Question of the nonlocality of the Aharonov-Casher effect

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(Received 13 June 1989)

The Aharanov-Casher effect is manifested in a (2+1)-dimensional model that screens the electromagnetic fields, in order to demonstrate that the effect is essentially nonlocal in its nature. The question of nonlocality is discussed by means of a nonrelativistic model for a superconductor. It is demonstrated that although the superconductor screens the electric field generated by an external charge it does not screen the modular electric field which is a constant of motion of the system. Consequently, a magnetic fluxon will accumulate the same phase as if the electric field were unscreened and the Aharonov-Casher effect will exist even in a force-free region.

I. INTRODUCTION

The work of Aharonov and Casher[1] reveals a new aspect of the Aharonov-Bohm effect.[2] Aharonov and Casher suggest that a neutral particle with a magnetic moment μ may exhibit a topological force-free interference effect, as a result of an interaction with a charged wire.[3] Consequently, the Aharonov-Bohm effect admits a duality, meaning that we can reverse the roles of the charged particle and the solenoid in the Aharonov-Bohm effect and still get a force-free interference effect.

Let us consider the duality in $2+1$ dimensions more explicitly. Both effects are related to a two-particle system consisting of one particle with a charge q and location r, and a second particle with a magnetic moment μ and location R. In the nonrelativistic limit when $\dot{r}/c \simeq \dot{R}/c \ll 1$ the corresponding two-particle Lagrangian is[1,8]

$$L=\tfrac{1}{2}m\dot{r}^{2}+\tfrac{1}{2}M\dot{R}^{2}+\frac{q}{c}\mathbf{A}(\mathbf{r}-\mathbf{R})\cdot\dot{\mathbf{r}}+\frac{\boldsymbol{\mu}}{c}\times\mathbf{E}(\mathbf{R}-\mathbf{r})\cdot\dot{\mathbf{R}}\ , \tag{1.1}$$

where $\mathbf{A}(\mathbf{r}-\mathbf{R})$ is the vector potential generated by the magnetic moment at the location of the charged particle, and $\mathbf{E}(\mathbf{R}-\mathbf{r})$ is the electric field generated by the charged particle at the location of the magnetic moment.

The duality is manifested by the numerical equality of the interaction terms

$$q\,\mathbf{A}(\mathbf{r}-\mathbf{R})=\mathbf{E}(\mathbf{R}-\mathbf{r})\times\boldsymbol{\mu}\ . \tag{1.2}$$

As a result of the equality, two observers O' and O'', situated at the rest frame of the magnetic moment and the rest frame of the charge, respectively, will calculate the same phase accumulated as a result of relative motion of the particles, but may interpret the nature of the interaction in a completely different way. While observer O' relates the phase $(Q'/\hbar c)\int \mathbf{A}\cdot d\mathbf{l}$ to the interaction of the charged particle with the vector potential generated by the magnetic moment, observed O'' will relate the phase $(1/\hbar c)\int \mathbf{E}\times\boldsymbol{\mu}\, dl$ to the interaction of the magnetic moment with a "vector potential" $\mathbf{E}\times\boldsymbol{\mu}$ generated by a charged stationary particle. Hence, according to observer O', the system accumulates phase as a result of a force-free interaction of the Aharonov-Bohm type which will be interpreted as a nonlocal effect; while, according to observer O'', although the interaction $\mathbf{E}\times\boldsymbol{\mu}$ does not generate a dynamical force (there is however a nonvanishing hidden momentum, see Ref. 4) on the magnetic moment, the accumulated phase can be interpreted as a result of a local interaction with a nonvanishing electric field.

According to the Galilean invariance of the interaction, both approaches are permitted and indeed give the same result. However, the resulting interpretations are not compatible. This suggests two possibilities: either the Aharonov-Bohm effect resulting from the interaction term $q\,\mathbf{A}(\mathbf{r}-\mathbf{R})$ is a result of a local interaction, or the interaction term $\mathbf{E}\times\boldsymbol{\mu}$, although it appears as a local interaction, also represents a nonlocal interaction and, consequently, the Aharonov-Casher effect must be interpreted as a nonlocal force-free effect[4] (here and in the following the term force-free is related to the situation where all the classical field strengths vanish).

