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Lattice Model for Confining "Bags"

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Two new lattice models, which do not appear to be related to gauge models, are proposed. The unique feature of these models is that they describe stable baglike structures. These models may be relevant to many fields of physics; in particular, some of the basic features of high-energy physics are obtained in these models.

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The purpose of this Letter is to present two quantum spin Hamiltonians that, while being simple variations of known models, exhibit a very rich and new physical structure. In particular, the basic excitations of a continuum limit of the models have similar properties to those of the well-known Massachusetts Institute of Technology "bag" model.[1]

The Hamiltonian is defined on a one-dimensional lattice, characterized by a distance a between adjacent sites:

$$\begin{aligned} H = & -h\sum_i \sigma_i^z - g\Big(\sum_i \sigma_{i-1}^{(+)}\sigma_i^{(+)}\sigma_{i+1}^{(-)} + \text{H. c.}\Big) \\ & + \frac{g}{8}\sum_i (\sigma_{i-1}^z - 1)(\sigma_i^z + 1)(\sigma_{i+1}^z - 1) + (\sigma_{i-1}^z + 1)(\sigma_i^z - 1)(\sigma_{i-1}^z + 1) \\ & + \Lambda^2 \sum_i \{(\sigma_i^z + 1)(\sigma_{i+1}^z + 1)(\sigma_{i+2}^z - 1)(\sigma_{i+3}^z - 1) + (\sigma_i^z - 1)(\sigma_{i+1}^z - 1)(\sigma_{i+2}^z + 1)(\sigma_{i+3}^z + 1)\}, \end{aligned} \tag{1}$$

where g and h are positive;

$$\sigma^{(+)} = \sigma^x + i\sigma^y, \; \sigma^{(-)} = \sigma^x - i\sigma^y, \tag{2}$$

σ^x, σ^y, and σ^z being the Pauli matrices; and where the summation over i is over all lattice sites. The unusual part of the Hamiltonian is the three-spin interaction representing divisions and unifications of local spin excitations. We define a defect as a triad of consecutive spins oriented along the Z axis, in such a way that the direction of the inner spin is opposite to the direction of the other two. The form of the Hamiltonian suggests that the defect may serve as a kind of fundamental building stone in the theory. All states that are free of defects are trivial eigenstates of the Hamiltonian. The motion of defects comes about as a result of the more fundamental processes of divisions and unifications of spin excitations. This situation is very different from the usual description where the motion of a particle is considered as the fundamental process and is described by destruction of a particle at a given site and its creation at an adjacent site. Another interesting feature of the motion described by our Hamiltonian is the fact that the spins are reversed in the wake of the passing defect.

The model exhibits the following interesting properties:

(1) The simplest excited states are entities that may be described as "dressed" defects. These are localized states in spite of the fact that the Hamiltonian is invariant under translations. In other words, the energy of the corresponding momentum eigenstate does not depend on the momentum. As such, these excitations cannot be considered as particles.

(2) There exist composite states, that play the role of free particles.

(3) The model shows the phenomenon of confinement[2] in the sense that movement of a defect from its initial site costs an energy that is linear in the distance covered by the defect. As a result, extension of the distance between two defects belonging to a composite state costs an energy that is linear in the length of the extension. It may be worthwhile to point out that this model is not a gauge model[2] since it has only two global symmetries and not local ones. (The d-dimensional analog has 2^d global symmetries.)

In the rest of this Letter we discuss in some detail the simplest excitations.

(a) *"Dressed" defects.*—By a "dressed" defect we mean an exact eigenstate of the system, that is a superposition of states coupled to the "bare" defect by repeated application of the Hamiltonian. We define

$$|\uparrow\uparrow\ldots\uparrow\underset{R}{\downarrow}\uparrow\uparrow\ldots\rangle = |\varphi_{0'}{}^R\rangle, \tag{3}$$

$$|\uparrow\uparrow\ldots\uparrow\downarrow\uparrow\downarrow\uparrow\ldots\rangle = |\varphi_0{}^R\rangle, \tag{4}$$

$$|\uparrow\uparrow\ldots\downarrow\underset{R}{\downarrow}\ldots\downarrow\uparrow\downarrow\uparrow\ldots\rangle = |\varphi_L{}^R\rangle, \tag{5}$$

where L denotes the distance to the right of R of the up spin sandwiched between two down spins. The general "dressed" defect state is

$$|\Psi^R\rangle = \Psi'^R|\varphi_{0'}{}^R\rangle + \sum_{L=-\infty}^{\infty} \Psi_L{}^R|\varphi_L{}^R\rangle. \tag{6}$$

The equations for the coefficients are

$$E\Psi_L{}^R = -g(\Psi_{L+a}{}^R + \Psi_{L-a}{}^R) + [(|L|/a+2)h + 2g + \Lambda^2]\Psi_L{}^R + \delta_{L0}g(\Psi_L{}^R - \Psi'^R) \tag{7}$$

and

$$E\Psi'^R = (h+g)\Psi_0{}^R - g\Psi'^R. \tag{8}$$

The continuum limit is obtained by considering

$$g = G/a^2 \tag{9}$$

and

$$h = Ha, \tag{10}$$

where G and H are finite constants, and taking the limit $a\to 0$. In this limit Eqs. (7) and (8) become

$$E\Psi(L,R) = -G\nabla_L{}^2\Psi(L,R) + H|L|\Psi(L,R) + \Lambda^2\Psi(L,R). \tag{11}$$

The eigenstates are localized states, since the central position, R, is unchanged and the dynamical variables are L and its conjugate momentum.

