Spin-$\frac{1}{2}$ Partons in a Dual Model of Hadrons

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Spin-$\frac{1}{2}$ partons are introduced into dual-parton theory. A nearest-neighbor antiferromagnetic interaction in longitudinal momentum is postulated and solved in the infinite-momentum frame. The resultant theory has a two-dimensional conformally invariant continuum limit and yields a dual theory of mesons and baryons which are magnonlike excitations of the spin lattice. For an even number of partons the model is equivalent to the Neveu-Schwarz meson theory. The odd-number case defines half-integer-spin states which are identified with baryons. The model is equivalent to a covariant model described previously. An important feature is the possibility of constructing current operators, although the gauge-invariance problem has not been solved yet. A major difficulty is the tachyonic nature of the "pion." Also, the nonspurious baryonic states have not yet been identified.

I. INTRODUCTION

There exists now a considerable amount of evidence that to a good approximation hadrons may be described as one-dimensional many-body systems.

Experimentally, the products of high-energy collisions may be ordered by their longitudinal momenta while the multiplicity of fragments obeys Feynman's distribution law,

$$dN \sim \frac{dP_L}{P_L} \text{ as } P_L \to \infty . \tag{1}$$

Deep-inelastic probes by photons indicate the presence of a large number of spin-$\frac{1}{2}$ partons whose electromagnetic interactions are adequately described by treating them as bare quanta.

On the theoretical side, both multiperipheral and dual-resonance models lead to a picture of hadronic matter characterized by an average parton distribution similar to Eq. (1), and correlations between the other dynamical variables which may be damped[1] at large separations of longitudinal momenta.

In the conventional dual-parton model[2] the basic starting point is quantum field theory. Hadronic processes and states are described by large planar Feynman diagrams. It is assumed that in collisions the role of the external particles is to provide sources and sinks for the conserved quantities – energy, momentum, angular momentum, isospin, etc. These latter quantities distribute themselves among a large number of degrees of freedom (partons) which are of course described by the propagators which build the diagram. If one assumes a fairly smooth flow then it is overwhelmingly probable that the total momentum flux through any infinitesimal portion of the diagram be infinitesimal. This observation, applied to the Euclidean continuation of the diagram, leads to the well-known approximation of ascribing Gaussian propagators to the partons. The result is a description of hadrons in terms of harmonic chains with nearest-neighbor coupling, and the Veneziano formulas.

It is our aim to include a description of parton spin in the dual-parton framework. It is apparent that the lack of such a description constitutes a serious flaw in the present models. Spin certainly exists, and the most natural description of hadronic matter should presumably be attempted using spin-$\frac{1}{2}$ constituents. The procedure is best described in the infinite-momentum frame,[3] which possesses some simplifying features with respect to the treatment of spin[4] and the physical interpretation of the equations.

A hadronic system will consist of a one-dimensional array of partons, ordered by a parameter $0<\theta<\pi$. In accordance with the conventional model, θ has a one-one correspondence with the average fraction of the (infinite) longitudinal momentum carried by the partons at θ,

$$\frac{P_L(\theta)}{P_L} = \alpha(\theta) = \lambda_0 \sin\theta . \tag{2}$$

The longitudinal momentum will be treated as a parameter and is not subject to dynamical fluctuations. In accordance with Feynman's assumption and the dual-parton model, the parton density is inversely proportional to the longitudinal momentum,

$$dN(\theta) = \frac{d\theta}{\pi\lambda_0 \sin\theta} . \tag{3}$$

Although it is not expected that the parton density be really infinite, this idealization will be assumed. Still, the model does retain a memory of the parameter λ_0 in that no wavelengths shorter than

$\pi\lambda_0 \sin\theta$ will be allowed in normal-mode expansions We note that this point has little influence on purely hadronic processes which are dominated by the region $\theta \to 0$ ("wee partons"), but will play an important role in the determination of currents, form factors, and deep-inelastic amplitudes.

