Complementarity between Local and Nonlocal Topological Effects

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In certain topological effects the accumulation of a quantum phase shift is accompanied by a local observable effect. We show that such effects manifest a complementarity between nonlocal and local attributes of the topology, which is reminiscent but different from the usual wave-particle complementarity. This complementarity is not a consequence of noncommutativity, rather it is due to the noncanonical nature of the observables. We suggest that a local/nonlocal complementarity is a general feature of topological effects that are "dual" to the Aharonov-Bohm effect.

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In the Aharonov-Bohm (AB) effect [1,2] a charge moves around a magnetic flux filament in a region with vanishing electromagnetic fields. The charge experiences no electromagnetic forces, yet it accumulates a topological quantum phase shift. Topological effects which are "dual" to the AB effect have been discovered for neutral particles. Aharonov and Casher [3–5] have shown that particles carrying a magnetic moment and moving around a straight wire with uniform charge density will experience no force but acquire a phase shift analogous to the AB phase [6–9]. More recently, it has been shown by He and McKellar [10] and by Wilkens [11] that a neutral particle carrying an electric dipole also exhibits similar dual topological effects [12,13]. Nevertheless, unlike the AB effect, in these dual cases the local fields along the trajectory of the particle do not vanish. Consequently, as was pointed out by Peshkin and Lipkin [14,15], the accumulation of the attendant quantum phase shift may be accompanied by a local observable effect.

In this Letter we suggest that such dual topological effects manifest a complementarity between the nonlocal and the local attributes of the topology, which is reminiscent but yet different from the usual wave-particle complementarity; by measuring a local observable we disturb the nonlocal phase information on the topology. However, the complementarity suggested here is not a consequence of noncommutativity; rather it is due to the noncanonical nature of the corresponding observables [16].

To illustrate this complementarity let us begin with the Aharonov-Casher (AC) effect [3]. In the AC effect the magnetic moment μ interacts with the electric field $\vec{E}$, the vector potential-like term, $\vec{\mu} \times \vec{E}$, which induces a phase

$$\phi_{\mathrm{AC}} = \frac{1}{\hbar} \oint \vec{\mu} \times \vec{E} \cdot \vec{d}l = \frac{\mu\lambda}{\hbar} n_{\mathrm{AC}} . \qquad (1)$$

Here μ is the projection of $\vec{\mu}$ in the direction of the line, λ is the charge per unit length, and n_{AC} is the winding number of the magnetic moment's trajectory around the line.

We note that several similarities exist between the AC and AB effects. Since in both cases the force vanishes, they are "force-free" effects [6]. Moreover, in both cases a "vector-potential" coupling gives rise to a topological phase; indeed for a closed trajectory, the AC phase and the AB phase are insensitive to the details of the path and are determined from the winding number alone.

However this similarity breaks down in one important aspect [7]. Since in the AB effect the particle couples to a gauge field, the locally accumulated phase is not gauge invariant. Only the phase accumulated in an interference experiment, on a closed loop, is a gauge invariant quantity. Therefore, the AB effect is sometimes described as being nonlocal. On the other hand, since in the present case the magnetic moment clearly couples directly to the field strengths $\vec{E}$, the locally accumulated phase is a gauge invariant and meaningful observable.

This naturally raises the question of the nonlocality of such effects [17–19]. In particular, Peshkin and Lipkin [15] have noted that a magnetic field is present in the local reference frame of the magnetic moment. Consequently, the magnetic moment vector precesses around the direction of the magnetic field by an angle that turns out to be proportional to the phase shift. Since the accumulation of the quantum phase shift is accompanied by a local precession, and since the latter is locally observable, they concluded that the AC is inherently local.

It was, however, implicitly assumed that the accumulated phase and the precession are two attributes of the effect, which are simultaneously meaningful. Indeed the autocorrelation operator suggested in [15] commutes with the phase operator (that we define below). This apparently implies that we can locally measure the rotation of the magnetic moment without disturbing the accumulated phase. Nevertheless, the precession and the accumulated phase are noncanonical variables [16], and *for noncanonical variables commutativity does not imply mutual observability*. In fact, we will show that by observing the local precession we necessarily induce an uncertainty in the accumulated phase, in a similar way as for ordinary canonical conjugate variables.

