A similar Comment on this Letter, which also pointed out the nonconservation of charge in the model, was submitted by C. R. Hagen, University of Rochester.

Comment on "New Model of Fractional Spin"

In a recent Letter, Liang and Ding[1] (LD) proposed a model for fractional spin. They claimed that a line-charge–neutron composite "uniquely determines the fractional [canonical] angular momentum quantization." To this end, LD considered a neutron with magnetic moment μ parallel to the line of charge whose linear density changed from zero to λ at time $t=0$.

The same line-charge–neutron composite can be created by bringing the charged line in from infinity. Aharonov and Casher[2] (AC) showed that the dynamics of a neutron moving around a line of charge with its magnetic moment constrained[3] to be parallel to the line (which we shall refer to as an AC composite) is identical to the dynamics of an electron moving around a magnetic-flux tube. Jackiw and Redlich[4] showed that for a flux-tube–electron [the Aharonov-Bohm (AB)] composite in a two-dimensional geometry the kinetic but not canonical angular momentum has fractional quantization. From the duality that has been established between the AB and AC configurations (which has recently been confirmed by the interference experiment[5]) it follows that for this case the line-charge–neutron composite also has no fractional quantization of the canonical angular momentum. Since the way of quantization cannot depend on the history of the system, we conclude that there must be a flaw in the LD analysis.

LD argue that there is a difference between the AB and AC composites in the time-varying situation: While the electron accelerates when the flux varies in time, the neutron has zero acceleration even when the linear charge density of the line is time dependent. In order to show that the acceleration of the neutron is zero, they calculate the electromagnetic field in the region outside the charged line starting with two equations:

$$\nabla\times\mathbf{B}=(1/c)\partial\mathbf{E}/\partial t\,, \qquad (1)$$

$$(\mu\cdot\nabla)\mathbf{B}=0\,. \qquad (2)$$

Equation (1) corresponds to the assumption that there is no current outside the line of charge. Equation (2) corresponds to the assumption of translational symmetry in the z direction, which makes this model two dimensional. However, the above two assumptions cannot be consistent with the requirement of charge conservation. Indeed, the mathematical solution of Eqs. (1) and (2) given by LD [their Eq. (8)] is not physically acceptable since it corresponds to a magnetic field $\mathbf{B}$ that is not single valued.

The correct analysis of the LD model must include a current feeding the line of charge. If we want to preserve their first assumption, Eq. (1), then the current must be along the line of charge and must have z dependence. The continuity equation yields $\mathbf{I}=-(d\lambda/dt)z\hat{\mathbf{z}}$. Then, the magnetic field is $\mathbf{B}=-(2z/cr)(d\lambda/dt)\hat{\boldsymbol{\phi}}$ and the acceleration of the neutron is[6] $\ddot{\mathbf{r}}=-(2\mu/mcr)\times(d\lambda/dt)\hat{\boldsymbol{\phi}}$. Thus, the kinetic angular momentum is not conserved but the canonical angular momentum is conserved [see LD, Eq. (7)], and, therefore, no fractional quantization of the canonical angular momentum arises.

Another possibility is to keep the LD assumption about z symmetry [Eq. (2)]. Then, there must be a current outside the line of charge, and, therefore, Eq. (1) should be replaced by

$$\nabla\times\mathbf{B}=(4\pi/c)\mathbf{j}+(1/c)\partial\mathbf{E}/\partial t\,.$$

One such possibility is that of a current feeding the line of charge along an infinite half plane, $\phi=0$, terminating at the line. The surface current then is $\mathbf{K}=-(d\lambda/dt)\hat{\mathbf{r}}$. From this and Maxwell's equations one obtains the magnetic field

$$\mathbf{B}=(2/c)(d\lambda/dt)\phi(\mathrm{mod}2\pi)\hat{\mathbf{z}}\,.$$

This agrees with Eq. (8) of LD everywhere except on the half plane $\phi=0$. The acceleration of the neutron indeed vanishes everywhere except on this half plane, where the acceleration is singular:

$$\ddot{\mathbf{r}}=-(4\pi\mu/mcr)(d\lambda/dt)\delta(\phi)\hat{\boldsymbol{\phi}}\,.$$

Thus, the kinetic angular momentum of the neutron is not conserved and we cannot proceed with the argument of LD in this case either. As before, $\oint L_z\,d\phi$ is a constant of motion. This indicates that the quantization of L_z is not affected by changing the charge density on the line.

When the Hamiltonian is single valued, $e^{i2\pi L_z}$ is a constant of motion. If initially the wave function is chosen to be single valued, then $e^{i2\pi L_z}$ is the unit operator and remains so even when the charge density on the line is changing. Hence the eigenvalues of L_z must remain integers.

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