The lth parton, at position θ_l, will be equipped with a set $\{\xi_l\}$ of dynamical variables. $\{\xi_l\}$ includes the transverse coordinates and momenta (x, p), the three spin matrices $\vec{\sigma}_l$, and the isospin matrices τ_l. Evidently,

$$[\xi_l, \xi_{l'}] = 0, \quad l \neq l'. \tag{4}$$

Interactions are assumed to be nearest-neighbor in θ. To find their form, note that for finite longitudinal momentum the total energy of two partons is

$$\begin{aligned} H_{12} &= (P_{12}{}^2 + p_{12}{}^2 + m_{12}{}^2)^{1/2} \\ &\to P_{12} + \frac{p_{12}{}^2 + M_{12}{}^2}{2P_{12}} \\ &= P_1 + \frac{p_1{}^2 + m_1{}^2}{2P_1} + P_2 + \frac{p_2{}^2 + m_2{}^2}{2P_2} + V_{12}, \end{aligned} \tag{5}$$

where V_{12} is the interaction energy. Assuming $P_1 = P_2$ and using Eq. (2),

$$V_{12} = \frac{M_{12}{}^2(\xi_1, \xi_2)}{2P\lambda_0 \sin\theta}. \tag{6}$$

Generalizing Eq. (6) to a chain of partons, and rescaling the infinite-momentum Hamiltonian [(transverse mass)^2] to get rid of the infinite time-dilation factor P, we have

$$\mathcal{H} = \sum_l \frac{1}{2\lambda_0 \sin\theta_l} U(\xi_l, \xi_{l+1}). \tag{7}$$

The continuum limit of Eq. (7) using the parton distribution (3) is

$$H = \int \frac{d\theta\, U(\xi(\theta), \partial_\theta \xi(\theta), \dots)}{2\pi(\lambda_0 \sin\theta)^2}. \tag{8}$$

Our choice for the spin-dependent part of U relies on the following considerations:

(a) Spin-orbit coupling is disregarded. This seems to be in line with the linearity of the Regge trajectories of the known hadronic states.

(b) The low-lying states of the system should have low values of spin. This means in particular a finite spin density and preference for anti-alignment of spins, so that a ferromagnetic lattice should be ruled out.

(c) Rotational invariance in the infinite-momentum frame necessitates only invariance under rotations around the preferred longitudinal (z) direction. In particular, note that the Pauli matrix σ_l^z is the parton helicity operator and generates rotations in the x-y plane. The matrices $\sigma_l^\pm$ are *not* rotation generators and play the role of helicity-flip operators.

(d) The resultant continuum limit of the model should lead to a conformally invariant theory. This, as is well known, is related to the assumed average isotropy of the original Feynman graph and is the ingredient which ensures crossing symmetry and duality.

(e) Lorentz invariance is very difficult to ensure *a priori* at infinite momentum. Rather, we shall show that the model has a covariant generalization, which has already been given elsewhere.[5]

The interaction we have found to obey all these conditions is the so-called x-y model[6]:

$$U_{\text{spin}}(\xi_l, \xi_{l+1}) = \tfrac{1}{2} g \sigma_l^\perp \cdot \sigma_{l+1}^\perp. \tag{9}$$

The complete solution will be presented in Sec. II. The end result is the transverse momentum-transfer part of the recent pion model of Neveu and Schwarz.[7] In addition, the model includes half-integer-spin states which are naturally identified with baryons.

We shall not deal with isospin in this paper. A model which includes parton isospin has been formulated and will be presented elsewhere.

II. SPIN WAVES AT INFINITE MOMENTUM

As mentioned in the Introduction, the infinite-momentum spin Hamiltonian is assumed to have the following form:

$$H_{\text{spin}} = \pi g \tfrac{1}{2} \sum_l \frac{\sigma_l^\perp \cdot \sigma_{l+1}^\perp}{\pi\lambda_0 \sin\theta_l}. \tag{10}$$

The time-dilation factor $\pi\lambda_0 \sin\theta_l$ is the average separation between neighboring partons, and the continuum limit will involve $\lambda_0 \to 0$. We may thus designate $\pi\lambda_0 \sin\theta_l$ by $d\theta_l$. The variables σ_l *commute* for different l's and anticommute for the same l. It is convenient to use the Jordan-Wigner trick and define a new set of variables by a nonlocal transformation. The new set will in fact define two bonafide Fermi-Dirac (F.D.) fields pertaining to even and odd points on the chain. The distinction between even and odd points is not surprising in view of the expected anti-alignment of neighboring spins in the low-lying states and the resultant noncontinuity of σ_l. Thus, define

