Implications of the triplet-Majoron model for the supernova SN1987A

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We address several possible implications of the triplet-Majoron (TM) model for supernova physics, such as cooling and rapid deleptonization. We find that recent observations of the neutrinos from SN1987A are consistent with the TM model even if the $\nu_e\nu_e$-Majoron coupling constant is $g_{ee} \simeq 0.7\times10^{-3}$ suggested by a recent $\beta\beta$-decay experiment. We find specifically that deleptonization via $\nu_e\nu_e \rightarrow \chi^0$, followed by conversion to $\bar{\chi}^0$, is unlikely to be important for this value of g_{ee}; however, if $g_{ee} \simeq g_{\mu\mu} \simeq g_{\tau\tau}$, neutrino species equilibration via $\nu_e\nu_e \rightarrow \nu_\mu\nu_\mu$ and $\nu_e\nu_e \rightarrow \nu_\tau\nu_\tau$ is likely to occur on a time scale of $\sim 10^{-4}$ sec. Rapid equilibration, followed by decay in flight via $\nu_\mu(\nu_\tau) \rightarrow \bar{\nu}_e + \chi^0$, can dramatically enhance the chance of observing neutronization neutrinos, even for $g_{\mu e}^2$ and $g_{\tau e}^2 \simeq 10^{-18}$. This last effect might be detected in nearby supernovas.

I. INTRODUCTION

The recent observations[1,2] of neutrino pulses from the supernova SN1987A are, in general, in qualitative agreement with the expectation from a standard type-II collapse.[3–5] This allows us to exclude some features of ν physics which could drastically affect the intensity, energy, and time structure of the pulses [such as decaying[6] or fairly massive ν_e's (Refs. 5 and 7–9)]. The triplet-Majoron (TM) model[10,11] is, in many ways, the most radical approach to new ν physics. While some of its possible implications for supernovas have been discussed already,[12,13] it is worthwhile to reexamine the issue now.

In this model the neutrinos obtain small Majorana masses from a tiny vacuum expectation value (VEV) of a new Higgs triplet $\chi^0\chi^-\chi^{--}$ each member carrying two units of lepton number:

$$\langle \chi^0 \rangle \equiv v \neq 0 \ . \tag{1}$$

This VEV spontaneously breaks the global lepton-number symmetry yielding a massless Goldstone pseudoscalar, $\mathrm{Im}\chi^0 = M^0$, the "Majoron," and a very light scalar partner, $\mathrm{Re}\chi^0 \equiv \rho^0$. M^0 and ρ^0 are Yukawa coupled to neutrinos via the terms $g_{ee}\chi^0\nu_e\nu_e, \dots, g_{\tau\tau}\chi^0\nu_\tau\nu_\tau, g_{e\mu}\chi^0\nu_e\nu_\mu, \dots, g_{\mu\tau}\chi^0\nu_\mu\nu_\tau$, and also couple very weakly $[\sim(vm_F/u^2)\chi^0 F\bar{F}]$ to non-neutrino fermions—quarks and leptons [$u = 250$ GeV is the Glashow-Weinberg-Salam (GWS) doublet Higgs VEV]. The potentially rich phenomenology generated by these interactions allowed restricting the parameters of the model[11,14] as follows:

$$v \le 10\text{–}100 \text{ keV}, \quad g_{ee} \le 10^{-3}, \quad g_{\mu\mu} \le 3\times10^{-2} \ , \\ g_{\tau\tau} \le 10^{-1}\text{–}10^{0} \ . \tag{2}$$

Recently an anomalous behavior of the electron-energy spectrum of the Pacific Northwest Laboratories/University of South Carolina (PNL/ USC) ^{76}Ge double-β-decay experiment has been reported.[15] If interpreted in terms of the decay $^{76}\mathrm{Ge} \rightarrow {}^{76}\mathrm{Se} + e^- + e^- + \chi^0$, then

$$g_{ee} \approx 0.7\times10^{-3} \ . \tag{3}$$

Taken with the bound[16] $\langle m_{\nu_e} \rangle \lesssim 2$ eV on the Majorana ν_e mass, this value implies via

$$m_\nu^{\mathrm{Maj}} = v g_{ee} \tag{4}$$

the bound

$$v \lesssim 3 \text{ keV} \ . \tag{5}$$

There are several ways in which the triplet-Majoron model, if correct, could affect the neutrino pulses from supernovas. In particular one might anticipate (i) quick excessive cooling of the collapsing core via Majoron emission, (ii) quick deleptonization of the core, (iii) interspecies decays

$$\nu_i \rightarrow \nu_j + \begin{bmatrix} M^0 \\ \rho^0 \end{bmatrix}$$

effecting the intensity and energy of the observed $\nu_e(\bar{\nu}_e)$ signal, and in some cases also its time structure, and (iv) if there are massive relic neutrinos, the neutrinos from the supernova may scatter sufficiently strongly on them so as to disperse the observed signal and degrade its energy. Points (i) and (ii) have to do with new effects inside the collapsing core and, hence, are dependent on the astrophysics of the collapse, whereas points (iii) and (iv) have to do with new effects in vacuum which are easier to ascertain. We consider here all these points and the limits implied by them on parameters of the model. We find that none can exclude the value $g_{ee} \simeq 0.7\times10^{-3}$.

