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Is the usual notion of time evolution adequate for quantum-mechanical systems? II. Relativistic considerations

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The measurement of nonlocal properties of relativistic quantum-mechanical systems, the compatibility (or lack of it) of two or more such measurements, and various other of their characteristics are considered; and with these notions in hand an old problem (which is to produce a covariant description of the state reduction associated with the measuring process) is attacked, and succumbs. The solution requires us to depart (not as we did in part I of the present work, but in an entirely different direction) from the usual picture of the time evolution of quantum states.

I. INTRODUCTION

Suppose that a certain particle may be in any one of three boxes[1] (which are located at the positions X_1, X_2, and X_3), and that this particle is initially prepared in the state

$$|\alpha\rangle \equiv |X_1\rangle + |X_2\rangle + |X_3\rangle \tag{1}$$

($|X_1\rangle$ here represents the state wherein the particle is in the box at X_1). At time t_1, an experiment at X_1 finds that the particle is not located at X_1, and at time t_2, an experiment at X_2 finds that the particle is not located at X_2; and these experiments are arranged such that the points (X_1,t_1) and (X_2,t_2) are separated by a spacelike interval. A Lorentz observer for whom (X_1,t_1) precedes (X_2,t_2) will describe the following sequence of states for the particle:

$$\begin{aligned} &|\alpha\rangle \quad \text{for } t<t, \\ &|\beta\rangle \equiv |X_2\rangle + |X_3\rangle \quad \text{for } t_1<t<t_2 , \\ &|X_3\rangle \quad \text{for } t>t_2 . \end{aligned} \tag{2}$$

An observer in some other frame, for whom (X_2,t_2) precedes (X_1,t_1), will describe, instead, this sequence:

$$\begin{aligned} &|\alpha'\rangle \quad \text{for } t'<t_2' , \\ &|\gamma\rangle \equiv |X_1'\rangle + |X_3'\rangle \quad \text{for } t_2'<t<t_1' , \\ &|X_3'\rangle \quad \text{for } t'>t_1' . \end{aligned} \tag{3}$$

The first observer will judge that the system is for a certain time ($t_1<t<t_2$) in the state $|\beta\rangle$ (more precisely, he will judge that, within that interval, any measurement of B, where $B|\beta\rangle=\beta|\beta\rangle$, will with certainty yield $B=\beta$); the second will judge that it is never in that state (nor in its Lorentz transform $|\beta'\rangle$) but, rather, is for a certain time in the state $|\gamma\rangle$ albeit that these two states and the two histories [(2) and (3)] of which they form parts are patently not Lorentz transforms of one another. There will in addition be an observer for whom (X_1,t_1) and (X_2,t_2) are simultaneous; for whom, that is, no intermediate interval exists at all.

We would like in this paper to address the question of whether any covariant quantum-mechanical description of such a system can be given. We have elsewhere[2,3] examined the attempts of various authors to deal with this question within the ordinary language of time evolution, and found them wanting; and we shall argue here on very general grounds that such a description cannot possibly take the form of a covariant function of space-time.

A somewhat more profound departure from the nonrelativistic language is necessary, in our view, and we shall describe it here in three stages: first, for a system subjected to no state reductions whatever; second, for a system subjected to only one such reduction; third, for a system subjected to a sequence of more than one state reduction, such as we have described above. It will emerge that each of these involves qualitatively different problems.

II. SYSTEM SUBJECTED TO NO STATE REDUCTIONS

Consider what can be said of a system which evolves from $t=-\infty$ to $t=+\infty$ without any disturbance whatsoever. Suppose that a particle is prepared in the state

$$|\delta\rangle \equiv |X_1\rangle + |X_2\rangle, \quad D|\delta\rangle = \delta|\delta\rangle \tag{4}$$

at $t=-\infty$ (and suppose, as we have above, that D is a constant of the motion) and evolves undisturbed until $t=+\infty$, whereupon it is verified once again to be in the state $|\delta\rangle$.

We have elsewhere (Ref. 3) described *Gedankenexperimente* whereby the state $|\delta\rangle$ can be verified in an arbitrarily short time entirely by means of local interactions between the particle and the measuring apparatus. The procedure for verifying that a particle is in the state $|\delta\rangle$ at some particular time t involves two simultaneous local interactions at X_1 and X_2 at t_1. If the system has been prepared in the state $|\delta\rangle$ this procedure will with certainty record that $D=\delta$, and furthermore it will *leave* the system in precisely that state at the end of the arbitrarily short interval wherein the interactions occur. Nothing prevents us from repeating the same procedure as often as we like, and so it can in principle be verified to any desired accuracy that the particle is in precisely the state $|\delta\rangle$ at all times $-\infty < t < \infty$ (Fig. 1).