$$\begin{aligned} \psi_\lambda^\mp &= \frac{1}{\sqrt{d\theta_\lambda}} \exp\left(\frac{i\pi}{2} \sum_{n=1}^{2(\lambda-1)} \sigma_n^z\right) \sigma_{2\lambda-1}^\mp, \\ \phi_\lambda^\mp &= \pm \frac{1}{\sqrt{d\theta_\lambda}} \exp\left(\frac{i\pi}{2} \sum_{n=1}^{2\lambda} \sigma_n^z\right) \sigma_{2\lambda}^\mp, \\ d\theta_\lambda &= 2d\theta_{2\lambda} = 2d\theta_{2\lambda-1}. \end{aligned} \tag{11}$$

Note that $\psi^+=(\psi^-)^\dagger$, $\phi^+=(\phi^-)^\dagger$. It is readily verified that the column $\chi=\binom{\psi}{\phi}$ satisfies the standard F.D. anticommutation rules,

$$[\chi_\lambda, \chi_{\lambda'}]_+=0, \qquad [\chi_\lambda, \chi^\dagger_{\lambda'}]_+=\frac{1}{d\theta_\lambda}\delta_{\lambda'\lambda}. \tag{12}$$

The continuum limit of Eq. (12) is obviously

$$[\chi(\theta), \chi(\theta')]_+=0, \qquad [\chi(\theta), \chi^\dagger(\theta')]_+=\delta(\theta-\theta'). \tag{13}$$

Substitution of Eq. (11) into the Hamiltonian (10) leads to the discrete form of the one-space–one-time massless free Dirac Hamiltonian. An important feature are the boundary terms whose form depends on whether the total number of partons N is even or odd,

$$H_{\text{spin}}=-\tfrac{1}{4}\sum_\lambda \{[\psi^+_\lambda\psi^-_\lambda(\phi^-_\lambda-\phi^-_{\lambda-1})-\phi^+_\lambda(\psi^-_{\lambda-1}-\psi^-_\lambda)] +\text{boundary terms}+\text{H.c.}\}. \tag{14}$$

The coupling constant g has been set equal to π^{-1}, and it is tacitly assumed that the fields pertaining to the zero and $N+1$ partons identically vanish. The boundary terms for the two cases are as follows.

Even $N=2L$:

$$-[(\psi^+_1\phi^-_1+\phi^+_1\psi^-_1)+(\psi^+_L\phi^-_L+\phi^+_L\psi^-_L)].$$

Odd $N=2L+1$:

$$-[(\psi^+_1\phi^-_1+\phi^+_1\psi^-_1)-(\psi^+_{L+1}\phi^-_L-\phi^+_L\psi^-_{L+1})]. \tag{15}$$

The formal continuum limit of H_{spin} is obviously

$$H_{\text{cont}}=-\frac{1}{4}\int_0^\pi d\theta\{[\chi^\dagger\alpha,(1/i)\partial_\theta\chi]+\text{H.c.}\} +\text{boundary terms}, \tag{16}$$

where the two-dimensional Dirac matrix α is defined as

$$\alpha=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}. \tag{17}$$

We emphasize that the continuum limit may be taken only after the long-wavelength normal modes of the discrete Hamiltonian have been proven to be continuous in θ.

The Hamiltonian (14) leads to Heisenberg equations for the fields (ψ,ϕ). The boundary terms result in boundary conditions on the proper frequencies. It will be seen that these latter differ in the odd and even systems.

Using the anticommutation rules (12), we have

$$i\partial_t\psi_\lambda+\frac{1}{d\theta_\lambda}(\phi_\lambda-\phi_{\lambda-1})=0, \qquad i\partial_t\phi_\lambda-\frac{1}{d\theta_\lambda}(\psi_{\lambda+1}-\psi_\lambda)=0. \tag{18}$$

The boundary terms are as follows.