II. COOLING BY MAJORON EMISSION

We consider first the possibility that emission of Majorons (M^0,ρ^0) can lead to quick excessive cooling of the collapsing core. The hot core could generate and emit particles X of mass m_X up to $kT \simeq 10$ MeV. This emission is an important factor for dissipating the energy stored in the core during a time t, if (a) the production of X should be strong enough so that during a time t the total number of X's produced will be comparable to N_{tot}, the total number of nucleons or electrons in the core, and (b) the mean free path of X should exceed R, the radius of the core, so that once it is produced the X will escape unscattered.

Once (a) and (b) are satisfied, then the X's will carry an appreciable part of the energy stored in the core. If t, the time scale for the X emission, is shorter than 1–10 sec, the expected (and measured) time scale for radiating the trapped ν's, then the X's may dominate the energy loss from the core. This would shorten and reduce the observed $\nu_e(\bar{\nu}_e)$ signal and hence is ruled out by the experiment. However, while condition (a) is satisfied for the $X = \chi^0 \simeq M^0, \rho^0$ production in the TM model, condition (b) is definitely violated. The reason is that M^0 and ρ^0 are members of $SU(2)_L$ *triplet*, rather than a *singlet*, as is the case for the singlet-Majoron model of Chikashige, Mohapatra, and Peccei,[17] and hence have strong neutral-current interactions with the nucleons. The quantum numbers of $\chi^0(\bar{\chi}^0) \propto \rho^0 \pm iM^0$ are those of $\nu\nu(\overline{\nu\nu})$, respectively, so that the Z^0 coupling to Majorons is twice that of the coupling to neutrinos and the cross section for coherent Majoron scattering off nuclei or neutron clusters in the core is four times that of neutrinos.

There is also a large cross section for forming Majorons:

$$\sigma[(e^+e^-) \rightarrow \text{virtual } Z^0 \rightarrow M^0 + \rho^0] = 2\sigma[(e^+e^-) \rightarrow \text{virtual } Z^0 \rightarrow (\nu_\mu \bar{\nu}_\mu)]$$

and

$$\sigma[(\nu_e \bar{\nu}_e) \rightarrow \text{virtual } Z^0 \rightarrow M^0 + \rho^0] = 2\sigma[\nu_e \bar{\nu}_e \rightarrow \text{virtual } Z^0 \rightarrow (\nu_\tau \bar{\nu}_\tau)] \ ,$$

corresponding to the fact that the Majorons contribute to the Z^0 width as two neutrino species.[11] Thus, we expect, regardless of the strength of g_{ee}, $g_{\mu\mu}$, etc., that a plasma of Majorons, neutrinos, electrons, positrons, and photons will form and that the Majorons will also stay trapped in the core for 1–10 sec, as will the neutrinos.

At each radius $r \lesssim R$, and for any time $t \lesssim t_{\text{trap}} \approx 1$–10 sec, we have a local temperature $T(r,t)$, and a corresponding Fermi-Dirac distribution of ν's of each species and a Bose-Einstein distribution for the Majorons. From the point of view of statistical mechanics, M^0 and ρ^0 count as 2 degrees of freedom, each ν species contributes twice $\frac{7}{8}$, e^+e^- contributes four times $\frac{7}{8}$, and γ contributes 2 degrees of freedom. Adding the Majorons changes the number of degrees of freedom from ≈ 11 to ≈ 13, a small and not very important change ($\leq 20\%$). Only the neutrinos and Majorons can leak out of the core. Within this subset, the 40% change of the number of degrees of freedom from 5.25 to 7.25 is still within the error of the estimated $\bar{\nu}_e \nu_e$ fluxes.

The effect of Majoron emission is most likely even smaller than 40%. First,

$$\sigma(\chi^0 N) \simeq 4\sigma(\nu_\mu N) \ ; \tag{6}$$

hence, the layer from which the Majorons are effectively emitted, the "Majoron sphere," is thinner than the corresponding "$\nu_\tau \bar{\nu}_\tau \nu_\mu \bar{\nu}_\mu$ sphere," and the Majorons are emitted from a cooler region further from the core and carry less energy. Similar calculations[3,18] made for $\nu_\mu \cdots \bar{\nu}_\tau$ vs $\nu_e \bar{\nu}_e$ emission suggest that we are likely to have only a 20%–30% increase in the total energy output.

There is also another mechanism for rechanneling Majorons into $\nu_e \bar{\nu}_e$, namely, the decay $\rho^0 \rightarrow \nu_e \bar{\nu}_e$ which dominates when

$$2m_{\nu\tau}, 2m_{\nu\mu} \geq m_{\rho^0} \geq 2m_{\nu_e} \ . \tag{7}$$

Since the M^0 does not decay, and since the average energy of the $\bar{\nu}_e$ and hence the cross section for their detection, is slightly reduced, this effectively rechannels $\sim$20–40% back into the observed energy of the $\bar{\nu}_e$'s. The net effect may then even be a slight enhancement of the detected signal.

The above conclusion will not be strongly affected if the $\rho^0 \rightarrow \nu_e \bar{\nu}_e$ decay is fast and often occurs inside the core. It will then be mainly part of the thermal equilibrium processes. However, whenever the decay occurs within one optical depth of the Majoron, that is while it is "on the way out of the core," the ν_e decay products having comparable optical depths are likely to escape as well.