So, it can be said of a particle such as the one we have described above that it is at all times in the state $|\delta\rangle$ in this sense: Any attempt to verify that $D=\delta$ at any time or at any set of times will *with certainty* find that $D=\delta$.

Suppose that a verification experiment is carried out for $|\delta\rangle$ involving interactions at X_1 and X_2 which are simultaneous in some particular Lorentz frame K. These cannot be simultaneous in any other frame (K', say), and it turns out (see Ref. 3) that within the interval separating these two interactions the evolution of the system is significantly disrupted by the measuring procedure, to wit: The system is not in the state $|\delta'\rangle$ within that interval (that is, if, within the interval, a verification experiment for $|\delta'\rangle$ were to be carried out by means of interactions at X_1' and X_2' simultaneous in K', it would not necessarily be found that $D=\delta'$), nor is the system necessarily a one-particle system at all within the interval. It is at that time, rather, in a mixture of states (including zero-particle and two-particle states); in a state correlated to the state of the apparatus (Fig. 2).

In the absence of the interactions simultaneous in K, however, the measurement in K' *would* with certainty confirm that $D=\delta'$, and could be repeated as often as desired so as to confirm to any desired accuracy that the particle is in the state $|\delta'\rangle$ at all times in K', $-\infty < t' < \infty$ (Fig. 3).

All of this begins to suggest something curious about the covariance of the state vector. A measurement which is judged by an observer in K to verify $|\delta\rangle$ without disturbing the system will necessarily disturb the system, during some finite interval, as judged by an observer in K'. The measuring process, so far as K is concerned, disrupts (as it were) the *transformation* properties of the state and disrupts its *covariance,* without in any way disrupting the history of the state itself. If, furthermore, *both* observers were to attempt to monitor the state history of the system in their own frames and in overlapping regions of space-time, these two monitoring procedures would disrupt one another as in Fig. 2.

Suppose (as we would like to distinguish more precisely between those qualities of the system which survive the measuring procedure and those which are disrupted by it) that we associate with each equal-time hypersurface t, in each Lorentz frame K, a separate and independent state $\Psi^k_{t_i}(X)$ of the particle. If the particle is permitted to evolve from $t=-\infty$ to $t=+\infty$ without any disturbances, then it happens that all of the $\Psi^k_{t_i}(X)$ conspire together to form a single covariant function of space-time $\Psi(X,t_i)$, such that

$$\Psi(X,t_i)=\Psi^k_{t_i}(X)=\Psi^{k'}_{t_i'}(X') , \tag{5}$$

wherein (X,t_i) is related to (X',t_i') by the appropriate Lorentz transformation. But if it happens that the system was initially prepared in $|\delta\rangle$ and subsequently $|\delta\rangle$ is *verified* by means of interactions simultaneous in K at time t_j, then, whereas (up to an overall phase) $\Psi^k_{t_i}(X)$ $\forall i,X$ will be entirely unaffected by such a procedure, $\Psi^{k'}_{t_i'}(X')$, as we have seen (supposing that t_i' and t_j intersect between X_1 and X_2, as in Fig. 2), will be entirely disrupted by it, so that (5) will here be invalid, and so much so, indeed, that the state at t_i' in K' will not be necessarily be any state of a one-particle system.

Heretofore we have taken our system to be confined to one (or to a superposition) of two small boxes at X_1 and X_2. Suppose now that we conceive of systems whose wave functions may have nontrivial values at many points in space, and of experiments which may entail many separate local interactions. It becomes possible now to imagine experiments wherein each of the separate interactions is spacelike separated from each other,[4] and wherein, nonetheless, the interactions do not all lie within any single equal-time hypersurface in *any* Lorentz frame. Such experiments surely do not measure anything about the system at any time in the frame (they do not measure anything, that is, along any of the surfaces we have just now considered). Suppose, then (as we should like to say precisely what about the system these are measurements of)

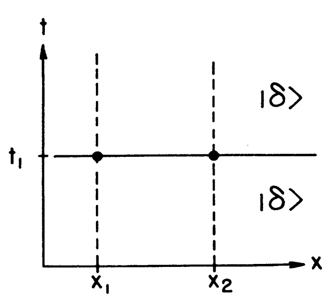

FIG. 1. A verification experiment for $|\delta\rangle$ is carried out at t_1.