Even:

$$i\partial_t\psi_1+\frac{1}{d\theta_1}\phi_1=0, \qquad i\partial_t\phi_L+\frac{1}{d\theta_L}\psi_L=0. \tag{19a}$$

Odd:

$$i\partial_t\psi_1+\frac{1}{d\theta_1}\phi_1=0, \qquad i\partial_t\psi_{L+1}-\frac{1}{d\theta_L}\theta_L=0. \tag{19b}$$

The inhomogeneity of the equations may be dealt with by subdividing the chain into $R\to\infty$ subchains such that $L(r)$, the number of partons in the rth subchain, still tends to infinity. The interparton separation $d\theta$ will be assumed constant in each subchain: $d\theta_\lambda=d\theta_r$. Further, the parton distribution is symmetric under the transformation $\theta\longleftrightarrow\pi-\theta$, so that $d\theta(r)=d\theta(R-r)$. Under these conditions, and neglecting boundary terms, the normal modes will have the following form:

$$\psi^k_{\lambda,r}=\cos\left[\left(\sum_{s<r}k_sL_s+k_r\lambda\right)-\alpha\right](b_ke^{-i\omega_kt}+c^\dagger_ke^{i\omega_kt}),$$

$$\phi^k_{\lambda,r}=-\sin\left[\left(\sum_{s<r}k_sL_s+k_r(\lambda+\tfrac{1}{2})\right)-\alpha\right]\times(b_ke^{-i\omega_kt}-c^\dagger_ke^{i\omega_kt}), \tag{20}$$

$$d\theta_r\omega_k=2\sin\tfrac{1}{2}k_r. \tag{21}$$

It is obvious that by setting $k_r=qd\theta_r$, namely, taking the long-wavelength limit, we have $\omega_k\to\omega_q=q$. The phase α and the eigenvalue q are found by substituting (20), (21), and the value of k_r into the boundary conditions, Eq. (19), and using the aforementioned symmetry.

Even:

$$2\sin\tfrac{1}{2}k_1\cos(k_1-\alpha)-\sin(\tfrac{3}{2}k_1-\alpha)=0, \qquad 2\sin\tfrac{1}{2}k_1\sin(q\pi-\tfrac{1}{2}k_1-\alpha)-\cos(q\pi-k_1-\alpha)=0. \tag{22a}$$

Odd:

$$2\sin\tfrac{1}{2}k_1\cos(k_1-\alpha)-\sin(\tfrac{3}{2}k_1-\alpha)=0, \qquad 2\sin\tfrac{1}{2}k_1\cos(q\pi-k_1-\alpha)+\sin(q\pi-\tfrac{3}{2}k_1-\alpha)=0. \tag{22b}$$

It is readily verified that Eqs. (22) are solved by

$$\alpha = \tfrac{1}{2}k_1 = \tfrac{1}{2}qd\theta_1 . \tag{23}$$

Even:

$$q = (\nu + \tfrac{1}{2}) \frac{\pi}{\pi - d\theta_1} \to \nu + \tfrac{1}{2},$$

Odd: (24)

$$q = \nu \frac{\pi}{\pi - d\theta_1} \to \nu , \quad \nu = 0, 1, 2 \ldots .$$

We re-emphasize, that for calculations involving the partons in the interior of the chain the wave number ν should be cut off at the θ-dependent value $(\pi\lambda_0 \sin\theta)^{-1}$, which defines the long-wavelength limit.

To summarize, the continuum limit of the normal-mode expansion is:

Even:

$$\psi(\theta, t) = \frac{1}{\sqrt{\pi}} \sum_{\nu=1}^{\infty} \cos(\nu - \tfrac{1}{2})\theta \, (b_\nu e^{-i(\nu-1/2)t} + c_\nu^\dagger e^{i(\nu-1/2)t}),$$

$$\phi(\theta, t) = -\frac{1}{\sqrt{\pi}} \sum_{\nu=1}^{\infty} \sin(\nu - \tfrac{1}{2})\theta \, (b_\nu e^{-i(\nu-1/2)t} - c_\nu^\dagger e^{i(\nu-1/2)t})$$

Odd: (25)

$$\psi(\theta, t) = \frac{1}{\sqrt{\pi}} \left[\sigma^- + \sum_{\nu=1}^{\infty} \cos\nu\theta (B_\nu e^{-i\nu t} + C_\nu^\dagger e^{i\nu t}) \right],$$

$$\phi(\theta, t) = -\frac{1}{\sqrt{\pi}} \sum_{\nu=1}^{\infty} \sin\nu\theta (B_\nu e^{-i\nu t} - C_\nu^\dagger e^{i\nu t}),$$

$$[b_\nu, b_{\nu'}^\dagger]_+ = [B_\nu, B_{\nu'}^\dagger] = \delta_{\nu\nu'}, \quad \text{etc.},$$
$$[B, \sigma]_+ = [C, \sigma]_+ = 0, \tag{26}$$
$$[\sigma^-, \sigma^+]_+ = 1 .$$