Let us digress a little to motivate the mass relations (7). In the TM model, the ν_e and ρ^0 masses are given by

$$m_{\nu_e} = v g_{ee} \tag{8}$$

and

$$m_{\rho^0} = v(\lambda_{\chi^4})^{1/2} \ , \tag{9}$$

where λ_{χ^4} is the quartic coupling of the χ triplet. There are two such invariant couplings,[10,11] but we disregard this finer detail in this analysis. The mass of the charged member χ^- is given in terms of the standard model VEV ($u = 250$ GeV) and the triplet-doubled quartic coupling

$$m_{\chi^-} = u(\lambda_{\rho^2\chi^2})^{1/2} \ , \tag{10}$$

in analogy with

$$m_\phi = \sqrt{\lambda_{\phi^4}}\, u \ , \tag{11}$$

the mass relation for the standard Higgs doublet. None of the dimensionless λ couplings is known at present, except for the requirement $\lambda_i \lesssim 1$. In addition it seems reasonable to assume that

$$\lambda^2_{\phi^2\chi^2} \gtrsim \lambda_{\phi^4} \lambda_{\chi^4} \ . \tag{12}$$

Such a relation could be motivated if the Higgs description is just an effective description, and the λ's represent the $\chi\chi^\dagger \rightarrow \chi\chi^\dagger, \phi\phi^\dagger \rightarrow \chi\chi$ and $\phi\phi^\dagger \rightarrow \phi\phi^\dagger$ amplitudes. From theoretical considerations, $m_\phi \lesssim 1$ TeV, while

$m_{\chi^-} \gtrsim 20$ GeV is the lower bound placed on charged scalars by e^+e^- experiments at DESY with PETRA. Strictly speaking, such a bound applies to stable charged scalars. The χ^- would decay via $g_{ee}\chi^-\bar{\nu}_e e^+$ into $\nu_e e^-$ with a rate $\Gamma_{\chi^-\to\nu e} \approx (g_{ee}^2/8\pi)m_{\chi^-}$. Thus $e^+e^- \to \chi^+\chi^-$ would yield an $e^+e^-\nu\bar{\nu}$ signal which may be less striking than that of stable χ^+,χ^-, and the bound may, in fact, be weaker. We find from (10)–(12) the reasonable lower bound

$$\lambda_{\chi^4} \geq 0.5\times 10^{-6} \text{ or } (\sqrt{\lambda} \geq 0.7\times 10^{-3}) \ . \tag{13}$$

From the ratio of Eqs. (9) and (8) we have then, using (13) and (3),

$$m_\rho / m_{\nu_e} = \lambda^{1/2}/g_{ee} \geq 1 \ . \tag{14}$$

Thus $m_\rho \geq 2m_{\nu e}$ is quite plausible and the weaker bounds on $g_{\mu\mu}, g_{\tau\tau}$ certainly allow $2m_{\nu_\tau}, 2m_{\nu_\mu} \geq m_\rho$.

In passing we note that in the singlet Majoron models,[17] the Majoron coupling, and hence their production rates, are so weak that they play no role in supernovas.

III. DELEPTONIZATION

We next address the issue of deleptonization, namely, that processes inherent to the TM model quickly deplete the initial large excess lepton number of the core, $N_L = N_e$ = total number of electrons. This could lead to a quicker collapse with higher entropy.[13]

In discussing deleptonization, two distinct issues should be clarified. The TM model, in its simplest original form,[10,11] does not separately conserve all the lepton flavors. The electronic lepton number $L_e = N_e$, which is the initial number of electrons, can therefore spread over all ν species. If the rates are fast enough, L_e can also be hidden in the bosonic sector via processes of the type $\nu\nu \to \chi^0$. The second issue has to do with genuine deleptonization, namely, the violation of overall lepton number $L_e + L_{\nu_e} + L_{\nu_\mu} + L_{\nu_\tau} + L_\chi$.

The Lagrangian of the model, i.e., all the basic quartic and Yukawa interactions, preserves overall L conservation $\{$with the assignments $L[\nu_i\ (\bar{\nu}_i)] = 1\ (-1)$ and $L[\chi^0\ (\bar{\chi}^0)] = 2\ (-2)\}$. The only L violations are spontaneously induced by $\langle \chi^0 \rangle = v \neq 0$ via $m_{\nu_i} \neq 0$ and via the mass splitting $m_{\rho^0} - m_{M^0} = m_{\rho^0}$ between the real and imaginary components of χ^0. Imperfect cancellations between the ρ^0 and M^0 exchanges generate the $\Delta L = 4$ process $\nu\nu \to \bar{\nu}\bar{\nu}$ with an amplitude $(m_\rho/E)^2$ smaller than the allowed $\nu\nu \to \nu\nu$ processes. The rates of such $\Delta L = 4$ processes are of order $(m_\rho/E)^4$. With $E \approx 1-10$ MeV, these rates are completely negligible.

