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Possibility of a sudden flip or disappearance of electromagnetic fields without photon emission

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We demonstrate in the sudden approximation that the rapid reversal or disappearance of the magnetic induction of the neutron's dipole moment is dominantly a nonradiative process. This has important consequences for vacuum n-$\bar{n}$ oscillations.

It is well known that in classical electrodynamics abrupt changes in the configuration of electromagnetic (EM) fields entail radiation of the "deficit fields" via electromagnetic radiation.[1] Similar semiclassical extensions have also been successfully applied to individual atomic or nuclear EM transitions, particularly insofar as average properties and/or cases involving large quantum numbers were considered.

It has recently been pointed out[2] that the proposed n-$\bar{n}$ oscillations in vacuum (which can occur in a large class of theories where the baryon number is not conserved[3]) entails a flip of the anomalous magnetic dipole moment of the neutron. The sudden reversal of the static field configuration is expected, according to the classical considerations, to be accompanied by photon emission. If this were indeed the case then we would face a dilemma since *CPT* requires that n and $\bar{n}$ be exactly degenerate, and hence there is absolutely no energy available for these photons; this will then "quench" such n-$\bar{n}$ vacuum oscillations (and in general $x^0 \to \bar{x}^0$ transition with x^0 a neutral system possessing some nonvanishing EM multipole). In the following we demonstrate that these classical considerations are not valid for the quantum case and sudden flips of EM fields occur to first order without photon emission. This is analogous to the case of the Mössbauer effect where the recoil of the nucleus is often accompanied by no vibrational excitation of the lattice. To be specific, let us consider the case of n-$\bar{n}$ oscillations. Initially we have a particular **B** field configuration which can be derived from a vector potential $\mathbf{A} = \boldsymbol{\mu} \times \nabla(1/r)$ via $\mathbf{B} = \nabla \times \mathbf{A}$.

The **A** field can in general be expanded in normal modes in some quantization box of volume $V = L^3$:

$$\mathbf{A}(x) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k},\alpha} q_{\mathbf{k}}^{\alpha} \epsilon_{\mathbf{k}}^{\alpha} e^{i\mathbf{k}\cdot\mathbf{x}} + \text{H.c.} \tag{1}$$

The $q_{\mathbf{k}}^{\alpha}$ are the dynamical degrees of freedom of the EM field. The latter behave like oscillators with energies

$$\epsilon_{\mathbf{k}} = \frac{1}{4\pi c^2} \frac{1}{2} (\dot{q}_{\mathbf{k}}^2 + \omega_{\mathbf{k}}^2 q_{\mathbf{k}}^2) \tag{2}$$

with frequencies $\omega_{\mathbf{k}} \equiv c\,|\mathbf{k}|$.

In the absence of any external sources of radiation, all the oscillators are in their (Gaussian) ground states:

$$\psi_{\mathbf{k}}^0 = N_{\mathbf{k}} \exp\left[\frac{-\omega_{\mathbf{k}}}{8\pi\hbar c^2} q_{\mathbf{k}}^2 \right], \tag{3}$$

where $N_{\mathbf{k}}$ are the appropriate normalizations. The vacuum state of the full EM field is then a direct product over all modes of the above Gaussians:

$$|\psi^0\rangle \equiv \prod_{\mathbf{k}} |\psi_{\mathbf{k}}^0(q_{\mathbf{k}})\rangle . \tag{4}$$

The introduction of a prescribed classical field $\mathbf{A}^{\text{class}}(\mathbf{x})$ amounts then to shifting each of these oscillators away from the previous (zero) averages in the vacuum so as to form the "coherent states" $|\alpha_{\mathbf{k}}\rangle$:

$$\psi_{\mathbf{k}}^0 \to \alpha_{\mathbf{k}} = N_{\mathbf{k}}^0 \exp\left[\frac{-\omega_{\mathbf{k}}}{8\pi\hbar c^2} (q_{\mathbf{k}} - \alpha_{\mathbf{k}})^2 \right] \tag{5}$$

with $\alpha_{\mathbf{k}}$ the corresponding Fourier coefficients of the external classical field

$$A^{\text{class}}(x) = \sum \alpha_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} + \text{H.c.} \tag{6}$$

The coherent states $|\alpha_{\mathbf{k}}\rangle$ have nonvanishing scalar products with $|\psi_{\mathbf{k}}^0\rangle$:

$$\langle \psi_{\mathbf{k}}^0 | \alpha_{\mathbf{k}} \rangle \equiv N_{\mathbf{k}}^2 \int dq_{\mathbf{k}} \exp\left[\frac{-\omega_{\mathbf{k}}}{8\pi\hbar c^2} [(q_{\mathbf{k}} - \alpha_{\mathbf{k}})^2 + q_{\mathbf{k}}^2] \right] = \exp\left[\frac{-\omega_{\mathbf{k}}}{8\pi\hbar c^2} \alpha_{\mathbf{k}}^2 \right].$$

This means that there is a probability $|\langle \psi_{\mathbf{k}}^0 | \alpha_{\mathbf{k}} \rangle|^2$ that the sudden introduction (or removal) of the external field at $t = 0$ will still allow the system to remain in the original (vacuum) ground state, i.e., with no photons emitted in the mode **k**.