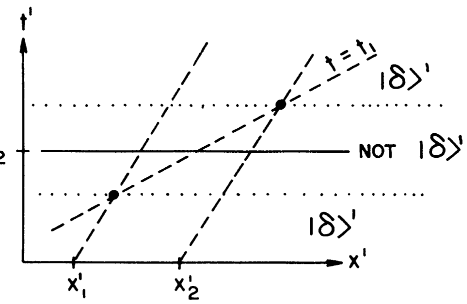

FIG. 2. A Lorentz-transformed version of Fig. 1.

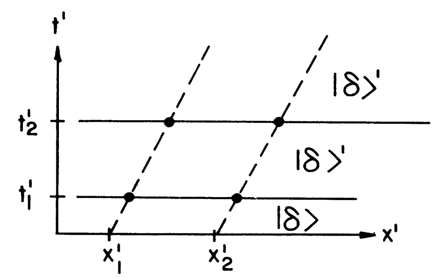

FIG. 3. A sequence of two verifications of $|\delta\rangle'$ are carried out by means of interactions simultaneous in K'.

that we associate with every spacelike hypersurface σ (of which every equal-time hypersurface in every frame is a small subset) a state $\Psi_\sigma(y)$, where y is some parameter of the surface σ.

We shall require a dynamics appropriate to this language; a dynamics, that is, not merely for evolving $\Psi(t)$ into $\Psi(t')$ for arbitrary times t and t', but, more generally, for evolving Ψ_α into Ψ_β for arbitrary spacelike hypersurfaces α and β. As above, there will (for the purposes of this present section) be two cases to consider.

Imagine first that the system is allowed to evolve without any disturbance. For *this* case, a dynamics of the kind we are seeking was written some 40 years ago by Dirac and Tomonaga and Schwinger roughly as follows. We begin with the basic covariant equation of motion:

$$i\hbar\partial_t \,|\,\psi(t)\rangle = H(t)\,|\,\psi(t)\rangle \tag{6}$$

and we now consider this to represent an infinite set of equations of the form

$$i\hbar\frac{\partial\,|\,\psi(t_\sigma(\bar{x}))\rangle}{\partial t_\sigma(\bar{x})} = \mathscr{H}(x)\Delta V\,|\,\psi(t_\sigma(\bar{x}))\rangle \;, \tag{7}$$

wherein the function $t_\sigma(\bar{x})\equiv\sigma(x)$ defines the position of the surface in space-time, $\mathscr{H}(x)$ is the local Hamiltonian density operator at X, and ΔV is an infinitesimal three-dimensional volume element at X. If (7) is integrated over a $t_\sigma(\bar{x})$, which describes an *equal-time* hypersurface, (6) is recovered. We now define an operation $\delta/\delta\sigma(x)$ which measures the variation of any quantity with respect to infinitesimal variations of the surface σ in the vicinity of the point X:

$$\frac{\delta}{\delta\sigma(x)}\,|\,\psi_\sigma\rangle \equiv \lim_{\Omega(x)\to 0}\frac{|\,\psi_{\sigma'}\rangle - |\,\psi_\sigma\rangle}{\Omega(x)} \;, \tag{8}$$

wherein $\Omega(x)$ is the four-dimensional volume separating σ and σ' (see Fig. 4), and now it follows from (7) that

$$i\hbar c\frac{\delta\,|\,\psi_\sigma\rangle}{\delta\sigma(x)} = \mathscr{H}(x)\,|\,\psi_0\rangle \;. \tag{9}$$

These are the elements of a description of the evolution of an isolated system in terms of *functionals* $(\psi_\sigma(y))$ on the set of spacelike hypersurfaces, rather than in terms of the more familiar *functions* $(\psi(x,t))$ of space-time. It happens (as before) that so long as the system is undisturbed, the various $\psi_\sigma(y)$ will all conspire together to form a single covariant function of space-time $\psi(x,t)$ such that

$$\psi(x,t) = \psi_\sigma(y) = \psi_\chi(z) \;, \tag{10}$$

where $(x,t)=(y)_\sigma=(z)_\chi$ is a point at which the surfaces σ and χ intersect, so that in this case all the features of the description in terms of functions can be subsumed (formally, at least) within the function $\psi(x,t)$, and insofar as isolated systems are concerned, these will constitute two equivalent languages.[5]

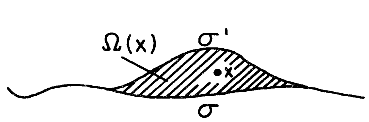

FIG. 4. Two spacelike hypersurfaces, σ and σ', which differ only in the vicinity of x.