The purpose of this work is to show that the second possibility is the correct one. We demonstrate that in $2+1$ dimensions it is possible to arrange both the charged particle and the magnetic moment to be in force-free regions, and yet get the usual phase shift and interference as if the electric field were unscreened. Consequently, we conclude that the concept of duality does not contradict the nonlocality of the Aharonov-Bohm effect and that its dual Aharonov-Casher effect is also essentially nonlocal in its nature.[5]

The plan of this paper is as follows. In Sec. II the effect is discussed in the context of a superconductor. We realize that in order to understand how the effect occurs in a force-free region we must examine the quantum behavior of the superconductor. The effective Hamiltonian will be derived in Sec. III. In Sec. IV we discuss the correct limit for the model to be a faithful description of a superconductor and find in this limit the ground state of the system. Section V will present the modular variables, and discuss their importance. In Sec. VI the magnetic fluxons will be included and demonstrated to accumulate the correct phase.

II. THE EFFECT IN THE PRESENCE OF A SUPERCONDUCTOR

It is well known that a superconductor screens all the electromagnetic fields exponentially; hence it is suitable as a force-free medium. A type-2 superconductor may support quantized localized magnetic fluxons with a flux $\Phi=hc/q$ which pertains to the medium's unit charge $q=2e$ (the Cooper pair). If we introduce into the superconductor an external particle with charge Q', that moves along a path enclosing a fluxon, it will accumulate the Aharonov-Bohm phase $(Q'/\hbar c)\int \mathbf{A}\cdot d\mathbf{l}$, where A is the vector potential generated by the fluxon,

$$q\,\mathbf{A}(\mathbf{r})=\frac{\hat{\mathbf{z}}\times\mathbf{r}}{|\mathbf{r}|^2}+O(e^{(-|\mathbf{r}|)/\lambda})\ . \tag{2.1}$$

Here $\mathbf{r}$ is a two-dimensional vector, $\mathbf{z}$ is a unit vector perpendicular to the x-y plane, and λ is the penetration depth. The phase will be nontrivial only for Q' which is unquantized with respect to q, $Q'\text{mod}q\neq0$.

It is possible to realize also the Aharonov-Casher effect by changing the roles of the charge and fluxon? On first observation it appears that the answer should be negative. The electric field generated by the external charge is screened; hence, the fluxon interacts locally with a field strength of $\exp(-r/\lambda)$, which cannot lead to the correct result. Nevertheless, there is the possibility that some part of the electric field still penetrates the superconductor, does not generate any dynamical force, but still is sufficient to cause the fluxon to accumulate the correct phase. Such a delicate effect cannot occur in a classical system, but is possible in a quantum system.

In order to understand the mechanism that makes the effect possible, we must include the quantum behavior of the shielder. This will be done in the following by means of a simple model for a superconductor in 2+1 dimensions (Fig. 1). Let us imagine a spherical superconductor with an external radius R_1 (and the corresponding boundary S_1). Insert the external charge Q' into a hole of radius R_2 (with a corresponding inner surface S_2) in the center of the superconductor. We assume that $R_2 \ll R_1$. In the following section we shall rely on the following assumptions and simplifications.

(1) The electric field inside the superconductor for $R_2 \ll r \ll R_1$ depends on the induced charge (Cooper pairs) near the inner surface S_2, through the Gauss law. We shall neglect any dependence on local fluctuations inside the superconductor. This can be justified by noticing that time averaging over a short time (relative to the time scale of the interference effect) leads to a cancellation of the local fluctuations. Hence, it will be sufficient for our purpose to formulate the quantum behavior of the induced charge.

(2) The origin of the nonlocal behavior is related to the

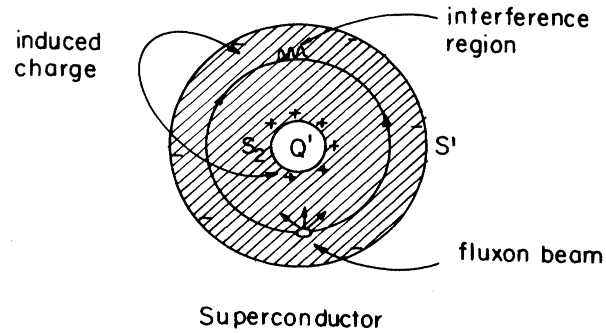

FIG. 1. Schematic description of the Aharonov-Casher effect in the presence of a superconductor.