We could, however, have $\Delta L = 2$ processes $\nu\nu \to \chi^0$ with the χ^0 loosing its lepton number by propagating far enough to become an incoherent mixture of ρ^0 and M^0. In the following we show that (a) under the condition pertaining to the supernova, the $\nu\nu \to \chi^0$ processes of order g^6, leading to a "leakage" of lepton number from the fermionic to bosonic sectors, are too slow to generate any appreciable effect, even for $g_{ee} = 0.7\times 10^{-3}$, and, furthermore, (b) even if the χ's were copiously produced, the $\chi_0 \to \bar{\chi}_0$ flip is quenched by interactions.

(a) The triplet-Majoron model allows for quick (second-order) species interconvection reactions such as (see Fig. 1)

$$\nu_e\nu_e \to \text{“}\chi^0\text{”} \to \nu_\mu\nu_\mu(\nu_\tau\nu_\tau)$$

with large cross sections,

$$\sigma(\nu_e\nu_e \to \nu_i\nu_i) = \frac{g_{ee}^2 g_{ii}^2}{32\pi E_\nu^2} \ .$$

This cross section exceeds $(10^{-38}-10^{-39})$ cm^2, even for $g_{ee} \simeq g_{ii} \simeq 0.7\times 10^{-3}$ and $E = 10$ MeV. The mean free path for neutrino densities $N_\nu \geq 10^{34}$ cm^{-3} is then smaller than 10^5 cm, guaranteeing practically instantaneous interconversions. The equilibration of the lepton number between all the species $N_{\nu_e} = N_{\nu_\mu} = N_{\nu_\tau} = L/3$ will reduce the Fermi pressure of the neutrinos and accelerate the collapse.

If processes leading to a net $\nu_e\nu_e \to \chi^0$ were fast enough, then lepton number can "hide away" in the bosonic sector. The χ^0 production processes are, however, of higher order in g_{ee}, as shown Figs. 2–4. The extra factor of $g_{ee}^2 \simeq 0.5\times 10^{-6}$, and the kinematics of three-body versus two-body phase space, suppress these g^6 processes by factors of $\sim 10^{-8}$ relative to the g^4 process of Fig. 1.

Alternative production processes involve the λ_{χ^4} coupling, as shown in Fig. 5. If we take seriously the lower bound $\sqrt{\lambda} \geq 0.7\times 10^{-3}$, then $\sqrt{\lambda} \geq g$ and the last process may be dominant. The actual cross section is

$$\frac{1}{1536\pi^3}\frac{g^2\lambda^2}{E_\nu^2} \approx 2\times 10^{-5}\frac{g^2\lambda^2}{E_\nu^2} \ .$$

For simplicity we take the g^6 cross sections shown in Figs. 2–4 also as

$$\sigma \approx g^6/E_\nu^2(10^{-3}-10^{-5}) \ . \tag{15}$$

A more detailed estimate corroborating this is described in the Appendix. The mean free path for $\chi^0 \to \nu\nu$ conversion is then

$$l_{\text{mfp}} = \frac{1}{N_\nu \sigma} \ .$$

The neutrino energy is essentially its Fermi energy, which controls the density

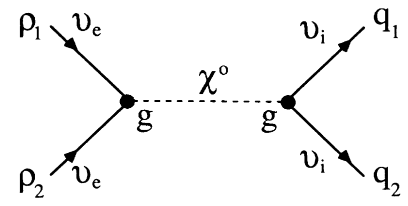

FIG. 1. Diagram of a process leading to equilibration of neutrino species (p_i and q_i refer to incoming and outgoing momenta, respectively).

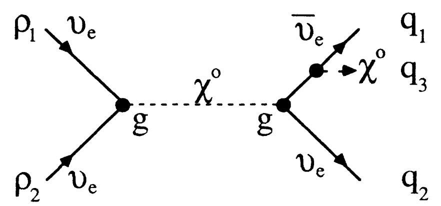

FIG. 2. Diagram of a process converting electron neutrinos partially to electron antineutrinos.

$$N_\nu = \frac{4\pi E_\nu^3}{3(2\pi)^3} .$$

The corresponding mean free path is

$$l_{\rm mfp} = (E_\nu g^6)^{-1} \times 2\pi^2 \times (10^3 - 10^5) \simeq (10^{12} - 10^{14})/(E_\nu/{\rm MeV})\ {\rm cm} .$$

Accordingly, $l_{\rm mfp}$ is much larger than $0.1c \simeq 3\times 10^9$ cm, the distance relevant for 0.1-sec deleptonization, to allow significant χ^0 production. This conclusion will be modified if $(\lambda_{\chi^4})^{1/2}$ is larger than its minimal $(0.7\times 10^{-3} \simeq g_{ee})$ by factors of between 10 and 100, or if $g_{\mu\mu}$ is larger than g_{ee} by similar factors.

The important point that we wish to emphasize is that a value $g_{ee} \sim 0.7\times 10^{-3}$ *per se* does not lead even to effective leakage of lepton number into the bosonic χ^0 component. Since such a leakage is the required first step for genuine deleptonization (via $\chi^0 \to \bar{\chi}^0$), the latter will certainly not happen. In particular, we show next that even when a χ^0 is produced, it cannot precess into $\bar{\chi}^0$ during the 0.1-sec standard deleptonization time.

