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Observability of the Sign Change of Spinors under 2π Rotations*

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We consider the observability of the sign change of spinors under 2π rotations. We show that in certain circumstances there are observable consequences of such rotations, contrary to the classical case. We suggest experiments which can in principle be done to demonstrate such effects.

IT is usually said that the sign change of spinors which have undergone 2π rotations is unobservable because physical quantities are quadratic in the wave functions.[1] The purpose of the present paper is to demonstrate that in certain kinds of situations, the sign change is observable.

Let us first make clear what is meant by 2π rotation in this context. Certainly a rotation of 2π of the entire universe is unobservable. For that matter, so is any rotation of the entire universe. Thus any meaningful distinction between 2π rotations and other rotations must refer to the relative rotation between one system and another. In classical physics, relative as well as absolute $2n\pi$ rotations are unobservable. We now present two *Gedanken* experiments to show that the peculiar change of sign of spinors under 2π rotations can lead to observable effects.[2]

Consider an electron, contained in a box with reflecting walls. The box is divided into two identical compartments, separated by an impenetrable partition. The partition has a hole in it which may be closed off by a shutter. The hole is initially open, so that the electron is free to go back and forth. Initially the electron is in a state with spin in the z direction described by the spinor

$$\begin{pmatrix}\psi_1(x)\\0\end{pmatrix}+\begin{pmatrix}\psi_2(x)\\0\end{pmatrix}=|\text{initial}\rangle\,, \tag{1}$$

where $\psi_1(x)$ is a wave function in compartment 1 and $\psi_2(x)$ is a wave function in compartment 2. The shutter is now closed, dividing the box into two isolated compartments which are subsequently separated to a large distance from one another. We assume that during the entire process, no forces were applied to the electron which would influence its spin. Uniform *equal* magnetic fields in the z direction are then applied to each box. The two boxes, each with its own magnetic field, may be considered as isolated systems, which can therefore undergo relative rotations. We consider the effect of rotating box 1 together with its attached magnetic field through $2n\pi$ around the x axis. A simple calculation shows that if the rotation is quasistatic, the part of the electron wave function in box 1 is multiplied by

$$e^{i(\mu H/\hbar)t}R_x(\omega t)\,. \tag{2}$$

Here, H is the magnitude of the magnetic field, μ is the electron magnetic moment, t is the time as measured from the start of rotation, $R_x(\theta)$ is the spin rotation operator for rotation about the x axis, and ω is the angular frequency of rotation.

The quasistatic nature of the rotation means that $\omega\ll\mu H/\hbar$. After the n complete 2π rorations, $\omega t=2n\pi$ and $R_x(\omega t)=(-1)^n$. Hence, the electron wave function in box 1 is multiplied by

$$(-1)^n e^{i2\pi\mu Hn/\hbar\omega}\,. \tag{3}$$

The part of the electron wave function in box 2 is multiplied by

$$e^{i2\pi\mu Hn/\hbar\omega}\,. \tag{4}$$

If shutters in the two boxes are now opened and the electron wave functions ψ_1 and ψ_2 are allowed to come together and interfere, the interference pattern will clearly depend upon the relative sign of ψ_1 and ψ_2, and therefore on the number of 2π rotations undergone by box 1.

In general, we may say that if fermions are coherently shared between two spatially isolated systems, then a relative rotation of 2π may be observable.

Imagine two systems having free electrons and exhibiting tunneling current when they are close together. We then separate the systems spatially and rotate one of them n times relative to the other and bring them together so that tunneling current can flow again. It turns out that the direction of current flow depends on n modulo 2.

To see this, note that the tunneling current is proportional to $\sin\alpha$, where α is the relative phase of each electron's wave function in system 1 and 2.[3] A $2n\pi$ rotation of system 1 alone will change the sign of the part of each wave function in system 1 by $(-1)^n$, leaving the wave function 2 unchanged.

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[1] A. Messiah, *Quantum Mechanics* (John Wiley & Sons, Inc., New York, 1961), Vol. I.

[2] In general, in such experiments the rotated systems will each contain an uncertain number of fermions, although the total number may still be definite.