Let us first show that the above complementarity is required for consistency of the AB effect with ordinary

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wave-particle complementarity [20]. Consider the usual AB interference experiment of charged particles around a solenoid enclosing a magnetic flux. We wish, however, to regard the effect of the charge on the internal degrees of freedom of the fluxon. For simplicity let space be only two dimensional; hence the solenoid is replaced by a magnetic moment $\vec{\mu}$ pointing in the "up" direction. The magnetic moment generates an AB vector potential; hence the charge acquires the usual AB phase shift that can be observed in an interference experiment. On the other hand, consider now the effect of the charged particle on the magnetic moment. In the rest frame of the magnetic moment, the moving charge generates the magnetic field $\vec{B} = \vec{v} \times \vec{E}$, where $\vec{v}$ is the velocity of the charge and $\vec{E}$ is its electric field. As noted above, this magnetic field causes a precession of the magnetic moment $\vec{\mu}$. By evaluating the angle of precession $\delta\varphi$ when the charge q is moving along either the upper or lower side of the magnetic moment we find $\delta\varphi \propto \phi_{\mathrm{AC}}$ (see below). Therefore, by measuring $\delta\varphi$ we can determine on which side of the fluxon the particle moved. Clearly, that contradicts the wave-particle complementarity principle, unless the measurement of precession destroyed the coherence of the two trajectories.

We shall next examine this process in more detail to see how in actuality this loss of coherence happens. The nonrelativistic Hamiltonian in three dimensions [3,8,9]

$$H = \frac{(\vec{P} + \vec{\mu} \times \vec{E})^2}{2m} - \frac{\mu^2 E^2}{2m} \tag{2}$$

describes a spin-half neutral particle carrying magnetic moment $\vec{\mu} = \mu\vec{\sigma}$, which moves in an electric field $\vec{E}$. In the AC effect, the electric field is generated by a straight wire with uniform charge density λ. If the particle moves in the plane orthogonal to the wire, and the momentum in the direction of the wire vanishes, one finds [9] that H effectively reduces to a two-dimensional Hamiltonian. Using polar coordinates $(r(x,y), \theta(x,y))$ where the charged wire is located at $r = 0$, and points in the z direction, we get

$$H = \frac{p_r^2}{2m} + \frac{(p_\theta + \xi\sigma_z)^2}{2mr^2}, \tag{3}$$

where $\xi \equiv \lambda\mu/2\pi$.

The Heisenberg equations of motion for the spin

$$\begin{aligned} \dot{\sigma}_x &= -\frac{2\xi}{\hbar m r^2} p_\theta \sigma_y, \\ \dot{\sigma}_y &= \frac{2\xi}{\hbar m r^2} p_\theta \sigma_x, \\ \dot{\sigma}_z &= 0. \end{aligned} \tag{4}$$

describe a precession of the spin around the z axis. When the magnetic moment moves between time t_0 to time t along a path joining points with angular coordinates $\theta(t_0)$ and $\theta(t)$, we find that (up to the trivial phase $\frac{2\xi}{\hbar m}\int_{t_0}^t \frac{dt'}{r^2}$), the precession is generated by the unitary operator

$$U(t,t_0) = e^{-i\frac{2\xi}{\hbar}\int_{t_0}^{t}\sigma_z\dot{\theta}(t')\,dt'}. \tag{5}$$

Indeed, this precession is induced by the magnetic field, $B_z = (\vec{v} \times \vec{E})_z = 2\xi\dot{\theta}$, experienced by the spin in its rest frame. If σ_z is constant, the angle of precession φ is hence $\varphi = 2\xi\sigma_z[\theta(t) - \theta(t_0)]/\hbar$.

Consider now the wave function ψ of the magnetic moment. For the above trajectory, it changes according to

$$\psi \rightarrow U(t,t_0)\psi = e^{-i\delta\phi_{\mathrm{AC}}}\psi, \tag{6}$$

where

$$\delta\phi_{\mathrm{AC}}(t,t_0) = \frac{\xi}{\hbar}\int_{t_0}^{t}\sigma_z\dot{\theta}(dt')\,dt'. \tag{7}$$

In the AC effect, ψ is an eigenstate of σ_z. If $\sigma_z = 1$, $\delta\phi_{\mathrm{AC}}(t,t_0) = \xi[\theta(t) - \theta(t_0)]/\hbar$. We will henceforth refer to $\delta\phi_{\mathrm{AC}}(t,t_0)$ in (7) as the "phase operator." In general it describes the phase accumulated along a definite trajectory for arbitrary σ_z.

Let us next examine what is the effect on a system when the spin precession is measured. The rotation implies that the spin at t_0 is related to the spin at some latter time t. Particularly we have the identity

$$C_\varphi(t,t_0) \equiv U^\dagger(t,t_0)\vec{\sigma}(t_0)U(t,t_0) - \vec{\sigma}(t) = 0, \tag{8}$$

which follows from the equation of motion of the spin. By observing that $C_\varphi(t,t_0)$ indeed vanishes we can verify that the spin rotates. One might think that since

$$[C_\varphi(t,t_0), \delta\phi_{\mathrm{AC}}(t,t_0)] = 0, \tag{9}$$

we should actually be able to observe simultaneously both the precession operator $C_\varphi(t,t_0)$ and the phase operator $\delta\phi_{\mathrm{AC}}(t,t_0)$.