fluctuation of the induced charge Q on S_2. A change in Q corresponds to a "jump" of a unit charge q from S_1 to S_2 and vice versa. This can be described as a collective radial fluctuation of the "macroscopic wave function" of the superconductor.[6]

III. THE EFFECTIVE HAMILTONIAN

The first step is to establish the effective Hamiltonian and the corresponding Hilbert space. At first the contribution of the fluxon is neglected. To this end, let us examine the energy dependence of the superconductor on the induced surface charge, $E=E(Q)$. This function must have a minimum at the value $Q=-Q'$, where Q' is the external charge. Hence, it is most likely to be of the form

$$E=\frac{1}{2C}(Q+Q')^2+\text{other contributions}\ . \tag{3.1}$$

The superconductor can be described as a spherical capacitor, where the inner and outer surfaces S_2 and S_1 play the role of the capacitor plates. The constant C plays the role of "capacitance" and is expected to depend on the geometry of the superconductor, $C=C(R_1,R_2)$.

Let q be the charge of a Cooper pair ($q=2e$), and let n be the number of Cooper pairs near S_2 (n is negative for Cooper holes). Using these definitions, we can write the energy related to the induced charge Q in the form

$$E=\frac{(nq+Q')^2}{2C}\ . \tag{3.2}$$

Proceeding to a quantized theory we define first the corresponding Hilbert space. The most natural choice is to use the states $|n\rangle$, which correspond to n Cooper pairs on the surface S_2. By Eq. (3.2) the matrix element of the Hamiltonian is given by

$$\langle n|H|n\rangle=\frac{(nq+Q')^2}{2C} \tag{3.3a}$$

and also

$$\langle n|H|n'\rangle = 0 \tag{3.3b}$$

for $n \neq n'$. This is not satisfactory since we expect to get fluctuations in n and therefore a nonvanishing "tunneling" amplitude. Hence, define the nondiagonal matrix element as

$$\langle n'|H|n\rangle = \frac{A}{2}\delta_{n,n'+1} + \frac{A}{2}\delta_{n+1,n'} , \tag{3.4}$$

where we have included only the first-order "tunneling" effect, and neglected higher orders. The coefficient A is unknown, but as we shall see, knowledge of its numerical value is unnecessary for our needs.

The Schrödinger equation in the energy representation is

$$H|E\rangle = E|E\rangle . \tag{3.5}$$

Multiplying from the left by the vector $\langle n|$, and substituting to the right of H the unity operator $\sum |n\rangle\langle n'|$, we reach

$$\sum_{n'} \langle n|H|n'\rangle\langle n'|E\rangle = E\langle n|E\rangle . \tag{3.6}$$

Define now the wave function u_n as $u_n = \langle n|E\rangle$, i.e., the amplitude of the superconductor to have energy E and n Cooper pairs on the inner surface. Then the Schrödinger equation for the wave function reads

$$i\hbar\frac{\partial u_n}{\partial t} = \frac{1}{2C}(nq+Q')^2 u_n - \frac{A}{2}(u_{n+1}+u_{n-1}) . \tag{3.7}$$

The last term on the right is of special significance since it represents the nonlocal jumps that were suggested in the previous section. Consequently, the stationary state of the superconductor will be an infinite superposition of the wave functions u_n and hence represents a state with an undefined induced charge.

In order to get a representation which diagonalizes the Hamiltonian, it is necessary to transform from the u_n to another basis. This can be done by the transformation

$$u_n(t) = \frac{1}{\sqrt{2\pi}}\int_{-\pi}^{+\pi} e^{-i(n+\alpha)\theta} U(\theta,t)\,d\theta , \tag{3.8a}$$

$$U(\theta,t) = \frac{1}{\sqrt{2\pi}}\sum_n e^{i(n+\alpha)\theta} u_n(t) , \tag{3.8b}$$

where α is an arbitrary constant, and θ takes the value $\theta \in (-\infty, +\infty)$. By definition $U(\theta)$ is a periodic function up to a phase

