true in w,
then, according to the maxims, you should say ψ rather than (φ ⏩ ψ)

If (φ ⏩ ψ) is true in w and $\Box\neg\varphi$ is true in w,
then, according to the maxims, you should say $\neg\varphi$ rather than (φ ⏩ ψ)

If (φ ⏩ ψ) is false in w, you shouldn't say (φ ⏩ ψ) at all in w.

The square of informational situation types for ($\varphi \blacktriangleright\!\!\blacktriangleright \psi$). (Veltman 1986)

(1) $\Box\varphi$ $\Box\psi$	(2) $\Box\varphi$ $\Diamond\psi\ \Diamond\neg\psi$	(3) $\Box\varphi$ $\Box\neg\psi$
(4) $\Diamond\varphi\ \Diamond\neg\varphi$ $\Box\psi$	(5) $\Diamond\varphi\ \Diamond\neg\varphi$ $\Diamond\psi\ \Diamond\neg\psi$	(6) $\Diamond\varphi\ \Diamond\neg\varphi$ $\Box\neg\psi$
(7) $\Box\neg\varphi$ $\Box\psi$	(8) $\Box\neg\varphi$ $\Diamond\psi\ \Diamond\neg\psi$	(9) $\Box\neg\varphi$ $\Box\neg\psi$

These are all the *possible logical* informational situations with respect to φ and ψ: φ can follow from the information, φ can be incompatible with the information, of both φ and $\neg\varphi$ can be compatible. The same for ψ.
That gives 9 combinations.

But now we argue:

FACT 1: In situation types (2), (3) and (6), ($\varphi \blacktriangleright\!\!\blacktriangleright \psi$) is **false**.
Hence, the assertion of ($\varphi \blacktriangleright\!\!\blacktriangleright \psi$) in situations of type (2), (3) or (6) **violates Quality.**

namely: ($\varphi \blacktriangleright\!\!\blacktriangleright \psi$) is false in w iff $\Diamond(\varphi \wedge \neg\psi)$ is true in w.

-In every world w of type 2, φ is true in every world in I_w (since $\Box\varphi$ is true in w).
In some world v in I_w $\neg\psi$ is true (since $\Diamond\neg\psi$ is true in w).
Hence in that world v in I_w $(\varphi \wedge \neg\psi)$ is true.
Hence $\Diamond(\varphi \wedge \neg\psi)$ is true in w.
Hence ($\varphi \blacktriangleright\!\!\blacktriangleright \psi$) is false in w.

-In every world w of type 6, $\neg\psi$ is true in every world in I_w (since $\Box\neg\psi$ is true in w).
In some world v in I_w φ is true (since $\Diamond\varphi$ is true in w).
Hence in that world v in I_w $(\varphi \wedge \neg\psi)$ is true.
Hence $\Diamond(\varphi \wedge \neg\psi)$ is true in w.
Hence ($\varphi \blacktriangleright\!\!\blacktriangleright \psi$) is false in w.

-In every world w of type 3, φ is true in every world in I_w and $\neg\psi$ is true in every world in I_w, hence in every world v in I_w $(\varphi \wedge \neg\psi)$ is true in v.
Now, we make the **plausible pragmatic assumption** that the information so far in w is consistent. This assumption says that $I_w \neq \emptyset$.
This means that there is a world v in I_w, and hence $\Diamond(\varphi \wedge \neg\psi)$ is true in w.
Hence ($\varphi \blacktriangleright\!\!\blacktriangleright \psi$) is false in w.

This means that the these cases are not compatible with Gricean Felicity:

(1) $\Box\varphi$ $\Box\psi$	(2)	(3)
(4) $\Diamond\varphi$ $\Diamond\neg\varphi$ $\Box\psi$	(5) $\Diamond\varphi$ $\Diamond\neg\varphi$ $\Diamond\psi$ $\Diamond\neg\psi$	(6)
(7) $\Box\neg\varphi$ $\Box\psi$	(8) $\Box\neg\varphi$ $\Diamond\psi$ $\Diamond\neg\psi$	(9) $\Box\neg\varphi$ $\Box\neg\psi$

FACT 2: In situations of type (1), (4) and (7), assertion of $(\varphi \blacktriangleright\!\blacktriangleright \psi)$ violates **quantity**.