(b) A χ^0 "precesses," while propagating in a vacuum, into $\bar{\chi}^0$ with frequency $\omega = m_\rho^2/E$. In the $\chi^0 = |\uparrow\rangle$ = "spin up," $\bar{\chi}^0 = |\downarrow\rangle$ = "spin down" basis, this is analogous to the effect of a Hamiltonian: $H = \boldsymbol{\sigma}\cdot\mathbf{B} = \sigma_x(m_\rho^2/E)$. For $E \approx 1-10$ MeV, and m_ρ in the range eV–keV, the oscillation length $2\pi E/m_\rho^2$ varies between 10^{-4} and 10^3 cm, and we have practically instantaneous flipping $\chi_0 \to \bar{\chi}_0$. The above result is modified, however, by the different interactions of χ^0 and $\bar{\chi}^0$ with the dense background of neutronization neutrinos ($N_{\nu_e} \gtrsim 10^{34}/{\rm cm}^3$). In the conditions prevailing in the supernova, the $\nu\chi^0$ scattering is stronger[12,13] than ν or χ^0 scattering via Z^0 exchange, and the latter process will be

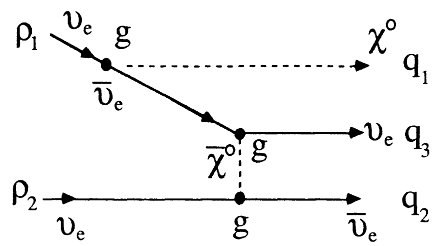

FIG. 4. Diagram of a process partially converting electron neutrinos to antineutrinos and χ^0.

neglected (except for its important role in confining the χ^0's).

The Born diagram of order g_{ee}/E contributes to $\bar{\chi}^0$ but not to χ^0 scattering on ν_e. This generates, via the optical theorem, an index of refraction difference

$$\delta_{\rm nref} \equiv \frac{2\pi[\bar{f}_\chi(0) - f_\chi(0)]N_\nu}{E^2} \tag{16}$$

between $\bar{\chi}^0$ and χ^0, with N_ν the number density of neutrinos. This, in turn, induces a difference between the diagonal components of the "Hamiltonian" describing the evolution of the two-component state vector

$$H = \begin{bmatrix} A & m_\rho^2/E \\ m_\rho^2/E & -A \end{bmatrix} = \frac{m_\rho^2}{E}\sigma_x + A\sigma_z , \tag{17}$$

with

$$A = E\delta_n = 2\pi[\bar{f}_\chi(0) - f_\chi(0)]N_\nu E^{-1} = 2\pi g_{ee}^2 N_\nu E^{-2} .$$

The evolution corresponds to precession around a resultant "**B**" tilted by an angle α (we will verify subsequently that α is small):

$$\alpha = \tan^{-1}\frac{m_\rho^2/2E}{A} \approx \frac{m_\rho^2 E}{g^2 N_\nu} \tag{18}$$

relative to the z axis (see Fig. 6).

The maximal buildup of a $\bar{\chi}^0$ probability admixed into the pure initial χ^0 state during the precession is given by

$$P = \sin^2\frac{\theta_{\max}}{2} = \sin^2\alpha \approx \frac{m_\rho^4 E^2}{g^4 N^2 (4\pi)^2} . \tag{19}$$

However, each time a "relevant" collision, such as a large-angle elastic scattering $\bar{\chi}^0\nu_e \to$ "$\bar{\nu}_e$" $\to \bar{\chi}_0\nu_e$ occurs,

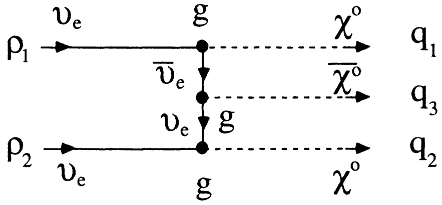

FIG. 3. Diagram of a process converting electron neutrinos to χ^0 and $\bar{\chi}^0$.

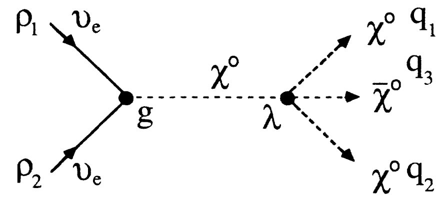

FIG. 5. Diagram of the process converting electron neutrinos to χ^0 and $\bar{\chi}^0$ via λ coupling.

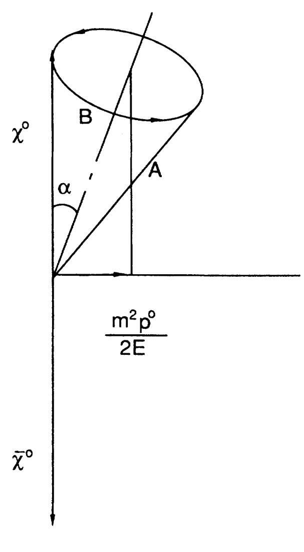

FIG. 6. Precession of the state of the Majoron about the vector B due to the interactions of χ^0 and $\bar{\chi}^0$ with the dense background of neutronization neutrinos.

the $\bar{\chi}^0$ component is scattered out of the beam and the precession and leakage starts all over again. The initial χ^0 probability is reduced after N_c collisions by $(1-\alpha^2)^{N_c} \approx e^{-N_c\alpha^2}$ and deleptonization happens when $N_c\alpha^2 \gtrsim 1$.

