Causality, memory erasing, and delayed-choice experiments

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A recent proposal of a "quantum eraser" by Ingraham [Phys. Rev. A **50**, 4502 (1994); **51**, 4295(E) (1995)] is analyzed. It is shown that Ingraham's predictions contradict relativistic causality and, therefore, cannot be right. A subtle quantum effect that was overlooked by Ingraham is explained.

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Recently, Ingraham [1] suggested "a delayed-choice experiment with partial, controllable memory erasing." Let us first sketch Ingraham's argument. He considered two atoms located close to each other. The atoms have four relevant levels: the ground state $|c\rangle$ and three excited levels, $|a\rangle,|b\rangle,|b'\rangle$. The first laser pulse puts the atoms into the superposition $(1/\sqrt{2})(|a_1c_2\rangle+|c_1a_2\rangle)$. The atom in the state $|a\rangle$ immediately emits a photon γ and ends up in the state $|b\rangle$. Then the quantum evolution of the system is

$$\frac{1}{\sqrt{2}}(|a_1c_2\rangle+|c_1a_2\rangle)\rightarrow\frac{1}{\sqrt{2}}(|b_1c_2\rangle|\gamma_1\rangle+|c_1b_2\rangle|\gamma_2\rangle). \tag{1}$$

We are looking for a possibility of the appearance of an interference between the states of the photon γ emitted from the two atoms on a distant screen. (To this end the wavelength of the γ photon has to be smaller than the separation between the atoms.) Since there is a "memory" in the atoms of which atom emitted the photon (the state $|b\rangle$), no interference pattern is expected.

Now we add a second pulse, after the first one, but before the photon γ reaches the screen. The second pulse excites the atom in the state $|b\rangle$ to the state $|b'\rangle$, which immediately decays to the ground state $|c\rangle$ and emits a photon ϕ. In the limiting case, when the wavelength of the photon ϕ is much larger than the distance between the atoms, the states ϕ_1 and ϕ_2 of the photon emitted from the two atoms are practically identical, $|\phi_1\rangle\simeq|\phi_2\rangle\equiv|\phi\rangle$. Therefore, the memory of which atom emitted the photon γ is erased. Indeed, the quantum evolution is

$$\frac{1}{\sqrt{2}}(|b_1c_2\rangle|\gamma_1\rangle+|c_1b_2\rangle|\gamma_2\rangle)\rightarrow\frac{1}{\sqrt{2}}(|b_1'c_2\rangle|\gamma_1\rangle+|c_1b_2'\rangle|\gamma_2\rangle)$$
$$\rightarrow\frac{1}{\sqrt{2}}|c_1c_2\rangle|\phi\rangle(|\gamma_1\rangle+|\gamma_2\rangle). \tag{2}$$

Consequently, in the experiment with these two laser pulses we expect to see an interference pattern on the screen.

If the above analysis were correct we could send signals faster than light in the following way. Consider an experiment performed on an ensemble of such pairs of atoms. Bob, who has to receive the signal, is prepared to measure the interference pattern on the screen. Alice, who is located near the atoms, decides to apply or not to apply the second laser pulse. Bob will see the interference pattern, only if Alice applied the second pulse. Thus, Bob will know Alice's decision with superluminal velocity.

The argument of Ingraham is based on the fact that before the second laser pulse the two states $|b_1c_2\rangle$ and $|c_1b_2\rangle$ are practically orthogonal, while after the second pulse they evolve into the states $|c_1c_2\rangle|\phi_1\rangle$ and $|c_1c_2\rangle|\phi_2\rangle$ which are practically identical since $|\phi_1\rangle$, the state of the photon emitted by the first atom, is almost identical to $|\phi_2\rangle$, emitted by the second atom. This process violates the unitarity of quantum theory and, therefore, it cannot take place.

But where is the mistake? The resolution is somewhat subtle. It is true that a single atom in the state $|b'\rangle$ immediately radiates photon ϕ. However, if we put another atom in the ground state very close to the first one (such that the scalar product of the states of the photon emitted from the two locations approximately equals 1), the probability to emit the photon reduces to 1/2 [2]. Indeed, while in the symmetric state $|\Psi_+\rangle=(1/\sqrt{2})(|b_1'c_2\rangle+|c_1b_2'\rangle)$ the atoms emit the photon ϕ immediately, in the antisymmetric state $|\Psi_-\rangle=(1/\sqrt{2})(|b_1'c_2\rangle-|c_1b_2'\rangle)$ the atoms cannot emit the photon. One can understand this phenomenon as a destructive interference between the states of the photon emitted by the two atoms in the antisymmetric state. The correct evolution, instead of Eq. (2) is

$$\frac{1}{\sqrt{2}}(|b_1'c_2\rangle|\gamma_1\rangle+|c_1b_2'\rangle|\gamma_2\rangle)$$

$$=\frac{1}{2}[(|\Psi_+\rangle+|\Psi_-\rangle)|\gamma_1\rangle+(|\Psi_+\rangle-|\Psi_-\rangle)|\gamma_2\rangle]$$

$$\to\frac{1}{2}[(|c_1c_2\rangle|\phi\rangle+|\Psi_-\rangle)|\gamma_1\rangle+(|c_1c_2\rangle|\phi\rangle-|\Psi_-\rangle)|\gamma_2\rangle]. \quad (3)$$

Since the states $(1/\sqrt{2})(|c_1c_2\rangle|\phi\rangle+|\Psi_-\rangle)$ and $(1/\sqrt{2})\times(|c_1c_2\rangle|\phi\rangle-|\Psi_-\rangle)$ are orthogonal, there is no memory erasing, no interference, and, of course, no superluminal communication.

Equation (3) was obtained for the limiting case in which $|\phi_1\rangle$ and $|\phi_2\rangle$, the states of the photons emitted by atom 1 or 2, respectively, are identical. In reality, $|\phi_1\rangle$ and $|\phi_2\rangle$ are not identical and, correspondingly, $|\Psi_-\rangle$ is a metastable state that finally decays emitting a photon $|\phi_-\rangle$. This photon, however, is emitted long after the photon $|\phi_+\rangle$ emitted by the symmetric state $|\Psi_+\rangle$, and the states $|\phi_-\rangle$ and $|\phi_+\rangle$ are orthogonal due to large spatial separation. Thus, in the realistic case Eq. (2) is replaced by

$$\frac{1}{\sqrt{2}}(|b_1c_2\rangle|\gamma_1\rangle+|c_1b_2\rangle|\gamma_2\rangle)$$

$$\to\frac{1}{2}|c_1c_2\rangle[(|\phi_+\rangle+|\phi_-\rangle)|\gamma_1\rangle+(|\phi_+\rangle-|\phi_-\rangle)|\gamma_2\rangle], \quad (4)$$

and we see again that the memory of which atom emitted the photon γ is not erased but stored in the photon states $(1/\sqrt{2})(|\phi_+\rangle\pm|\phi_-\rangle)$.

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[1] R.L. Ingraham, Phys. Rev. A **50**, 4502 (1994); **51**, 4295(E) (1995).

[2] R.H. Dicke, Phys. Rev. **93**, 99 (1954).