$$P_{\mathbf{k}}^0 \equiv |\langle -\alpha_{\mathbf{k}} | \alpha_{\mathbf{k}} \rangle|^2 = \left| N_{\mathbf{k}}^2 \int dq_{\mathbf{k}} \exp\left[\frac{-\omega_{\mathbf{k}}}{8\pi\hbar c^2} [(q_{\mathbf{k}} - \alpha_{\mathbf{k}})^2 + (q_{\mathbf{k}} + \alpha_{\mathbf{k}})^2] \right] \right|^2 = \exp\left[\frac{-\omega_{\mathbf{k}}}{2\pi\hbar c^2} (\alpha_{\mathbf{k}})^2 \right] \tag{7}$$

is the probability (in the sudden approximation) of not emitting a photon in the mode **k** when the classical field $\mathbf{A}^{\rm class}(x)$ is abruptly flipped (and consequently $\alpha_{\mathbf{k}} \rightarrow -\alpha_{\mathbf{k}}$). The probability that no photon will be emitted in *any* of the **k** modes is the corresponding product

$$P(\text{zero photons}) = \prod_{\mathbf{k}} P_{\mathbf{k}}^0 = \prod_{\mathbf{k}} \exp\left[\frac{-\omega_{\mathbf{k}}}{2\pi\hbar c^2}(\alpha_{\mathbf{k}})^2\right] = \exp\left[-\sum_{\mathbf{k}} \frac{\omega_{\mathbf{k}}}{2\pi\hbar c^2}(\alpha_{\mathbf{k}})^2\right]. \quad (8)$$

For the specific case of the magnetic dipole field $\alpha_{\mathbf{k}}$ is given by the Fourier transform of $\mathbf{A} = \boldsymbol{\mu} \times \nabla(1/r)$:

$$\alpha_{\mathbf{k}} = \frac{\mathbf{k}\times\boldsymbol{\mu}}{\sqrt{V}} \int \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{r} = \frac{\mathbf{k}\times\boldsymbol{\mu}}{\sqrt{V}} \left[\frac{1}{|k|^2}\right]. \quad (9)$$

Using $\sum_{\mathbf{k}} \rightarrow V\int d^3\mathbf{k}$ we can rewrite Eq. (8) as

$$P(\text{zero photons}) = \exp\left[\frac{-V\int d^3k\, \omega_{\mathbf{k}}}{2\pi\hbar c^2}[\alpha(k)]^2\right] = \exp\left[\frac{-\mu^2}{4\pi\hbar c^2}\int \frac{(\mathbf{k}\times\boldsymbol{\mu})\cdot(\mathbf{k}\times\boldsymbol{\mu})}{|k|^4}\omega_{\mathbf{k}}\right] \quad (10)$$

$$= \exp\left[\frac{-\mu^2 c}{2\hbar}\int_0^{k_{\max}} k\,dk \int \sin^2\Theta\, d\cos\Theta\right] = \exp\left[\frac{-\mu^2}{2\hbar c}\frac{2}{3}k_{\max}^2\right]. \quad (11)$$

The UV cutoff $k_{\max}$ is required because of the singular behavior of the dipole field $\mathbf{A}(r) \sim \mu/r^2$. The latter is clearly an artifact of the utilization of a pointlike neutron. In reality we have an extended neutron of radius ~Fermi ~ (1/0.2 GeV^{-1}) and $k_{\max} \cong 0.2$ GeV is an appropriate cutoff. To be conservative and to avoid extra parameters we will use

$$k_{\max} \sim \frac{cm_n}{\hbar/g_n} \sim 0.5\ \mathrm{M}_n \sim 0.5\ \text{GeV}.$$

Substituting $\mu_n = g_n e\hbar/m_n c$, we finally find

$$P(\text{zero photon}) = \exp\left[-\frac{e^2}{\hbar c}\frac{1}{3}\right] \sim e^{-\alpha/3} \sim 1 - \frac{\alpha}{3} + \cdots.$$

Thus, we find that contrary to the naive expectation from classical arguments, the probability of emitting zero photons is large and deviates from 1 only by corrections of order α. These corrections do indeed tend to diminish somewhat the overlap probability (and consequently the n-$\bar{n}$ vacuum transition). However, the effect is an order-α radiative correction as suggested by perturbative, Feynman-diagrammatic considerations.

Let us conclude with a few additional comments. (i) With minor modifications the above arguments can be readily extended to any multipole. (ii) The classical result that flipping a macroscopic dipole moment necessarily entails radiation is readily regained if we consider a large number N of magnetic moments which rigidly flip together so that $\mu_{\rm macroscopic} = N\mu$. In this case $P(0) = \exp(-N^2\alpha/3)$ is indeed vanishing small. (iii) When the effective number of quanta in a cloud surrounding a source is large these could "quench" the possible disappearance or flip of the source factors e^{-N}.[4] (iv) We have so far considered quantum fluctuation of the radiation field but treated the source classically. Even if the neutron was infinitely heavy, the $[\sigma_i, \sigma_j]$ commutation relation causes fluctuations of the direction of the dipole moment $\boldsymbol{\mu}$. While the exact treatment of the fluctuating spin coupled to the radiation field is impossible, we do expect that the extra fluctuations tend to make the states before and after the $\boldsymbol{\mu}$ flip even more similar. This in turn will tend to make P(zero photons) even closer to unity.

Our interest in this problem stemmed originally from the possible quenching of charge-violating decays such as $e \rightarrow \nu + \gamma$ due to the Coulomb cloud surrounding the initial electron. The above discussion appears to carry over to the monopole case as well. However, there are two important differences—the infrared divergences of the expression for P(zero photons) and the need to emit longitudinal photons as well.[5]

Leaving this more subtle issue we believe that the above demonstration, though treating very basic and fairly well-known issues, is useful in clarifying the absence of quenching due to EM effects of $n \rightarrow \bar{n}$ transitions (and possibly in other hydrogen-antihydrogen, muonium-antimuonium transitions) as well.

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