The cross section for the elastic scattering is

$$\sigma_{ee}(\bar{X}_0\nu_e \rightarrow \bar{X}_0\nu_e) \approx \frac{g^4}{32\pi E^2} . \tag{20}$$

The time scale relevant for deleptonization is $t \simeq 10^{-3}-10^{-1}$ sec. This is the time for collapse from $\rho \approx 10^{12}$ gm/cm^3 to just subnuclear densities $\rho \approx 10^{14}$ gm/cm^3, or the time for emitting most of the lepton excess.[3,18,19] During this time, χ^0 travels $l=ct =3\times10^7-3\times10^9$ cm and suffers $N_c = l\sigma N_\nu$ collisions. Deleptonization will occur within this interval only if

$$N_c\alpha^2 = \frac{g^4 N_\nu}{32\pi E^2}\frac{m_\rho^4 E^2 l}{N_\nu^2 g^4 (2\pi)^2} = \frac{m_\rho^4 l}{N_\nu 128\pi^3} \gtrsim 1 . \tag{21}$$

For $10^{12} \le \rho \le 10^{14}$ g/cm^3 and $10^{-2} \le Y_e \lesssim 1$, we have $N_\nu = (10^{38}-10^{34})$ cm^{-3}, and using 1 keV cm$=0.5\times10^8$ we have then

$$N_c\alpha^2 = \frac{l\,10^{32}}{4\times10^3(10^{38}-10^{34})16}(m_\rho/\text{keV})^4$$
$$=(10^{-3}-10^3)[m_\rho(\text{keV})]^4 . \tag{22}$$

Thus, it simply suffices to take $m_\rho \lesssim 0.1$ keV to completely quench the quick genuine deleptonization. Note that

$$\alpha = \frac{2\times10^6(m_\rho/\text{keV})^2(E/\text{MeV})}{10^{38}-10^{34}}$$
$$\approx(10^{-2}-10^{-6})\times(m_\rho/\text{keV})^2(E/\text{meV})$$

is, in fact, very small justifying the small-α approximations made above. Deleptonization could be avoided by symmetry restoration[12] when core temperatures exceed $v_0 \approx 100$ keV. Since this is a subtle issue we prefer the direct argument presented above.

IV. DELEPTONIZATION AND THE OBSERVED NEUTRINO SIGNAL

We have not discussed so far the question of how deleptonization, or equilibration of lepton number between the different ν species, affects the collapse and the final observed $\bar{\nu}_e$ pulse. This largely astrophysical issue has been analyzed recently[20] and here we confine ourselves to only a few remarks. It is generally believed that the bulk of the energy flux is emitted via the thermalized $\bar{\nu}_e\nu_e \cdots \bar{\nu}_\tau\nu_\tau$ neutrinos and *not* via the early neutronization (conventionally ν_e) neutrinos. Also, the detected signal is mainly due to the late thermal $\bar{\nu}_e$'s. This signal depends on very general global properties of the system, the total gravitational energy accumulating in the collapsing core, the size of the core, and on the number of equilibrating bosonic and fermionic degrees of freedom. These parameters fix the surface area and temperature of the radiating system and, hence, the luminosity, time duration, and energy spectrum of the observed $\bar{\nu}_e$ signal. The variation of temperature for different species, i.e., the lower temperature of $\bar{\nu}_e$ which, because of its shorter mean free path, is emitted from the cooler outer region, is an important correction of this picture.[3,18,19]

The preceding arguments suggest that it is not likely that the Majoron model drastically changes these basic parameters. The quick equipartition of species via $\nu_i\nu_i \rightarrow \chi^0 \rightarrow \nu_j\nu_j$ reduces the Fermi pressure, causing a quicker collapse. It also equalizes the optical depths, spectra, and time structures of all emitted ν_i species. The effective combination "ν_{eff}"$\approx\frac{1}{3}(\nu_e+\nu_\mu+\nu_\tau)$ suffers less absorption than the neutronization ν_e's which conventionally prevail at the very early stages. The enhanced emission of neutronization neutrinos may deplete the remaining energy available during thermal emission by more than the canonical 5–10%. However, in the later thermal stage, $\bar{\nu}_e,\bar{\nu}_\mu,\bar{\nu}_\tau$ will be all emitted with the same temperature, which is higher than the temperature for separate emission of the more strongly absorbed $\bar{\nu}_e$. This, and the quicker collapse, causing more complete energy trapping, offsets the effect of enhanced energy loss via neutronization neutrinos. The net result is likely to be a mild change. Finally we should emphasize that even if the consequences of the species equilibration were truly disastrous, they still need not conflict directly with $g_{ee} \approx 0.7\times10^{-3}$ suggested by experiment. We could have, for example, an extended variant of the TM model with separate χ_e^0 coupling to ν_e's with a vanishing cross section for $\nu_e\nu_e \rightarrow \nu_i\nu_i$.