(b) *Bare particles.*—The simplest bare particle in one dimension is a cluster of two bare defects. This state is coupled by repeated application of the Hamiltonian to all states of the form

$$|\chi_{R_1R_2}\rangle = |\uparrow\uparrow\ldots\uparrow\underset{R_2}{\downarrow}\uparrow\downarrow\downarrow\ldots\downarrow\uparrow\underset{R_1}{\downarrow}\uparrow\ldots\rangle, \tag{12}$$

where the smallest distance between R_1 and R_2 is three units, and to the special state

$$|\Gamma^{R'}\rangle = |\uparrow\ldots\uparrow\downarrow\underset{R'}{\downarrow}\uparrow\downarrow\downarrow\uparrow\ldots\rangle. \tag{13}$$

The general "dressed" particle is of the form

$$|\Psi\rangle = \sum_R \lambda^R|\Gamma^R\rangle + \sum_{R_1,R_2}\Psi(R_1,R_2)|\chi_{R_1R_2}\rangle. \tag{14}$$

The equations determining the coefficients are, apart from the boundary equations,

$$E\Psi(R_1,R_2) = -(G/a^2)[\Psi(R_1,R_2+a) + \Psi(R_1,R_2-a) + \Psi(R_1+a,R_2) + \Psi(R_1-a,R_2) - 4\Psi(R_1,R_2)] + H[|R_2-R_1|-a]\Psi(R_1,R_2). \tag{15}$$

In the continuum limit the equation becomes

$$E\Psi(R_1,R_2) = -G\nabla_{R_1}{}^2\Psi(R_1,R_2) - G\nabla_{R_2}{}^2\Psi(R_1,R_2) + H|R_1-R_2|\Psi(R_1,R_2). \tag{16}$$

The boundary equations imply in this continuum limit that the partial derivative with respect to the difference coordinate $R_2 - R_1$ must vanish when $R_2 - R_1 = 0$. Equation (16) together with the boundary condition is the same as that describing two identical bosons interacting via a linear potential. The particlelike solutions of Eq. (16) satisfy Galilean invariance and have an internal structure similar to the structure of a pair of bosons bound by a linear potential.

Two such particles interact only when they touch one another. When they collide they may exchange internal and external energies, but we have not solved this problem so far.

This theory describes dynamical violation of Galilean invariance which occurs at energies above Λ^2. Since Λ^2 is a free parameter it can be chosen sufficiently large so that this violation will occur at much higher energies than those observed so far. But at energies below that, the system describes normal nonrelativistic particles that *seem* to be each a bound state of defect pairs. Although this picture resembles in its essential features the accepted quark picture, it differs in one important aspect. While an individual quark behaves like a particle with infinite mass, in our case an individual defect has a finite energy (of the order of Λ^2) but it does not have the properties of a particle as discussed earlier.

The violation of Galilean invariance may be avoided by considering the following model:

$$H = g\sum_i (1-\sigma_{i-1}^z)(1+\sigma_i^x)(1+\sigma_{i+1}^z) + (1+\sigma_{i-1}^z)(1+\sigma_i^x)(1-\sigma_{i+1}^z) - h\sum_i \sigma_i^z . \tag{17}$$

This model describes clusters of reversed spins, that can grow or shrink by reversing spins at the boundaries, which also makes possible the motion of the clusters. The one-cluster sector is described again by the wave equation (16), where R_1 and R_2 denote the boundaries of the cluster.

The particlelike excitations in more than one dimension will be bags each enclosing a volume of reversed spins. This structure ensures that a spin reversed by the passage of a given boundary point will be reversed again by another boundary point. The bags in the first case are made of defect layers, while in the second case they are just the boundaries of clusters of reversed spins. The motion of the bags corresponding to the second model seems to resemble the motion of free particles in d dimensions (while the motion of bags corresponding to the first model is less tractable). This fact, together with the fact that the potential energy of the bag is proportional to its volume, suggests that the only excitations in our model in d dimensions have the features of the phenomenological bag model.

Baglike structure might be encountered in other fields of physics or even in biology and this model suggests a way in which such structures can be explained in terms of microscopic interactions.

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[1]A. Chodos, R. Jaffe, K. Johnson, C. Thorn, and V. Weisskopf, Phys. Rev. D 9, 3471 (1974).

[2]K. G. Wilson, Phys. Rev. D 10, 2445 (1974); J. Kogut and L. Susskind, Phys. Rev. D 11, 395 (1975); A. M. Polyakov, Phys. Lett. 59B, 82 (1975).