Namely, in these situations $\Box\psi$ is true.
Since ψ is a pragmatically stronger statement than $(\varphi \blacktriangleright\!\blacktriangleright \psi)$, you should, by quantity, in such situations assert ψ rather than $(\varphi \blacktriangleright\!\blacktriangleright \psi)$.

This means that the these cases are not compatible with Gricean Felicity:

(1)	(2)	(3)
(4)	(5) $\Diamond\varphi$ $\Diamond\neg\varphi$ $\Diamond\psi$ $\Diamond\neg\psi$	(6)
(7)	(8) $\Box\neg\varphi$ $\Diamond\psi$ $\Diamond\neg\psi$	(9) $\Box\neg\varphi$ $\Box\neg\psi$

FACT 3: In situations of type (7), (8) and (9), assertion of $(\varphi \blacktriangleright\!\blacktriangleright \psi)$ violates **quantity**.

Namely, in these situations $\Box\neg\varphi$ is true.
Since $\neg\varphi$ is a pragmatically stronger statement than $(\varphi \blacktriangleright\!\blacktriangleright \psi)$, you should, by quantity, in such situations assert $\neg\varphi$ rather than $(\varphi \blacktriangleright\!\blacktriangleright \psi)$.

This means that the these cases are not compatible with Gricean Felicity:

(1)	(2)	(3)
(4)	(5) $\Diamond\varphi$ $\Diamond\neg\varphi$ $\Diamond\psi$ $\Diamond\neg\psi$	(6)
(7)	(8)	(9)

From this we conclude:

CORROLLARY: The only situations where $(\varphi \blacktriangleright\!\blacktriangleright \psi)$ is asserted in accordance with **Quality** and **Quantity** are situations of type (5).

Hence:

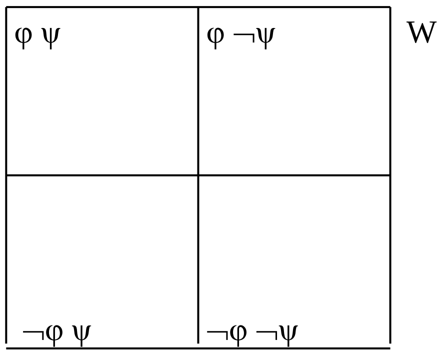
> Assertion of $(\varphi \blacktriangleright\!\!\blacktriangleright \psi)$ conversationally implicates $\Diamond\varphi$, $\Diamond\neg\varphi$, $\Diamond\psi$, $\Diamond\neg\psi$.
> These implicatures are called the **clausal implicatures** of $(\varphi \blacktriangleright\!\!\blacktriangleright \psi)$.

Relevance.

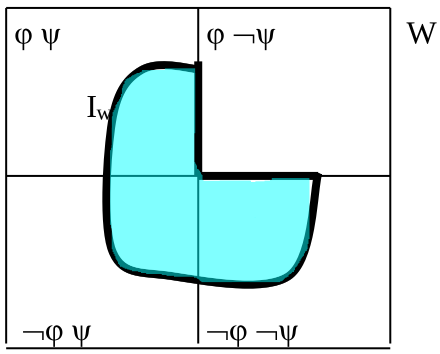
Situation types (1), (4), (7), (8), (9) are situation types where the conditional (φ ⏩ ψ) is true for **irrellevant** reasons:
the conditional is true, not because there is a relevant connection between the truth of φ and the truth of ψ, but because of some facts about φ or some facts about ψ.

Situation type (5) excludes these irrelevant reasons: in situations of type (5), the conditional is true because all φ-worlds in I_w are ψ-worlds.