However, the above commutativity is a dynamical result, i.e., it is valid only by virtue of equations of motion (4). To define $C_\varphi(t,t_0)$ and $\phi_{\mathrm{AC}}(t,t_0)$ one has to specify the Hamiltonian (2). For such noncanonical variables [16], commutativity does not imply mutual observability.

To observe $C_\varphi(t,t_0)$, we have to couple to the system twice, first at time t_0 and then at a later time t. Since the coupling of the system to a measuring device at time t_0 changes the Hamiltonian (2) (because we must add to the Hamiltonian new terms describing the interaction of the system with a measuring device), the spin component σ_z will no longer be constant, and the accumulated phase (7) will change by an uncertain amount.

Let us examine in more detail the uncertainty produced in $\delta\phi_{\mathrm{AC}}(t,t_0)$ when $C_\varphi(t,t_0)$ is measured. To this end, we couple at $t = t_0$ to σ_i and at some later time t to the rotated spin $U^\dagger\sigma_i U$. To be able to observe a precession of a single spin, we must be sure that the spin has rotated by a sufficiently large angle, say $\varphi = \pi/2$. Choosing $i = x$, we get for this case

$$C_{\pi/2}(t,t_0) = \sigma_y(t) - \sigma_x(t_0) = 0. \tag{10}$$

Because $C_{\pi/2}(t,t_0)$ is of order unity it must be observed with precision

$$\Delta C_{\pi/2}(t, t_0) \ll 1\,. \tag{11}$$

Hence $\sigma_x(t_0)$ is effectively measured with precision $\Delta\sigma_x \ll 1$. During the time interval (t, t_0), σ_z then becomes uncertain by $\Delta\sigma_z \approx 1$. The consequent uncertainty in the AC phase

$$\Delta\phi_{\mathrm{AC}}(t, t_0) = \frac{\xi}{\hbar}[\theta(t) - \theta(t_0)]\Delta\sigma_z \approx \pi/4 \tag{12}$$

is hence sufficiently large to erase the phase information. This achieves our goal of showing that by measuring the precession we destroy the coherence.

More generally, we will be able to infer that $C_\varphi(t, t_0) = 0$ only statistically. Consider, for example, the limiting case that the spin has precessed by only a small angle $\varphi \ll 1$. Let $|x\rangle$ be the eigenstate of σ_x, and denote the rotated eigenstate by $|\varphi\rangle$. Hence $|\langle\varphi\,|\,x\rangle| \simeq 1 - \varphi^2$. To verify the precession, one has to repeat the experiment over a sample of $N \sim \frac{1}{\varphi^2}$ spins, all initially in the same $|x\rangle$ state, and measure separately for each spin the operator $C^i_\varphi(t, t_0)$. The total phase accumulated by the N spins, $\phi^N_{\mathrm{AC}} = \sum_{i=1}^N \phi^i_{\mathrm{AC}}$, will become uncertain by

$$\Delta\phi^N_{\mathrm{AC}} = \frac{\xi}{\hbar}[\theta(t) - \theta(t_0)]\sum_{i=1}^{N}\Delta\sigma^i_z \approx \varphi\sqrt{N}/2 \sim 1/2\,, \tag{13}$$

where the relation, $\phi = 2\xi[\theta(t) - \theta(t_0)]/\hbar$, was used, and we have assumed that the uncertainties $\Delta\sigma^i_z \approx 1$, for each spin, are independent. Therefore, if we verify that the N spins precess, the total accumulated phase becomes uncertain. This verifies our claim also for this case.

Next consider a special case where the spinor nature has a special role. Suppose that the spin rotates around the $\hat{z}$ axis, by either $\varphi = +\pi$ or $\varphi = -\pi$. In both cases, of either a clockwise or a counterclockwise rotation, σ_x changes to $-\sigma_x$ and

$$C_\pi = \sigma_x(t) + \sigma_x(t_0) = 0\,. \tag{14}$$

In space-time these two alternatives correspond to a magnetic moment moving along either a clockwise or a counterclockwise path around the charged wire with $\theta(t) - \theta(t_0) = \pm\hbar\pi/2\xi$. Since both paths give rise to the same rotation of the spin, they cannot be distinguished by measuring $C_\pi(t, t_0)$. Therefore, in this particular case, consistency with ordinary wave-particle complementarity does not require that coherence must be lost. So it may appear that this provides a counterexample to our claim.