The boundary conditions are summarized by

Even:

$$\phi(0) = \partial_\theta \psi(0) = \partial_\theta \phi(\pi) = \psi(\pi) = 0 .$$

Odd: (27)

$$\phi(0) = \partial_\theta \psi(0) = \partial_\theta \psi(\pi) = \phi(\pi) = 0 .$$

Note the existence of a zero-frequency mode designated σ in the odd system. Its appearance stems from the invariance of H_{spin} under a 180° rotation around any transverse axis. Because of the half-integral helicity of the odd system, the ground state has to be degenerate. Obviously, the operators $\sigma^\pm$ are isomorphic to the Pauli matrices, and a natural choice for the ground state is $\sigma^z = \pm 1$. This, in fact, is borne out by counting the number of normal modes and comparing it with the total number of states, which is just 2^N.

The continuum limit of Eq. (18) is of course the two-dimensional Dirac equation,[8]

$$i(\partial_t + \alpha \partial_\theta)\chi = 0, \tag{28}$$

or, define the Dirac matrices

$$\gamma^t = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^\theta = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \quad \gamma^5 = \gamma^t \gamma^\theta = \alpha , \tag{29}$$

to get

$$(\gamma^t \partial_t + \gamma^\theta \partial_\theta)\chi = 0 . \tag{30}$$

The Dirac equation leads to two conservation laws, associated with γ^5 and 1. The charge density associated with the gauge transformation $\chi \to e^{i\alpha}\chi$ is

$$\tfrac{1}{2}[\chi^\dagger, \chi] = \tfrac{1}{2}\{[\psi^\dagger, \psi] + [\phi^\dagger, \phi]\} . \tag{31}$$

Comparison with the definition (11) leads to

$$\tfrac{1}{2}[\chi^\dagger, \chi] = \rho^z(\theta), \tag{32}$$

where $\rho^z(\theta)$ is the helicity density at the point θ. Thus, the Dirac "charge" is simply the conserved total helicity of the system. Evidently, the normal-mode operators create or destroy a unit of helicity and the fermionic excitations are just spin waves.

In the even system the ground state is a singlet, while in the odd case the ground state is a degenerate doublet.

As is well known, the two-dimensional massless Dirac equation is conformally invariant. This is most easily demonstrated by using the linear combinations $\psi \mp i\phi \equiv \chi_\pm$, which satisfy

$$\partial_{t \pm \theta} \chi_\pm(\theta, t) = 0 . \tag{33}$$

The γ matrices in this representation are

$$\gamma^t = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \quad \gamma^\theta = \frac{1}{i}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \alpha = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{34}$$

Continuing the Dirac equation to the Euclidean region $t \to -i\tau$, and defining $z = \theta + i\tau$, we find

$$\partial_{\bar z} \chi_+ = \partial_z \chi_- = 0 \tag{35}$$

so that $\chi_\pm$ are analytic inside the strip $0 < \theta < \pi$, and the equation is invariant under the conformal transformation,

$$z \to \omega(z),$$
$$\frac{\partial}{\partial z}\omega(z) = re^{i\phi}, \tag{36}$$
$$\chi \to re^{i\alpha\phi/2}\chi .$$

Furthermore, the helicity current J^z is just

$$J^z(\theta) = \tfrac{1}{2}[\chi^\dagger \alpha, \chi] . \tag{37}$$

The boundary conditions may be compactly summarized by the requirement that no helicity flux crosses the boundaries,

$$J^z(0) = J^z(\pi) = 0 . \tag{38}$$

Evidently J^z is form-invariant under (36) so that the boundary conditions remain invariant too.

The model is thus completely solved and satisfies all the required conditions. The main feature is the fermionic character of the low-lying excitations which originates from the nonlocal definition of the fields χ. These, in turn, though just an artifice in the discrete problem, are the correct variables for the continuum limit because of their continuity properties, which are not shared by the original spins.