In the picture below we partition the set of all worlds into four parts: the set of worlds where φ and ψ are both true, the set of worlds where φ and ψ are both false, the set of worlds where φ is true but ψ false, and the set of worlds where ψ is true but φ false:



Now we let w be any world in W of type (5) where (φ ⏩ ψ) is true.
This means that I_w lies inside W in the following way:
If (φ ⏩ ψ) is uttered in accordance with the maxims,

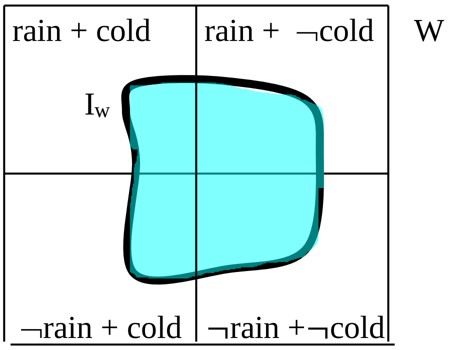


The information I_w does not overlap the set of worlds where $(\varphi \wedge \neg\psi)$ is true.
Now, there must be a **reason** why your information is structured this way.
As you can see from the picture, it's not because you already know that ψ is true, or that you already know that φ is false. You know neither.
The reason will have to do with some independent connection between φ and ψ that you assume:

For instance, assume (φ ⏩ ψ) is *If it rains it is cold*.

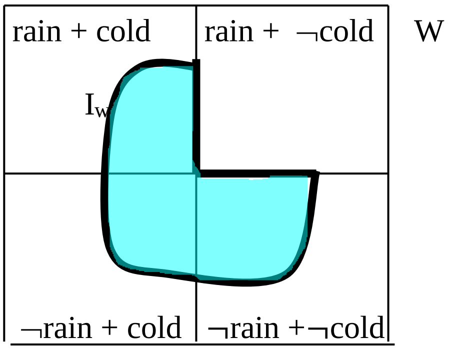
-The reason may be something like the following:
a.) I_w is restricted to worlds that respect the laws of nature, one of them being that: rain is produced by excessive humidity.
b.) I_w is restricted to worlds that respect a meteorological fact about Holland: in Holland, excessive humidity only happens at low temperatures.
c.) I_w is restricted to worlds where you are in Holland.
With these three assumptions about I_w, I_w will not contain worlds where it rains but isn't cold. Hence *If it rains it is cold* is true in w (relative to I_w, of course).

So: If your information contains the information that you are in Israel, your informational situation may be:



And your information does not support: If it rains it is cold.

But, if your information contains the information that you are in Holland, it will look like:



And *if it rains it is cold* is true in w (relative to I_w)

The reason may be something quite different, for instance:

You know that (i.e. in every world in I_w it is true that) John owns a mackintosh, a leather motorcycle coat, and an afghan coat and he always wears one of them.

But you also know that:

-When it is cold, John wears his leather coat when it rains and his Afghan coat when it doesn't rain (they are both warm).

-When it is warm, he wears his macintosh when it rains,

-but when it is warm and dry he wears his leather coat, because that is when he rides his motorcycle (a family heirloom that he wouldn't ride in the rain or in the cold).

The picture that represents this information is indicated below:

A	B
rain **cold** LEATHER COAT	**dry** **cold** AFGHAN COAT
MACINTOSH **rain** **warm**	LEATHER COAT **dry** **warm**
C	D

Now look at the following dialogue:

A: Is it cold out, dear?
B [Looking out of the window, seeing John pass by wearing his leather coat]
If it rains, it is.

The information as updated with the proposition that John is wearing his leather coat is:

A	B
rain **cold** LEATHER COAT	
	LEATHER COAT **dry** **warm**
C	D

In other words: you know now that it is either cold and rainy or dry and warm.
If it rains, it is not dry and warm, hence cold.

Given these informational options, it is indeed true that if it rains it is cold.

Pragmatic relevance versus semantic relevance.

We do not **semantically** require the truth of ($\varphi \blacktriangleright\!\blacktriangleright \psi$) to express a relevant connection between the truth of φ and the truth of ψ.
This means: we do not make ($\varphi \blacktriangleright\!\blacktriangleright \psi$) **false** in situations of type (1), (4), (7), (8), (9) for two important reasons:

1. ($\varphi \blacktriangleright\!\blacktriangleright \psi$) is **pragmatically incorrect** in these situations anyway.
So we don't have to put relevance into the semantics to explain the 'funnyness' of an inference: "ψ, hence ($\varphi \blacktriangleright\!\blacktriangleright \psi$)."