$$U(\theta+2\pi) = e^{2\pi i\alpha} U(\theta) . \tag{3.9}$$

If we choose the normalization of the wave function u_n to be

$$\sum_n |u_n(t)|^2 = 1 , \tag{3.10a}$$

we get, for the normalization of $U(\theta,t)$,

$$\int_{-\pi}^{+\pi} U^*(\theta,t)U(\theta,t)\,d\theta = 1 . \tag{3.10b}$$

Substituting Eq. (3.8) into (3.7), and using the periodicity of the wave functions $U(\theta,t)$, gives the Schrödinger equation in the θ representation:

$$i\hbar\frac{\partial U(\theta,t)}{\partial t} = \frac{1}{2C}\left[\left[-iq\frac{\partial}{\partial\theta} - \alpha q + Q'\right]^2 - A\cos\theta\right] U(\theta,t) . \tag{3.11}$$

Define charge and number operators:

$$\hat{N} = -i\frac{\partial}{\partial\theta} , \tag{3.12a}$$

$$\hat{Q} = -iq\frac{\partial}{\partial\theta} . \tag{3.12b}$$

The operator Q is the canonical conjugate momentum of θ. The physical meaning of Q becomes clear if we calculate its expectation value

$$\langle\hat{Q}\rangle = q\alpha + q\sum_n n|u_n|^2 . \tag{3.13}$$

Clearly, $\hat{Q}$ represents the induced charge on S_2 plus a constant charge. If we choose the constant α to take the value $\alpha = Q'/q$, the operator represents the total charge $Q_t = Q_{\text{induced}} + Q_{\text{external}}$, and the Hamiltonian takes its simplest form

$$H_{\text{eff}} = \frac{1}{2C}\hat{Q}^2 - A\cos\theta . \tag{3.14}$$

The last specific representation ("gauge") of the Hamiltonian emphasizes the role of the external charge in the model. The external charge does not appear in the classical equations of motion as is evident from Eq. (3.14) (or by constructing the effective Lagrangian and noticing that the effect of Q' for any α is to introduce a total time derivative $d\theta/dt$ which does not change the equations of motion). Its only effect is to change the structure of the state of the system. From Eq. (3.9) we see that in this "gauge" the external charge determines the phase of the wave function resulting from the transformation $\theta \to \theta + 2\pi$.

However, in the following we choose to use the value $\alpha = 0$ for which there is a clear distinction between the induced fluctuating superconductor's charge and the external charge. Hence, for the rest of the paper $\hat{Q}_t = \hat{Q} + Q'$.

Another consequence of the nature of the eigenfunction and of the symmetry of the Hamiltonian under $\theta \to \theta + 2\pi$ is that the energy must be a periodic function of Q':

$$E(Q') = E(Q'+q) \quad\text{or}\quad E = f\left[\cos\left[\frac{2\pi Q'}{q}\right]\right] . \tag{3.15}$$

This means that energetically the superconductor does not distinguish between Q' and $Q'+q$. Consequently, since the external electric field is screened, the most the fluxon can sense is some modular function $\cos(2\pi Q'/q)$. We shall see that indeed this is the case.

IV. THE GROUND STATE

The operators $\hat{Q}$ and θ satisfy the usual canonical commutation relation

$$[\hat{Q},\theta]=-iq \tag{4.1}$$

and therefore obey an uncertainty relation

$$\Delta\theta\Delta Q\geq q \ . \tag{4.2}$$

The last relation is analogous to the phase-charge uncertainty in the BCS model. According to this theory, the wave function is the ground state of the system and obeys the relations $\Delta\theta\rightarrow 0$, and $\Delta Q \gg q$. Hence, we shall require the ground state of the effective Hamiltonian to satisfy the same relations.