V. NEUTRINO MIXINGS AND DECAYS

We next address possible novel effects involving the propagation of the neutrinos from the supernova to

Earth. The observed signal reflects effects happening both in the core and during propagation (in the "thin" progenitor and/or vacuum). The standard interflavor mixing [vacuum and/or Mikheyev-Smirnov-Wolfenstein (MSW) type] could *a priori* be quite strong in the TM model. It does not, however, strongly affect an already maximally mixed system of all flavors. $\nu_i \to \bar{\nu}_i$ oscillations are also possible via the Majorana mass, though with negligible m_ν/E amplitude.[21]

In certain extensions of the minimal TM model, where the neutrino masses are not generated exclusively via $\langle \chi^0 \rangle \neq 0$, diagonalizing the mass matrix by going to the physical basis of neutrinos propagating in vacuum, does not necessarily entail diagonalization of the χ^0 couplings, and decays of the type $\nu_i \to \bar{\nu}_j + \chi^0$, could happen for $m_i > m_j$. Even if we do not have $\nu_e \to \nu_\mu$, ν_τ, with almost a complete loss of the ν_e and $\bar{\nu}_e$ signals,[6] the $\nu_\mu(\nu_\tau) \to \bar{\nu}_e + \chi^0$ decays could have some dramatic consequences for the early ($t \lesssim 0.1$ sec) portion of the signal. Instead of having only neutronization neutrinos ν_e (or $\nu_\mu \nu_\tau$ as well in the TM case) which can be detected only via the fairly low cross-section $\nu e \to e'\nu'$ reactions, we would have $\bar{\nu}_e$'s which are detected via the higher cross-section reaction $\bar{\nu}_e p \to e^+ n$ and a significantly enhanced early signal. This feature is not inconsistent with the observations if the first two detected ν's are interpreted as $\bar{\nu}_e$ rather than ν_e. We should emphasize that only very weak couplings $g_{\mu e} \gtrsim 10^{-10}$ are required for $m_{\nu_\mu} \geq 1$ eV in order to ensure that (i) the decay path $l_D = c 8\pi E_\nu / g^2 m_\nu^2$ is shorter than the distance to the supernova (1.5×10^{23} cm), and that (ii) the extra delay in arrival time of the $\bar{\nu}_e$, due to the early phase of propagation as the heavier species,

$$\Delta t_1 = \frac{m^2}{2E^2} t_D = \frac{4\pi}{g^2 E} ,$$

is shorter than 0.1 sec.

A similar value of Δt results in the case when the ν_μ is emitted at angle θ with respect to the Earth-supernova axis and then the ν_e is redirected to Earth during the $\nu_\mu \to \nu_e + \chi^0$ decay (see Fig. 7). The decay angle, $\beta \geq \theta$, is kinematically restricted by m/E. The extra path length is $\Delta l = (1-\cos\theta) l_D \approx (\theta^2/2) l_D \leq \frac{1}{2}(m/E)^2 l_D$ and the corresponding time delay is, again,

$$\Delta t \lesssim \tfrac{1}{2}(m/E)^2 t_D = \frac{4\pi}{g^2 E} .$$

In the triplet-Majoron model, interactions of the neutrinos with the 3-K Majoron background may disperse the signal and degrade its energy.[12,22] The cross section for scattering a supernova neutrino of energy E on the background Majoron of energy = temperature = T is

$$\sigma = g^4/32\pi E T = g^4/(32\pi \times 10 \text{ MeV} \times 10^{-3} \text{ eV}) \simeq g^4 \times 10^{-16} \text{ cm}^2 .$$

The number of collisions suffered in transit from the supernova is $n\sigma l = 5 \times 10^{10} g^4$, where a cosmological density, $n = 10^3/\text{cm}^3$, was used. The requirements of no signal dispersal due to such collisions implies $g^2 = 2 \times 10^{-5}$,

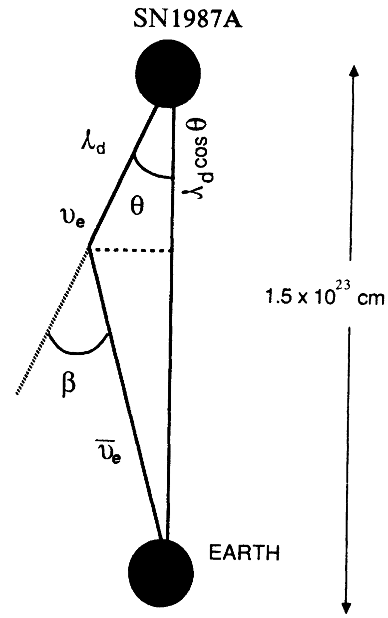

FIG. 7. Geometry of neutrino decay in flight by Majoron emission, resulting in an antineutrino reaching Earth.

which is inferior to the lower bounds from the double-beta-decay experiments. In the minimal TM model, the primordial neutrinos annihilate into massless Majorons[11,12] and cannot contribute to the dark matter. This can be avoided in models where the Majorons are massive due to an explicit lepton symmetry breaking.[12]

A Majoron model with a massive neutrino background was recently considered by Raffelt and Silk.[23] The interactions of the neutrinos prevent free streaming, making them much better dark-matter candidates. In this case the local density of the massive neutrinos ν_H or the massive Majorons in the halos of the Large Magellanic Cloud (LMC) galaxy as well as our own, is roughly 10^6 times larger than the cosmological density $n = 10^3/\text{cm}^3$, used in the previous estimate for the Majoron background. The requirements of no signal dispersal in this case allows only 10^{-6} times lower cross sections. This implies $g_{ee} g_{HH}$ (or g_{ee}^2) $< 2 \times 10^{-8}$ where g_{ee}, g_{HH} are the Majoron couplings to ν_e and ν_H, respectively.