[3] P. W. Anderson, Rev. Mod. Phys. 38, 298 (1966). Note, however, that for superconductors this effect would be absent since the charge carriers are bosons.

Thus sinα, and therefore the current, will have a sign depending on n mod 2. An odd number of rotations will reverse the current relative to the current in the unrotated situation.

In order to have a large current, the many-electron state must be symmetric in the coordinate α, which means symmetric in the direction of current flow and therefore antisymmetric in the other coordinates.

In a future paper we shall discuss the relevance of these considerations to the fermion superselection rule.[4–6]

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[4] G. C. Wick, A. S. Wightman, and E. P. Wigner, Phys. Rev. **88**, 101 (1952).

[5] R. F. Streater and A. S. Wightman, *TCP, Spin and Statistics and All That* (W. A. Benjamin, Inc., New York, 1964).

[6] Y. Aharonov and L. Susskind (to be published).

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Steady State of Cosmic-Ray Nuclei—Their Spectral Shape and Path Length at Low Energies

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The steady state of cosmic-ray nuclei in interstellar space is discussed. It is shown that for a steady-state situation (or for any mode of propagation in which the allowed path lengths between the source and observer have a wide distribution), the generally used matter-slab approximation for the interstellar matter traversed by cosmic rays leads to erroneous conclusions. The steady-state energy spectra of heavy nuclei are found to have negative slopes down to energies ~50 MeV/N, if the injection spectra are like a rigidity power law; this offers an explanation for the apparently surprising observation of flat spectra for heavy nuclei down to energies ~50 MeV/N. Further it is found that the L/M ratio cannot keep on increasing at low energies but must decrease continuously below a few hundred MeV/N, even for energy-independent fragmentation cross sections; this also is in accord with recent experimental results.

INTRODUCTION

TRAVERSAL through interstellar space modifies both the spectral shape and the chemical composition of cosmic rays. It is well recognized that an understanding of these modification processes combined with the experimentally observed properties of cosmic rays near the solar system can provide useful information about their propagation history. One of the gross parameters characterizing this propagation is the amount of matter traversed by cosmic rays at the time of their arrival. This is usually determined by measuring the relative abundances of nuclei such as Li, Be, B (L nuclei), which have a cosmic-ray abundance much larger than their universal abundance and hence should be products of the fragmentation suffered by heavier nuclei, mainly C, N, O (M nuclei). The relative abundance L/M should be a measure of the number of collisions suffered by M nuclei and hence of the amount of matter traversed by cosmic rays. The basic advantage of such a procedure is that the L/M ratio is affected very little by solar modulation.

The analysis procedure adopted by most of the authors (see for example Appa Rao and Kaplon[1] and Balasubrahmanyan *et al.*[2]) is briefly as follows: It is assumed that the amount of matter traversed by particles of a given rigidity is unique with zero or a small spread;[2] in other words, a slab of matter (hydrogen) of definite thickness is assumed to exist between the source of cosmic rays and the observer. Then, starting with a reasonable source spectrum, a diffusion equation is set up in which the various fragmentation processes and energy loss due to ionization are properly taken into account. In this way the changes in the composition and the spectral shape are calculated and a comparison with the experimental values of L/M (or He^3/He^4) ratio[3] gives the appropriate value of the thickness of the matter slab, which is then called the amount of matter traversed by cosmic rays.

Here we would like to emphasize that such an approximation is not valid and leads to an erroneous interpretation of the experimental data at low energies. Lack of any anisotropies in the observed flux of cosmic rays has led to the conclusion that after production their directions of motion are randomized by the irregular magnetic fields existing in the galaxy. Thus for particles of any given rigidity there must exist a host of allowed trajectories, of widely varying lengths, connecting the point of origin and the point of observation; hence, the distribution in the amount of matter (in

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[1] M. V. K. Appa Rao and M. F. Kaplon, Nuovo Cimento **27**, 700 (1963).

[2] V. K. Balasubrahmanyan, E. Boldt, and R. A. R. Palmeira, Phys. Rev. **140**, B1157 (1965).

[3] Unlike the L/M ratio, the He^3/He^4 ratio is considerably affected by solar modulation, owing to the large difference in the A/Z ratios of He^3 and He^4.