2. We use **this** semantics, the maxims, and the above square of situation types to explain the usage of **rhetorical conditionals**.

USING ROWS AND COLUMNS IN RHETORICAL CONDITIONALS
(Veltman 1986)

BASIC ASSUMPTION 1: (Except in irony, which we are not studying here)
We assume that the assertion of $(\varphi \blacktriangleright\!\!\blacktriangleright \psi)$ is made in accordance with the maxim of Quality, whether or not the conditional is rhetorical or not.

BASIC ASSUMPTION 2:
Rhetorical conditionals are relevance connection violations:
Their assertion signals that there isn't a relevant connection between the truth of φ and the truth of ψ.
This means that the assertion of a rhetorical conditional signals that situations of type (5.) do not obtain.

Rhetorical Conditionals:

COL. 1	**COL. 2**	**COL. 3**	
(1) $\Box\varphi$ $\Box\psi$	(2)	(3)	**ROW 1**
(4) $\Diamond\varphi$ $\Diamond\neg\varphi$ $\Box\psi$	(5)	(6)	**ROW 2**
(7) $\Box\neg\varphi$ $\Box\psi$	(8) $\Box\neg\varphi$ $\Diamond\psi$ $\Diamond\neg\psi$	(9) $\Box\neg\varphi$ $\Box\neg\psi$	**ROW 3**

Note:
On row 1: (1) is the only available type of situation.
On row 2: (4) is the only available type of situation.
On column 2: (8) is the only available type of situation.
On column 3: (9) is the only available type of situation.
Situation type (7) is the only type of situation that is not the only available type at any row or column.

ASSUMPTION 3:
Rhetorical conditionals use rows and columns.
They signal that situation type (5) does not obtain, and instruct you to find a row or column, where only one type of situation is available, and derive the consequences from that.

Consequently: situation type (7) is not available for rhetorical conditionals.
The idea here is that situation type (7) gives an ambiguous instruction.
We get:

The square of informational possibilities for rhetorical conditionals:

COL. 1	**COL. 2**	**COL. 3**	
(1) $\Box\varphi$ $\Box\psi$	(2)	(3)	**ROW 1**
(4) $\Diamond\varphi$ $\Diamond\neg\varphi$ $\Box\psi$	(5)	(6)	**ROW 2**
(7)	(8) $\Box\neg\varphi$ $\Diamond\psi$ $\Diamond\neg\psi$	(9) $\Box\neg\varphi$ $\Box\neg\psi$	**ROW 3**

All examples are from Veltman 1986.

TYPE 1
(1) She's on the wrong side of fourty, if she is a day. **ψ, if φ**
(2) If there's anything I can't stand, it's getting caught in rushhour traffic **if φ, ψ**

Analysis of (1):
Obviously, she is at least a day: $\Box\varphi$ is true in w.
We look in the table, and see that only type 1 is compatible with this.
We conclude: $\Box\psi$ is true in w.
Thus (1) is a rhetorical way of saying ψ: *She's over fourty*.

Analysis of (2):
Obviously, there is **at least something** I can't stand (since I am human).
Again, $\Box\varphi$ is true in w.
We conclude: (2) is a rethorical way of saying ψ: *I can't stand being caught in rushhour traffic*.

COL. 1	**COL. 2**	**COL. 3**	
(1) □φ □ψ	(2)	(3)	**ROW 1**
(4) ◊φ ◊¬φ □ψ	(5)	(6)	**ROW 2**
(7)	(8) □¬φ ◊ψ ◊¬ψ	(9) □¬φ □¬ψ	**ROW 3**

TYPE 9

(3) If this is true, then I'm the empress of China. **If φ, then ψ**
(4) If this happens, I'll eat my hat. **If φ, then ψ**
(5) I'll be hanged, if that happens. **ψ, if φ**
(6) If this is true, I am a Dutchman/a monkey's uncle. **If φ, then ψ**

Analysis of (3): (the other cases are similar)
Obviously, I am **not** the empress of China.
□¬ψ is true in w.
We look in the table and see that only type (9) is compatible with this.
We conclude: □¬φ is true in w.
Thus (3) is a rhetorical way of saying ¬φ: *This isn't true*.