VI. DISCUSSION AND CONCLUSIONS

We have considered a broad spectrum of possible implications of the triplet-Majoron model for supernovas and the neutrino signals emerging from stellar collapse. We conclude that the Majoron model is consistent with the recent observations of neutrino bursts from SN1987A. It is quite remarkable that even the fairly weak g_{ee} couplings suggested by one of the $\beta\beta$ experiments could have non-negligible consequences. A particularly amusing scenario involving first a quick equilibration of all neutrino flavors, followed by decays in flight ($\nu_\mu, \nu_\tau \to \bar{\nu}_e + \chi^0$), could render the neutronization pulse observable in future detections of neutrinos from supernovas in our own Galaxy.

One final comment should be addressed to the strong $\nu\nu$ interactions mediated by Majoron exchanges an issue discussed in Refs. 12 and 24. Such interactions cannot self-trap the neutrinos. Had it not been for the confining effect of the nucleons, the ν's would leave the core instantaneously with the sound velocity $c/3$, just as a pressurized gas once the enclosing container is removed. The strong $\nu\nu$ scattering cross sections suggest that the motion of the neutrinos will be much more coherent with the neutrinos having at each point a well-defined velocity field $\mathbf{v}(\mathbf{r},t)$.

Thus the neutrinos may behave as a viscous fluid rather than a gas. An amusing speculative possibility is that this could modify the neutrino convection. During the early stage of stellar collapse, with only neutrinos present, this may affect the momentum transferred to the nuclei of the envelope. We can envision then not only coherent scattering of neutrinos on nucleons in a nucleus, or some arbitrary cluster, but also doubly coherent processes involving the simultaneous scattering of several neutrinos, enhancing the cross section further. It is conceivable that these effects could eventually help explain the "blow off" of the stellar mantle in a type-II supernova.

APPENDIX

The cross section for the process of Fig. 2 is

$$\sigma(\nu_e(p_1)+\nu_e(p_2)\rightarrow\bar{\nu}_e(q_1)\tilde{\chi}(k)\nu_e(q_2)) = [2(2E)^2(2\pi)^5]^{-1}\int d\phi_3\,|M|^2 ,$$

where

$$|M|^2=g^6\frac{1}{64}\left[\frac{W-2\omega_1}{W-2\omega_2}\right](1/E^2) ,$$

where $2E=W$ is the c.m. energy and (ω_1) ω_2 are the c.m. energies of the final (antineutrino) neutrino. The angular part of the three-body phase-space element, $d\phi_3$, separates yielding

$$\sigma(\nu\nu\rightarrow\bar{\nu}\nu\chi^0)=\{g^6/[2^{14}(\pi^3E^2)^2]\}\times\int_0^{w/2}\int_0^{w/2}d\omega_1 d\omega_2\frac{W-2\omega_1}{W-2\omega_2} .$$

The $d\omega_2$ integral has a logarithmic divergence due to the $\omega_2\approx W/2$ region, i.e., the collinear configuration with $\mathbf{k}$ parallel to $\mathbf{q}_1$. In this case, the lightlike four-momenta k and q_1 add up to a lightlike $k+q_1$, yielding the on-shell pole $(q_1+k)^{-2}$. The wave packets of the final $\bar{\nu}$ and χ^0 move collinearly with exactly the same speed $v=c$, and never separate to form a genuine three-body state. The singularity is avoided by introducing a finite ν_e mass so that σ becomes

$$\sigma(\nu\nu\rightarrow\bar{\nu}\nu\chi^0)=\frac{g^6}{2^{14}\pi^3E^2}\ln(E/m_{\nu_e}) .$$

Even if we take m_{ν_e} as small as 10^{-33} eV (corresponding to a ν_e Compton wavelength=Hubble radius), $\sigma\leq10^{-3}g^6/E^2$. The expression for $|M|^2$ of the $\nu_e\nu_e\rightarrow\chi^0\chi^0\bar{\chi}^0$ process of Fig. 3 is $|M|^2=\frac{1}{4}q_1\cdot q_2/(P_1\cdot q_1)(P_2\cdot q_2)$. The integral $\int d\phi_3$ no longer separates into the integration over the Euler angles describing the orientation of the final $\mathbf{q}_1,\mathbf{q}_2,(\mathbf{q}_3=-\mathbf{q}_1-\mathbf{q}_2)$ plane and the $d\omega_1 d\omega_2$ integration. The double collinear configuration with $\mathbf{q}_1$ parallel to $\mathbf{p}_1$ (and $|\mathbf{q}_1|\leq|\mathbf{p}_1|=W/2$) and $\mathbf{q}_2$ parallel to $\mathbf{p}_2\,|\mathbf{q}_2|\leq|p_2|=W/2$ will not occur in the physical region. Since $\mathbf{p}_1$ and $\mathbf{p}_2$ are antiparallel, so will $\mathbf{p}_1-\mathbf{q}_1$ and $\mathbf{p}_2-\mathbf{q}_2$ and hence p_1-q_1 and p_2-q_2 could not add to a null vector q_3. Thus we have a single $\ln(E/m_{\nu_e})$ factor, and the above estimate holds yielding again $\sigma\leq g^6 10^{-3}/E^2$.

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