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Comment on " Time asymmetry in quantum mechanics: a retrodiction paradox"

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Abstract

It is shown that the "retrodiction paradox" recently introduced by Peres [Phys. Lett. A 194 (1994) 21] arises not because of the fallacy of the time-symmetric approach as he claimed, but due to an inappropriate use of retrodiction.

In a recent Letter [1] Peres claimed that our time-symmetric approach to quantum theory [2,3], in which retrodictions and predictions are on an equal footing, leads to a paradox. We shall show that the paradox arises due to a particular use of retrodiction by Peres, and no such paradox arises in our approach.

A trivial time asymmetry of a quantum measurements is illustrated by the following example. Assume that the x component of the spin of a spin-$\frac{1}{2}$ particle was measured at time t, and was found to be $\sigma_x = 1$. While there is a symmetry regarding prediction and retrodiction for the result of measuring σ_x after or before the time t (in both cases we are certain that $\sigma_x = 1$), there is an asymmetry regarding the results of measuring σ_y. We can predict equal probabilities for each outcome, $\sigma_y = \pm 1$, of a measurement performed after the time t, but we cannot claim the same for the result of a measurement of σ_y performed before the time t. The difference arises from time asymmetric basic preconception, namely the usual assumption that there is no "boundary condition" in the future, but there is a boundary condition in the past: the state in which the particle was prepared before the time t. For more details see Section II of Ref. [2].

The time symmetric approach is applicable in a situation when we consider a quantum system at a time between two complete measurements which yield *two* boundary conditions. Then, the time symmetry of the formalism of quantum theory [4] together with the symmetry of having boundary conditions *both* in the past and in the future allow us to apply our time symmetric approach. We describe the system by a two-state vector consisting of the state evolving from the measurement in the past and the backward-evolving state evolving from the complete measurement in the future. We are making inferences about the results of measurements (if performed) at the time t using both prediction and retrodiction. The situation considered by Peres essentially falls into this latter category, and therefore we can apply our approach for its analysis.

Peres considers an ensemble of pairs of separated spin-$\frac{1}{2}$ particles, initially (before time t) prepared in a singlet spin state. This pre-selected ensemble is divided into four sub-ensembles according to the randomly chosen measurement performed after the time t. Peres focuses on the sub-ensemble in which σ_{Ax}

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was measured. Although in this case no complete measurement has been performed on the pair of particles, we are in the position to infer, from the measurement before the time t (the singlet state) and the measurement after the time t (the known x component of the spin of particle A), the result of a measurement of σ_x of particle B. From the correct inference that the outcome of the measurement of σ_{Bx} at time t is known with certainty Peres concludes that particle B was, from the beginning, in one of the eigenstates: either $|\uparrow_x\rangle$ or $|\downarrow_x\rangle$. In our approach, this conclusion is incorrect. Initially and at time t, the pair of particles A and B is in a pure forward-evolving singlet state and no measurement at a time later than t can change this. Therefore, particle B cannot be described by a pure forward-evolving quantum state.

Peres encounters a retrodiction paradox because he uses the standard approach of a single forward-evolving state. In this approach the only possibility in which the result of a measurement can be known with certainty is when the system is in an eigenstate of the measured variable. In the two-state vector approach, when we use both prediction and retrodiction, there are many situations in which neither the forward-evolving state nor the backward-evolving state are eigenstates of the measured variable but, nevertheless, the result of the measurement at time t (if performed) is known with certainty. See Refs. [5,6] for relevant examples.

Retrodiction is not "rewriting history" as Peres claims. It comes not instead of but in addition to prediction. The combination of the two allows Lorentz-invariant time symmetric quantum description which is something more than just a mathematical tool for calculating probabilities.

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