III. VERTICES AND AMPLITUDES

Although the model is best described in terms of the χ fields, external probes will interact locally with the individual partons. It is thus necessary to extract the spin operator from the F.D. fields. This is done by inverting Eq. (11) and substituting integrations for sums,

$$\sigma^-_{\text{odd}}(\theta)=\sqrt{d\theta}\,\exp\left(\frac{i\pi}{2}\int_0^{\theta^-} d\theta'[\chi^\dagger(\theta'),\chi(\theta')]\right)\psi^-(\theta),$$

$$\sigma^-_{\text{even}}(\theta)=-\sqrt{d\theta}\,\phi^-(\theta)\times\exp\left(\frac{i\pi}{2}\int_{\theta^+}^{\pi} d\theta'[\chi^\dagger(\theta'),\chi(\theta')]\right)e^{-i\pi S},\tag{39}$$

$$\sigma^+(\theta)=[\sigma^-(\theta)]^\dagger .$$

The operator S is just the conserved total helicity of the system,

$$S=\tfrac{1}{2}\int_0^\pi d\theta[\chi^\dagger(\theta),\chi(\theta)] .\tag{40}$$

S has an integer eigenvalue in the even case and a half-integer one in the odd case. It is easily verified that $\sigma(\theta)$ satisfies the required commutation and anticommutation rules. In particular, the σ's commute at different points.

Assume now that a pion current carrying transverse momentum k interacts with the system. The interaction energy with a parton at the point θ will have to include the momentum shift operator $e^{ik\cdot x(\theta)}$, the time dilation factor $(\lambda_0\sin\theta)^{-1}$, and the rotationally invariant helicity-flip operator $k\cdot\sigma$. (The expression $k\cdot\sigma$ and similar expressions are to be interpreted as $k_x\sigma_x+k_y\sigma_y$.) Thus

$$V(k,\theta)=\gamma e^{ik\cdot x(\theta)}\,\frac{k_+\sigma^-(\theta)+k_-\sigma^+(\theta)}{\lambda_0\sin\theta},\tag{41}$$

$$k_\pm\equiv k_x\pm ik_y .$$

Summing over all partons with the density function Eq. (3),

$$V(k)=\frac{\gamma}{\pi\lambda_0^{\,2}}\int\frac{d\theta\, e^{ik\cdot x(\theta)}}{(\sin\theta)^2}\{k_+[\sigma_o^-(\theta)+\sigma_e^-(\theta)]+\text{H.c.}\}.\tag{42}$$

The poles of this expression occur at the $(\text{mass})^2$ values of the particles in this channel, while the residues are the vertices. As usual, $e^{ik\cdot x(\theta)}$ has to be normal ordered and using the cutoff $(\lambda_0\sin\theta)^{-1}$, a factor $(\sin\theta)^{-k^2}$ appears under the integral sign. Moreover, the expressions (39) for $\sigma(\theta)$ are perfectly regular except for the factor $\sqrt{d\theta}=(\pi\lambda_0\sin\theta)^{1/2}$, which shifts the exponent of $\sin\theta$ by a half unit of energy. As is well known, the poles are due to the divergence at $\theta=0,\pi$, and will thus occur at

$$k^2=-\tfrac{1}{2}+\text{integer} .\tag{43}$$

Noting that the reference state is the well-known tachyon of dual theory at $(\text{mass})^2=-1$, it is seen that the positions of the poles agree with the spectrum of the even system. The residue of the first pole [called "pion" by Neveu and Schwarz (N-S)] is therefore the following.

Even:

$$R(k)=G\{:e^{ik\cdot x(0)}:[k_+\psi^-(0)+\text{H.c.}]+:e^{ik\cdot x(\pi)}:[k_+\phi^-(\pi)e^{-i\pi S}+\text{H.c.}]\},\tag{44}$$

Odd:

$$R(k)=G\{:e^{ik\cdot x(0)}:[k_+\psi^-(0)+\text{H.c.}]+:e^{ik\cdot x(\pi)}:[ik_+\psi^-(\pi)e^{-i\pi S}+\text{H.c.}]\}.$$

Note that the factor i in the odd case combines with the imaginary $e^{-i\pi S}$ (which anticommutes with ψ) to ensure the Hermiticity of the vertex. G is an overall coupling constant which includes the λ_0 factors.