However, by observing C_π, we are still unable to distinguish between a nontrivial or a trivial phase in an interference experiment, i.e., we cannot detect a nontrivial topological effect. To see that, let us compare two cases: first consider an AC effect where the charged line generates the phases $\phi_{\mathrm{AC}} = \pi/2$ on one path and $-\pi/2$ for the other. This yields a relative nontrivial π phase. In the second setup we arrange a special charge distribution which generates the *same* $\pi/2$ phase for *both* paths. The relative phase in the second case is trivial, however, since the spin rotates by π, $C_\pi = 0$ in both cases, and we cannot distinguish by performing this measurement between the cases of a topological and a nontopological effect.

Similar reasoning applies for the case of a magnetic moment moving through a region with a homogeneous but time dependent magnetic field $B(t)$. The corresponding phase shift

$$\phi_{\mathrm{AC}} = \frac{1}{\hbar}\int \vec{\mu}\cdot\vec{B}(t')\,dt' \tag{15}$$

was observed by Allman *et al.* [5]. This effect is sometimes referred to in the literature as the Scalar-AB effect, because the interaction term $\vec{\mu}\cdot\vec{B}$ in (2) is analogous to the qV term in the AB Hamiltonian $(p - qA)^2/2m + qV$, which gives rise to the potential-AB effect. Since with the inclusion of a magnetic field the AC Hamiltonian is $H_{\mathrm{AC}} = (\vec{p} + \vec{\mu}\times\vec{E})^2/2m - \vec{\mu}\cdot\vec{B}$, perhaps it is more natural to identify this phase as a "potential-AC effect." It can be readily shown that our arguments for the AC effect follow, by replacing the particle's rest-frame magnetic field, $\dot{\theta}(t)$, with $B(t)$.

So far we have shown a complementarity relation in the cases of the AC and the potential AC (or Scalar-AB) effects. Nevertheless, the gedanken experiment suggested earlier indicates that a similar complementarity relation exists for other dual effects. Such is the topological effect for electric dipoles [10,11,13], which is manifested via a vector potential $\vec{d}\times\vec{B}$ or a "potential" $\vec{d}\cdot\vec{E}$ [11]. It was noted that the effect for electric dipoles can be obtained from the AC setup by a Maxwell duality transformation [12]. To connect our gedanken experiment with these cases, we will, however, make use of a different type of duality [3]

$$\text{charge} \leftrightarrow \text{magnetic moment} \tag{16}$$

which transforms the magnetic filament and the charge in the AB effect to a charged wire and a magnetic moment in the AC effect and vice versa. This duality is closely related to the Galilean invariance of the nonrelativistic charge-magnetic moment system. The total accumulated phase depends only on the relative motion of the charge and the magnetic moment. Hence by a duality transformation we transform from the rest frame of the magnetic moment (in the AB effect) to the rest frame of the charge (in the AC effect). As we already have shown, since the phase is "common" to the charge and the magnetic moment, the consistency of the AB effect with ordinary wave-particle complementarity necessitates a local/nonlocal complementarity for the "companion" dual effect.

We next note that the potential electric-dipole effect generated by the $\vec{d}\cdot\vec{E}$ term [11] is dual to the nonlocal potential AB effect: let two charged plates of a "capacitor" initially overlap each other. A potential difference between the two sides of the capacitor is then formed when a charged particle passes close to the capacitor (so that $\vec{E} \simeq 0$), by changing temporarily the distance between the plates. The charge then experiences no force but accumulates the AB phase $\frac{q}{\hbar}\int V(t')\,dt'$.

Consider now the duality transformation

$$\text{charge} \leftrightarrow \text{electric dipole} \tag{17}$$

which replaces the capacitor (viewed as a planar density of electric dipoles) by a homogeneously charged plate, and the moving charge by a time dependent electric dipole $\vec{d}(t)$. In a sense this again corresponds to a transformation from the capacitor rest frame to that of the charge. Hence the electric dipole effect is the dual companion of the potential AB effect. The electric dipole experiences no forces, yet it acquires the phase $\phi_D = \frac{1}{\hbar}\int \vec{d}(t')\cdot\vec{E}\,dt'$. However, viewing the dipole as formed by an extended (time dependent) charge distribution in an external electric field, it will induce a corresponding time dependent nonvanishing internal stress. The consistency of the potential AB effect then requires that the local internal effects should be complementary to the accumulated phase. It would be interesting to understand the details of the complementarity relation for this case and the related vector-potential dipole effect.

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