COL. 1	COL. 2	COL. 3	
(1) $\Box\varphi$ $\Box\psi$	(2)	(3)	**ROW 1**
(4) $\Diamond\varphi$ $\Diamond\neg\varphi$ $\Box\psi$	(5)	(6)	**ROW 2**
(7)	(8) $\Box\neg\varphi$ $\Diamond\psi$ $\Diamond\neg\psi$	(9) $\Box\neg\varphi$ $\Box\neg\psi$	**ROW 3**

TYPE 4

(7) There's coffee in the pot, if you want some. **ψ, if φ**

(8) I paid back that fiver, if you remember. **ψ, if φ**

(9) If I may interrupt you, you're wanted on the telephone. **if φ, ψ**

Analysis:
Note first that all these cases are clearly relevance violations:
you wanting coffee, doesn't **make** there to be coffee in the pot,
unless you're a magician, which you're not.

These cases are all cases, where **politeness** considerations require us to assume that:

$\Diamond\varphi$ and $\Diamond\neg\varphi$ are true in w.

-Maybe you want coffee, maybe not (it would be impolite of me to assume that I know what you want).
-You may remember, you may not (don't even think I am suggesting that you know it very well).
-Maybe I am allowed to interrupt you, maybe not (of course, I can't look in the mind of really busy people).

We look in the table, and see that only type (4) is compatible with this.
We conclude: $\Box\psi$ is true in w.
Hence (7)-(9) are rhetorical ways of saying ψ:

-*There is coffee in the pot*.
-*I did pay back that fiver*.
-*You're wanted on the telephone.*

COL. 1	COL. 2	COL. 3	
(1) $\Box\varphi$ $\Box\psi$	(2)	(3)	**ROW 1**
(4) $\Diamond\varphi$ $\Diamond\neg\varphi$ $\Box\psi$	(5)	(6)	**ROW 2**
(7)	(8) $\Box\neg\varphi$ $\Diamond\psi$ $\Diamond\neg\psi$	(9) $\Box\neg\varphi$ $\Box\neg\psi$	**ROW 3**

TWO MORE CASES OF TYPE 4

(10) This is the best book of the month, if not the year. **ψ, if $\neg\varphi$**

This case is similar, but can be argued also in a different way.
We assert: **$(\neg\varphi \blacktriangleright\!\blacktriangleright \psi)$**

The situation type is once again: $\Diamond\varphi$ and $\Diamond\neg\varphi$ are true in w.
Maybe it is the best book of the year, maybe not.
So we could, following type 4, conclude $\Box\psi$.

But this time you also know something else:

$(\varphi \blacktriangleright\!\blacktriangleright \psi)$ is trivially true, true in all worlds.

If it is the best book of the year, it is the best book of the month.

This means that in asserting $(\neg\varphi \blacktriangleright\!\blacktriangleright \psi)$, we can conclude, with quality and the above fact:

$(\neg\varphi \blacktriangleright\!\blacktriangleright \psi) \wedge (\varphi \blacktriangleright\!\blacktriangleright \psi)$ is true in w.

But: $(\neg\varphi \blacktriangleright\!\blacktriangleright \psi) \wedge (\varphi \blacktriangleright\!\blacktriangleright \psi) \Rightarrow \Box\psi$
So also in this way we conclude: $\Box\psi$ is true in w:
(10) is a rhetorical way of saying ψ: *This is the best book of the month.*

COL. 1	COL. 2	COL. 3	
(1) $\Box\varphi$ $\Box\psi$	(2)	(3)	**ROW 1**
(4) $\Diamond\varphi$ $\Diamond\neg\varphi$ $\Box\psi$	(5)	(6)	**ROW 2**
(7)	(8) $\Box\neg\varphi$ $\Diamond\psi$ $\Diamond\neg\psi$	(9) $\Box\neg\varphi$ $\Box\neg\psi$	**ROW 3**

(11) If there's anything you need, my name is Marcia.

