Reply to the Comment of Cohen and Peres

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We describe what sort of physical information many-time experiments produce, and how that information can be verified by further such experiments.

Cohen and Peres[1] are mistaken. Apparently they have misunderstood what many-time observables are, and how they are measured, and how such measurements can be verified by further experiments. Perhaps it will be useful to other readers of our original paper[2] to carefully set things right.

The measurement of a familiar single-time observable (carried out at time t_0, say) provides experimentally verifiable information about the future and the past of t_0 (if the Hamiltonian of the system is zero, for example, the same measurement, carried out either in the future or the past of t_0, will invariably yield the same result). Cohen and Peres correctly point out that this is not the case of measurements of many-time observables. If a many-time observable is measured at t_1 and t_2, and if the same observable is measured for the same system at t_a and t_b (with $t_1 < t_2 < t_a < t_b$ or $t_a < t_b < t_1 < t_2$, and with the Hamiltonian of the system taken to be zero), these two measurements "will, in general, yield different results." We, ourselves, made a point of that in Ref. 2; indeed (as the reader shall presently be reminded) it is *precisely that* property of many-time observables which makes such observables interesting.

Cohen and Peres suppose that what they have pointed out implies that many-time measurements produce *no* experimentally verifiable information. They are wrong. Such a measurement (at t_1 and t_2, say) provides experimentally verifiable information about the results of other many-time measurements at t_c and t_d, where $t_1 < t_c < t_d < t_2$ or $t_c < t_1 < t_2 < t_d$. Let us recall how that comes about. Suppose that a measurement of $s_z(t_1) + s_x(t_2)$ is carried out by means of the interaction Hamiltonian (1) of Ref. 1, and that another such measurement is carried out at t_c and t_d (with, say, $t_c < t_1 < t_2 < t_d$) by means of another such Hamiltonian, of the same form. The measuring apparatus is initially prepared [in accordance with (2) of Ref. 1] thus,

$$p_1 + p_2 = 0 \ , \quad q_1 - q_2 = 0 \ , \tag{1}$$

$$p_c + p_d = 0 \ , \quad q_c - q_d = 0 \ . \tag{2}$$

Since, for times $t_c \leq t \leq t_1$, $\dot{s}_z = 0$, and since, for times $t_2 \leq t \leq t_d$, $\dot{s}_x = 0$, it follows that $\dot{p}_c(t_c) = \dot{p}_1(t_1)$ and $\dot{p}_d(t_d) = \dot{p}_2(t_2)$. When all the interactions are complete, then, it will be the case that

$$p_1 + p_2 = p_c + p_d \ , \tag{3}$$

and so (as the results of these measurements are recorded in the above p sums) these two measurements will invariably produce *the same result.* Thus, in this fashion, the result of one multiple-time measurement can always be *confirmed* by another, carried out at times t_c and t_d, so long as $t_c < t_1 < t_2 < t_d$ or $t_1 < t_c < t_d < t_2$. The fact that $\dot{s}_x \neq 0$ during the first interval, and that $\dot{s}_z \neq 0$ during the second (which, in the event that $t_1 < t_2 < t_c < t_d$, would destroy the correlation between the results of those two measurements) is, for the present case, of no consequence whatever.

The result of a multiple-time measurement at t_1 and t_2 can also be verified by two single-time measurements at t_c and t_d, so long as $t_c < t_1 < t_2 < t_d$ or $t_1 < t_c < t_d < t_2$. Suppose that a z-spin measurement is carried out at time $t_c < t_1$, with the result $s_z = +\frac{1}{2}$. Thereafter an $s_z(t_1) + s_x(t_2)$ measurement is carried out (as described above). Arguments of the same form as those presented above imply that if the result of the multiple-time measurement is zero, then it must be the case at t_d that $s_x = -\frac{1}{2}$. Thus, a multiple-time measurement can be verified by means of the correlation it produces between two single-time measurements.

Just as a single-time measurement carried out at t_0 will, in general, yield no information about the result of multiple-time measurements carried out at t_m and t_n, where $t_m < t_0 < t_n$, a multiple-time measurement at t_1, t_2 will, in general, produce no information about the results of measurements carried out either entirely in the future or entirely in the past of the interval $t_1 \leq t \leq t_2$. The types of information produced by single- and multiple-time measurements are thus *complementary* to one another (and *that* is what is of interest about multiple-time measurements). Multiple-time measurements certainly produce no less, nor less verifiable information than single-time ones.

[1] E. Cohen and A. Peres, preceding paper, Phys. Rev. D **31**, 1525 (1985).

[2] Y. Aharonov and D. Z. Albert, Phys. Rev. D **29**, 223 (1984).