It is readily verified that the even vertex is isomorphic with the transverse part of the N-S "pion" vertex, so that we shall not deal with it further. We only wish to comment on the factor $(-)^S$ which appears in the vertex at $\theta=\pi$. The point is that in computing amplitudes involving a number of external lines, these lines are attached to the fundamental "trunk" and their relative "times" are integrated over. Each time an external creation or destruction operator crosses another there is a sign change because of the fermionic nature of the excitations. When crossing from one side of the strip $(\theta=0)$ to the other $(\theta=\pi)$, this sign change is just $(-)^S$ where S is the total number of fermions in the state, which is equal to its helicity. This factor may in fact be absorbed into the usual redefinition of the F.D. field's time-ordered products to include the relative sign changes.

We shall now compute the scattering amplitude of a "pion" off a ground-state baryon for transverse momentum transfer, and exhibit its relativistic invariance. The vertex to be used is the "odd" version of Eq. (44). Assuming an incoming "pion" momentum of k_1 and an outgoing "pion" k_2, we have to multiply the usual dual amplitude originating from $e^{ik\cdot x}$ by the contraction of the two χ fields. Using the normal-mode expansion (25), we find for the s-t term

$$\langle 0|k_1\cdot\psi(0)k_2\cdot\psi(\tau)|0\rangle = \frac{1}{\pi}\left[k_1\cdot k_2\left(1+2\frac{e^{i\tau}}{1-e^{i\tau}}\right)+(k_1\times k_2)_z\sigma^z\right]; \tag{45}$$

multiplying by the standard dual integrand and using $2k_1\cdot k_2=-t-1$, $X\equiv e^{i\tau}$, we find

$$\left[-2(t+1)\int X^{-s+M^2}(1-X)^{-t-2}-(t+1)\int X^{-s+M^2-1}(1-X)^{-t-1}+2i\sigma^z(k_1\times k_2)_z\int X^{-s-M^2-1}(1-X)^{-t-1}\right]. \tag{46}$$

This expression is relativistically invariant if it is identical to the infinite-momentum limit of the covariant pion-nucleon amplitude

$$A(s,t)+(\not k_1-\not k_2)B(s,t)$$

when $k_{1,2}$ are transverse. In terms of the infinite-momentum variables the amplitude should thus have the form

$$A(s,t)[i(p'-p)\times\sigma^\perp+2M] + B[2s-2M^2+t+1+2i(k_1\times k_2)_z\sigma^z]. \tag{47}$$

Using the identity

$$\int X^{-s+M^2}(1-X)^{-t-2}=\frac{s-M^2}{t+1}\int X^{-s+M^2-1}(1-X)^{-t-1},$$

we find that (46) is in fact identical with (47) upon taking

$$A=0,\quad B=\int X^{-s+M^2-1}(1-X)^{-t-1}. \tag{48}$$

M is the ground-state baryon mass, which up to now is undetermined. The t-u term is obtained by $s\longleftrightarrow u$. The s-u term is obtained by evaluating one vertex at $\theta=0$ and the other at $\theta=\pi$ with the result

$$A=0,\quad B=-\int X^{-s+M^2-1}(1-X)^{-u+M^2-1}.$$

As remarked in the Introduction, the infinite-momentum model described above has a manifestly covariant generalization. As this version has already been presented elsewhere we shall only summarize it briefly.

The idea is to represent the parton by Dirac spinors and construct a diagram development generator from a sum of nearest-neighbor γ-matrix interactions. The interaction which leads to the model has the following form:

$$\sum_i(v_{2l-1}\cdot v_{2l}+a_{2l}\cdot a_{2l+1})\frac{1}{d\theta_l}, \tag{49}$$

where

$$v^\mu+\frac{i\gamma^\mu}{\sqrt{2}},\quad a^\mu=\frac{i\gamma^5\gamma^\mu}{\sqrt{2}}.$$