(11) has two natural uses, one of type (5) (a normal conditional), one of type (4).

Type 5: (11) is really short for (11')

(11') If there's anything you need, call for me, Marcia.

My name is Marcia is like an **appositive** on a suppressed consequent: *call for me.*
(11') is a normal relevant conditional with an appossitive.

Type 4:
Your needing something doesn't make my name Marcia.
I am a polite waitress, so of course I don't assume that I know whether you will be needing something or not.
But this restaurant is in California, and in restaurants in California waitresses are not just serving machines, but real persons, who have names (but note, only first names, we are, after all, in America).
(11) is my polite way of telling you that my name is Marcia.

COL. 1	COL. 2	COL. 3	
(1) $\Box\varphi$ $\Box\psi$	(2)	(3)	**ROW 1**
(4) $\Diamond\varphi$ $\Diamond\neg\varphi$ $\Box\psi$	(5)	(6)	**ROW 2**
(7)	(8) $\Box\neg\varphi$ $\Diamond\psi$ $\Diamond\neg\psi$	(9) $\Box\neg\varphi$ $\Box\neg\psi$	**ROW 3**

TYPE 8

(12) If it doesn't rain tomorrow, then it's going to pour. **If $\neg\varphi$, then ψ**

(13) [Muhammed Ali:] If I don't beat him, I'll thrash him. **If $\neg\varphi$, then ψ**

These cases are similar to example (10).
You can argue that I don't know whether it is going to pour tomorrow or not.
Even Ali doesn't know whether he is going to thrash him or not.
So: $\Diamond\psi$ and $\Diamond\neg\psi$ are true in w.
This is only the case in situation type (8), so you can conclude $\Box\neg\neg\varphi$, since the antecedent of the conditional was $\neg\varphi$. $\Box\neg\neg\varphi \Leftrightarrow \Box\varphi$, so we conclude $\Box\varphi$.

As in the case of example (10), there is another way of analyzing the case:

($\psi \blacktriangleright\!\blacktriangleright \varphi$) is trivially true, true in all worlds.

If it is going to pour, it is going to rain.
If I'll thrash him, I'll beat him.

So, again, from quality and the above fact we conclude:

$(\neg\varphi \blacktriangleright\!\blacktriangleright \psi) \wedge (\psi \blacktriangleright\!\blacktriangleright \varphi)$ is true in w.

But: **$(\neg\varphi \blacktriangleright\!\blacktriangleright \psi) \wedge (\psi \blacktriangleright\!\blacktriangleright \varphi) \Rightarrow \Box\varphi$.**

So we conclude: $\Box\varphi$ is true in w.
Hence, (12) and (13) are rhetorical ways of saying φ (the negation of the antecedent):

-It's going to rain.
-I'll beat him.

CONCLUSION: The modal semantics of the conditional and the informational interpretation of the maxims explains the relevance constraint on the normal use of the conditional, and it explains the interpretations of rhetorical conditionals.

Summary of rows and columns for rhetorical conditional:

TYPE	ASSERTION	BACKGROUND	CONCLUSION	CONVEYS
1	$(\varphi \blacktriangleright\!\!\blacktriangleright \psi)$	$\Box\varphi$	$\Box\psi$	ψ
4	$(\varphi \blacktriangleright\!\!\blacktriangleright \psi)$	$\Diamond\varphi\ \Diamond\neg\varphi$	$\Box\psi$	ψ
8	$(\varphi \blacktriangleright\!\!\blacktriangleright \psi)$	$\Diamond\psi\ \Diamond\neg\psi$	$\Box\neg\varphi$	$\neg\varphi$
9	$(\varphi \blacktriangleright\!\!\blacktriangleright \psi)$	$\Box\neg\psi$	$\Box\neg\varphi$	$\neg\varphi$

Given our assumptions, these are the only things that can be conveyed by the assertion rhetorical conditionals.