The ground state of the effective Hamiltonian can be constructed easily as a linear combination of localized wave functions at all the minima of the potential. First, construct a ground-state wave function $u_0(\theta)$ in the well around $\theta=0$. The Hamiltonian can be approximated as a harmonic oscillator

$$H_{\mathrm{osc}}=\frac{1}{2C}(\hat{Q}+Q')^2+\frac{A}{2}\theta^2 \tag{4.3}$$

and the solution is well known to be

$$u_0(\theta)=e^{-i\theta Q'/q}h_0(\theta) \ , \tag{4.4a}$$

where

$$h_0(\theta)=\left[\frac{\sqrt{AC}}{\pi q}\right]^{1/4}e^{-(\sqrt{AC}/2q)\theta^2} \ . \tag{4.4b}$$

Second, construct a superposition of the unperturbed wave functions at the separate wells

$$U(\theta)=\sum_N u_0(\theta-2\pi N)$$
$$=\sum_N e^{-i(\theta-2\pi N)Q'/q}h_0(\theta-2\pi N) \ . \tag{4.5}$$

The solution is periodic in θ, in agreement with Eq. (3.9).

Using this result we find for the uncertainties of the canonical variables

$$(\Delta\theta)^2=\frac{1}{2}\frac{q}{\sqrt{AC}} \ , \tag{4.6a}$$

$$(\Delta Q)^2=\tfrac{1}{2}q\sqrt{AC} \ . \tag{4.6b}$$

Hence, taking the limit $\sqrt{AC}\gg q$ leads to the required conditions $\Delta\theta\rightarrow 0$ and $\Delta Q\gg q$.

V. MODULAR VARIABLES

Having derived the wave function of the system, we can now examine the behavior of the total charge $Q_t=Q_{\mathrm{induced}}+Q_{\mathrm{external}}$. For the expectation value of $(Q_t)^n$ we find

$$\langle(\hat{Q}_t)^n\rangle=\int_{-\pi}^{+\pi}U(\theta)^*(\hat{Q}+Q')^nU(\theta)d\theta$$
$$=\int_{-\pi}^{+\pi}h_0(\theta)^*\hat{Q}^n h_0(\theta)d\theta \ . \tag{5.1}$$

Notice that only the part that is localized at the well around $\theta=0$ contributes to the integral. The contribution of the other parts can be neglected since according to (4.6) the spread $\Delta\theta\rightarrow 0$. Consequently, the expectation value of the total charge is independent of the external charge Q' (odd moments of Q_t vanish). Applying the Gauss law in two spatial dimensions we obtain the following operator equality for the radial electric field inside the superconductor:

$$\hat{E}=\frac{1}{2\pi r}\hat{Q}_t=\frac{1}{2\pi r}(\hat{Q}+Q') \ , \tag{5.2}$$

where r is a c number. Since the last result for $\langle\hat{Q}_t\rangle$ applies also for the expectation value of $\hat{E}$, we conclude that in the superconductor (for $R_2\ll r\ll R_1$) the field strength vanishes. Hence the interior of the superconductor can be regarded as a force-free region.

The expectation value of any moment of $\hat{Q}_t$ is by the right-hand side of Eq. (5.1) independent of Q'. Can we construct a function of $\hat{Q}_t$ that is sensitive to the external charge? The ground state, in the required limit of $\sqrt{AC}\gg q$, is a superposition of orthogonal functions localized at $\theta=2\pi N$. The dependence on the external charge Q' appears only in the relative phase of these functions. Hence, only functions with a nonvanishing matrix element between $u_0(\theta)$ and $u_0(\theta+2\pi)$ can have an expectation value which depends on Q'. Any finite polynomial will not do. Therefore, we are led to consider a function of the form $e^{2\pi i(\hat{Q}_t/q)}$, which is a unitary shifting operator of $u(\theta)$ to $u(\theta+2\pi)$. For this function

$$e^{2\pi i(\hat{Q}_t/q)}U(\theta)=e^{2\pi iQ'/q}U(\theta) \ . \tag{5.3}$$

The operator $e^{2\pi i(\hat{Q}_t/q)}$ will be referred to as the modular operator[7] of $\hat{Q}_t$. The corresponding expectation value will be Q' dependent:

$$\langle e^{2\pi i(\hat{Q}_t/q)}\rangle=e^{2\pi iQ'/q} \ . \tag{5.4}$$

The operator $\hat{Q}_t$ (or $\hat{E}$), or any moment of $\hat{Q}_t$, is not a constant of motion since it does not commute with the Hamiltonian. Nevertheless, the modular operator of $\hat{Q}_t$ satisfies[7]