Defining even and odd fields as before, by using γ^5 instead of σ^z,

$$\psi_l^\mu=\frac{1}{\sqrt{d\theta}}\left(\prod_{n=1}^{2l-1}i\gamma_n^5\right)v_{2l-1}^\mu,\qquad \phi_l^\mu=\frac{1}{\sqrt{d\theta}}\left(\prod_{n=1}^{2l-1}i\gamma_n^5\right)v_{2l}^\mu, \tag{50}$$

one finds that the four-vector $\chi^\mu=\binom{\psi^\mu}{\phi^\mu}$ satisfies the two-dimensional Dirac equation. The rest goes on as before, except that the vertex is now defined by

$$V(k,\theta)=G\frac{e^{ik\cdot x(\theta)}}{\pi\lambda_0\sin\theta}k\cdot a(\theta). \tag{51}$$

The zero-frequency mode of the odd system is now of the form

$$\Gamma^\mu:\ [\Gamma^\mu,\Gamma^\nu]_+=-g^{\mu\nu}. \tag{52}$$

Evidently Γ^μ should be identified with the Dirac matrices of the spin-$\frac{1}{2}$ ground-state baryon. In fact, due to the axial-vector character of ψ, $(\sqrt{2}/i)\Gamma^\mu$ is just $\gamma^5\gamma^\mu$ for the ground state. Evidently a constraint on the allowed states of the baryon system is the (space-time) Dirac equation,

$$p\cdot\Gamma|\ \rangle=(i/\sqrt{2})M\Gamma^5|\ \rangle, \tag{53}$$

where M is the mass operator. In fact, the N-S subsidiary condition, which in our language limits the external vertices to $k\cdot\gamma$, may be understood qualitatively upon noticing that an external particle which satisfies (53) will not violate this equation upon colliding with another particle, if their interaction mass is proportional to $p\cdot\gamma$. This of course is just a heuristic argument, not a complete explanation of the constraints.

IV. DISCUSSION

We have presented a dual-parton model which incorporates parton spin. The model has both mesonlike and baryonlike states, and its meson sector corresponds to the Neveu-Schwarz model.

There are several obvious flaws in our picture. First, the ground-state "pion" is a tachyon. Second, baryon states which do not decouple from the spurious ground-state scalar have not yet been identified. On the plus side there exists in this model a well-defined procedure of identifying the vector and axial-vector current operators and thus calculating electromagnetic and weak form factors and amplitudes. This of course would involve the solution of the gauge-invariance problem, which we have not dealt with in this paper. There is the further problem of extending this type of model to include isospin and chirality. A model which includes parton isospin but no chirality constraints will be presented elsewhere. We wish to end with a conjecture that chiral and isospin constraints together may be the ingredient needed to generate a more physical spectrum.

Note added in proof. The problem of coupling photons has been solved and it has been proved that a deep-inelastic photon couples only through transverse components, in contrast to the usual model where it couples only through longitudinal components.

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PHYSICAL REVIEW D VOLUME 5, NUMBER 4 15 FEBRUARY 1972

Impact-Parameter Representations in Potential Scattering*

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Starting from the Lippmann-Schwinger equation the off-energy-shell generalizations of the Glauber amplitude and the Blankenbecler-Goldberger amplitude are derived for potential scattering. The Lippmann-Schwinger equation for partial waves treated as a Fredholm equation is solved and the high-energy limit studied.

I. INTRODUCTION

The eikonal approximation of Glauber[1] has been extensively used in analyzing data involving high-energy scattering. An alternative form for the scattering amplitude at high energies was proposed by Blankenbecler and Goldberger.[2] A great deal of theoretical work has recently been done on the formal aspects of these impact-parameter representations of scattering amplitudes in potential scattering.[3,4] In a recent paper Lévy and Sucher[5] derived the Glauber amplitude in potential scattering (and relativistic scattering) for the on-energy-shell case using the "eikonal approximation" for the propagators. In the present work we have studied the high-energy scattering and derived both the Blankenbecler-Goldberger amplitude and the Glauber amplitude in the potential scattering for the off-energy-shell scattering. The transition to the on-energy-shell case is trivial. Our study makes clear the set of approximations one has to make to get one or the other amplitude.

II. DERIVATION OF BLANKENBECLER-GOLDBERGER AND GLAUBER AMPLITUDES FOR OFF-ENERGY-SHELL SCATTERING

In this section we derive the Blankenbecler-Goldberger (B-G) amplitude and the Glauber (G) amplitude for off-energy-shell scattering. Our starting point is the Lippmann-Schwinger[6] equation,