In general, if assert a conditonal $(\varphi \blacktriangleright\!\!\blacktriangleright \psi)$ and we **signal** by rhetorical means that situation type (5) is unavailable, then:
-we convey ψ, if we make clear that φ is compatible with the information (cases 1 and 4).
-we convey $\neg\varphi$, if we make clear that $\neg\psi$ is compatible with the information (cases 8 and 9).

This generalization relies on the plausible assumption (that we made earlier) that I_w is not empty. In that case, $\Box\varphi$ entails $\Diamond\varphi$, and $\Box\neg\psi$ entails $\Diamond\neg\psi$, so that we can reduce case (1) to case (4), and case (9) to case (8).

In other words, on the assumption that I_w is not empty, there are really two main situation types:

(1) We assert $(\varphi \blacktriangleright\!\!\blacktriangleright \psi)$ in w.
(2) We signal that situation type (5) is unavailable.
(3) $\Diamond\varphi$ is true in w.
We conclude: ψ
A rhetorical conditional conveys the consequent, if the antecedent is compatible with the information.

(1) We assert $(\varphi \blacktriangleright\!\!\blacktriangleright \psi)$ in w.
(2) We signal that situation type (5) is unavailable.
(3) $\Diamond\neg\psi$ is true in w.
We conclude: $\neg\varphi$
A rhetorical conditional conveys the negation of the antecedent, if the negation of the consequent is compatible with the information.

This works, on the assumption of rows and columns that we made (assumption 3):

Rhetorical conditionals ignore the cases where both the consequent and the negation of the antecedent follow from the information (cases of type 7).

One would think that the latter is, because in cases of type (7), the instruction is ambiguous: the hearer wouldn't know whether the speaker wants him or her to to use the negation of the antecedent to conclude the consequent, or the consequent to conclude the negation of the antecedent.

In the philosophical literature rhetorical conditionals are called Bisquit conditionals, based on an early example in the literature. Rhetorical conditionals is a better term, so I use that here.

Similar arguments can be made for rhetorical disjunctions:

Informationally, again, you should only say $\varphi \vee \psi$ is $\Box(\varphi \vee \psi)$ ist true.
We derive here too that the normal utterance situation for a conditional is is the situation: $\Diamond\varphi$, $\Diamond\neg\varphi$, $\Diamond\psi$, $\Diamond\psi$.

Look at the square:
The square of informational situation types for $\Box(\varphi\vee\psi)$.

(1) $\Box\varphi$ $\Box\psi$	(2) $\Box\varphi$ $\Diamond\psi$ $\Diamond\neg\psi$	(3) $\Box\varphi$ $\Box\neg\psi$
(4) $\Diamond\varphi$ $\Diamond\neg\varphi$ $\Box\psi$	(5) $\Box(\varphi\vee\psi)$ $\Diamond\varphi$ $\Diamond\neg\varphi$ $\Diamond\psi$ $\Diamond\neg\psi$	(6) $\Box(\varphi\vee\psi)$ $\Diamond\varphi$ $\Diamond\neg\varphi$ $\Box\neg\psi$
(7) $\Box\neg\varphi$ $\Box\psi$	(8) $\Box(\varphi\vee\psi)$ $\Box\neg\varphi$ $\Diamond\psi$ $\Diamond\neg\psi$	(9) $\Box\neg\varphi$ $\Box\neg\psi$

In situations 1,2,3,4,7 $\Box(\varphi\vee\psi)$ is true because a stronger statement is true.
In situation 9 $\Box(\varphi\vee\psi)$ is false.

In situation 5, 6, and 8 we would violate quality unless we restrict ourselves to the part where $\Box(\varphi\vee\psi)$ is true. So we do that (in red)
But then in situation 6 also $\Box\varphi$ is true and in 8 $\Box\psi$ is true, so these too are situations where you should have made a stronger statement.
Hence, here too, the only situation where you can utter the disjunction in agreement with the maxims is situation (5).

Here the natural rhetorical case is a situation where $\Box\neg\psi$ will allow you to conclude that $\Box\varphi$, and hence φ:

(1) I'll find him, or my name isn't Sherlock Holmes.