$$[e^{2\pi i(\hat{Q}_t/q)},e^{i\theta}]=0 \tag{5.5}$$

and hence

$$[e^{2\pi i(\hat{Q}_+/q)},H_{\mathrm{eff}}]=0 \ . \tag{5.6}$$

The modular operator $e^{2\pi i(\hat{Q}_t/q)}$ is a constant of motion. Moreover, from Eq. (5.3) we have $\hat{Q}\bmod q=0$. This is not surprising since it means that the total induced charge is quantized with respect to the charge of a Cooper pair, and by Eq. (5.6) this quantization is constant in time. Consequently, although the induced charge screens the external charge ($\langle\hat{Q}_t\rangle=0$) it cannot screen the modular charge ($Q'\bmod q$) and the result is a nontrivial phase for the expectation value of the modular operator in Eq. (5.4).

Similarly, define a quantum of electric field (a c number) and a modular electric modular field operator as

$$e_0 = \frac{q}{2\pi r} \quad \text{and} \quad e^{2\pi i (\hat{E}/e_0)} , \tag{5.7}$$

respectively, where e_0 pertains to the field generated by a single particle of the medium. We obtain

$$\begin{aligned}\langle e^{2\pi i (\hat{E}/e_0)} \rangle &= \langle e^{2\pi i (\hat{Q}_t / q)} \rangle \\ &= e^{i 2\pi Q'/q} \\ &= e^{i(1/c\hbar)\int \mathbf{E}\times \mathbf{u} dl} .\end{aligned} \tag{5.8}$$

Notice that the expectation value of the modular electric field on the left-hand side is equal numerically to the phase shift in the unscreened Aharonov-Casher effect on the right-hand side. The modular electric field will not vanish if and only if Q' is unquantized, $Q' \bmod q \neq 0$, which is the same condition for which Q' displays the Aharonov-Bohm effect. This transparency of the superconductor to the modular electric field is exactly the property that enables the fluxon to accumulate the correct phase, since as we shall see in the following section the solution for the Shrödinger equation that includes the contribution of the fluxon involves a phase which is proportional to the modular electric field.

VI. THE INTERACTION OF THE FLUXON

Finally we would like to demonstrate that the fluxon accumulates the usual phase although its motion is entirely confined to a force-free region $R_2 \ll r \ll R_1$ (Fig. 1).

In order to include the contribution of the fluxon to the effective Hamiltonian we shall assume that the fluxon has an effective mass M and momentum $\mathbf{P}$. Then, the interaction with the electric field is given by the replacement[1]

$$\mathbf{P} \to \mathbf{P} + \frac{1}{c} \mathbf{E} \times \boldsymbol{\mu} . \tag{6.1}$$

Since the "vector potential" $\mathbf{E}\times\boldsymbol{\mu}$ has only a nonvanishing azimuthal component we can simplify the problem and regard only the azimuthal motion of the fluxon described by the angle $\phi \in (-\pi, \pi)$. Substituting the corresponding operator for E [Eq. (5.2)] and the explicit value for $\boldsymbol{\mu}$, ($|\boldsymbol{\mu}| = hc/q$), we reach the following effective Hamiltonian:

$$H_{\text{eff}} = \frac{1}{2C} \hat{Q}_t^2 - A \cos(\theta) + \frac{1}{2M} \left[P_\phi - \frac{h}{2\pi r} \frac{\hat{E}}{e_0} \right]^2 . \tag{6.2}$$

Let us substitute a solution of the form

$$\Psi(\theta,\phi) = e^{i\phi(\hat{E}/e_0)} \psi(\theta,\phi) . \tag{6.3}$$

We find that the function $\psi(\theta,\phi)$ satisfies the "free" equation of motion

$$\left[\frac{1}{2C} \hat{Q}_t^2 - A \cos(\theta - \phi) + \frac{1}{2M} P_\phi^2 \right] \psi(\theta,\phi) = E \psi(\theta,\phi) . \tag{6.4}$$

There is a path dependence in the potential term, the motion of the fluxon moves the location of the wells.

An explicit solution for ψ can be found in the adiabatic approximation ($\hbar^2/2Mr^2 \gg q^2/2C$ is satisfied). The wave function takes the form

$$\psi(\theta,\phi) = \rho(\theta)\eta(\phi) \tag{6.5}$$

and the Schrödinger equation (6.4) separates into two equations:

$$\left[\frac{1}{2C} \hat{Q}_t^2 - A \cos(\theta - \phi) \right] \rho_\phi(\theta) = E_{\text{SC}}(\phi) \rho_\phi(\theta) , \tag{6.6a}$$

$$\frac{1}{2M} P_\phi^2 \eta(\phi) = (E - E_{\text{SC}}) \eta(\phi) . \tag{6.6b}$$

In our approximation the motion of the fluxon is assumed to be adiabatic [in accordance with assumption (1) of Sec. II], its corresponding wave function $\eta(\phi)$ is well localized and hence the angle ϕ in Eq. (6.6a) is regarded as a slowly varying parameter. For $\phi = 0$ the solution has already been computed in Sec. IV. When the fluxon moves, the solution [Eq. (4.5)] changes according to the shift of the wells and takes the form

$$\rho(\theta) = \sum_N u_0(\theta - \phi - 2\pi N) . \tag{6.7}$$

Hence, the total wave function Eq. (6.3) is given by

$$\Psi(\theta,\phi) = e^{i\phi(\hat{E}/e_0)} \sum_N u_0(\theta - \phi - 2\pi N) \eta(\phi) . \tag{6.8}$$

This is in general not a permissible solution for the Schrödinger equation (6.2), since unless $E \bmod e_0 = 0$ ($Q' \bmod q = 0$) it is a nonsingle-valued function of ϕ. However, for a simply connected region it is still possible to use this solution.

Consider a coherent beam of fluxons prepared at $\phi = \phi_0$. The beam is split into two localized wave packets $\Psi = \Psi_1 + \Psi_2$, where Ψ_1 represents the beam on one side of the external charge and Ψ_2 on the opposite side. Each of the beams travels in a simply connected region and hence can be described by the solution (6.8). Therefore we have

$$\Psi_1 = e^{i\Delta\phi_1(\hat{E}/e_0)} \psi_1(\theta - \Delta\phi_1, \Delta\phi_1) , \tag{6.9a}$$

$$\Psi_2 = e^{i\Delta\phi_2(\hat{E}/e_0)} \psi_2(\theta - \Delta\phi_2, \Delta\phi_2) , \tag{6.9b}$$

where $\Delta\phi_i = \phi_i - \phi_0$. The final wave function at the interference region ($\Delta\phi_1 = \pi$ for ψ_1, $\Delta\phi_2 = -\pi$ for ψ_2) is given by

$$\Psi_1+\Psi_2=e^{i\pi(\hat{E}/e_0)}\psi_1(\theta-\pi,\pi)+e^{-i\pi(\hat{E}/e_0)}\psi_2(\theta+\pi,-\pi) \tag{6.10a}$$

$$=e^{2\pi i(\hat{E}/e_0)}\psi_1'+\psi_2' \tag{6.10b}$$

where ψ' stands for $e^{-i\pi(\hat{E}/e_0)}\psi(\theta+\pi,\pi)$ and we have used the single valuedness of ψ.

Clearly, the interference between the two beams depends on the value of the modular electric field. According to the discussion in Sec. V although the expectation value of the electron vanishes, the modular electric field is a constant of motion of the system and by Eqs. (5.3) and (5.8) takes the value $e^{2\pi iQ'/q}$. Hence, the interference between the two fluxon beams will depend of the phase difference $2\pi Q'/q$.

In conclusion, we have shown that because of the transparency of the superconductor to the modular electric field, the magnetic fluxon will accumulate the same phase as if the electric field were unscreened. Hence, in this case the Aharonov-Casher effect must also be interpreted as a force-free nonlocal effect. We also note that the nontriviality of the modular electric field [Eq. (5.8)] is a general property that will occur whenever $Q'\text{mod}q\neq 0$ (where q is a unit charge of the medium and Q' is an external charge); hence, we expect other models that manifest the Aharonov-Bohm effect and screen the electromagnetic fields (e.g., Higgs model in 2+1 dimensions) to manifest also the Aharonov-Casher effect through a similar mechanism.

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[8]As far as we know the first calculation of the phase shift for neutrons passing through a region of electric field was carried out by J. Anandan, Phys. Rev. Lett. **48**, 1660 (1982). However, the topological aspect of this interaction was not discussed.