Suppose, on the other hand, that a certain state is verified along σ by means of local interactions disturbed along that surface. In *that* event, the states along various surfaces which intersect σ will be disrupted (by analogy, say, with Fig. 2, but the topology here may be somewhat more complicated).[6] Equation (10) will be untrue. If we should not like to encounter difficulties of this kind, we must include the measuring apparatus as a dynamical part of our system, as a part, that is, of what is described by the states ψ_0. This is certainly possible, as no collapse is entailed here; the measuring apparatus, since it functions only to *verify* a preexisting state, operates throughout deterministically and in accordance with the equations of motion. But now the same difficulties arise with respect to verifications of the state of this *composite* system, and if they are similarly avoided there, they will arise on the *next* level. Let it suffice for the moment to say that this begins to suggest something curious about the relationship between these two descriptions. In the next section, distinctions between these descriptions will emerge which are far more profound.

Finally, we remark in passing that there is in this another instance of a very general principle which heretofore we have encountered in the nonrelativistic theory: There is more that can be said with certainty about a system than can possibly be verified by any set of measurements on it; whatever we verify of the nonlocal properties of a system disrupts an infinity of other such properties which would otherwise *with certainty* have had a definite value.

III. SYSTEM SUBJECTED TO ONE STATE REDUCTION

Suppose that the particle confined to two boxes of which we have spoken above is prepared at $t=-\infty$ in the state $|\,\delta\rangle$ of Eq. (4), that at $t=0$ the particle is found to be in the box at $X_1=0$ by means of a detector which has been positioned there, and which interacts locally with the particle, and that at $t=+\infty$ another measurement confirms that the particle remains in that box.

Now we would like to assign a covariant state history to this particle. We first observe that since the measurements at $t=-\infty$ and $t=0$ do not commute, since, that is, it is the case that any measurement of position within the interval $-\infty<t<0$ would with certainty yield $X=X_1=0$

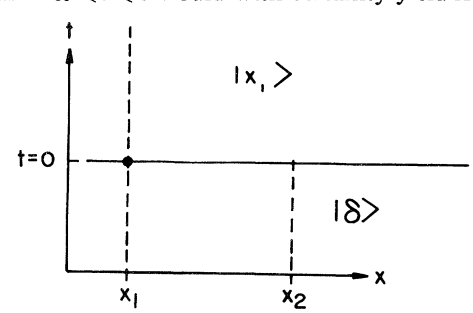

FIG. 5. The conventional, nonrelativistic, reduction postulate.

and that any measurements of D [of Eq. (4)] would with certainty yield $D=\delta$ (such as we have described in the course of our nonrelativistic considerations), *two* state histories are associated with this system. One of these is trivial: It tells that for all time ($-\infty<t<+\infty$) the system remains in the state $|X_1\rangle$, it does not entail any collapse (save at $t=-\infty$), and according to this account the measurement at $t=0$ verifies a preexisting state by means of a single local interaction at X_1, so that in this case no problems whatsoever of covariance will arise,[7] and a description in terms of a single function of space-time will completely suffice.

The other is more problematic. According to *this* account, the state associated with the particle will change instantaneously at $t=0$ from $|\delta\rangle$ to $|X_1\rangle$ (Fig. 5).[8] What is paradoxical in this is that the $t=0$ hypersurface across which the state changes will not be an equal-time hypersurface in any other frame. Thus if the collapse is, say, stipulated to occur instantaneously in some particular frame, then it will not occur instantaneously in any other. On the other hand, the statement that the detector has located the particle and thereby measured its position is apparently an entirely covariant one, so that the position measurement cannot be said to be attached to any particular frame.

Apparently we must design a new reduction postulate for the relativistic case, and to this end it has been proposed[9] that the relativistic reduction processes be taken to occur not instantaneously but along the backward light cone of the measurement event (Fig. 6). This process is first of all manifestly covariant (since the light cone will transform into itself, under Lorentz transformations), and indeed Hellwig and Kraus (Ref. 9) have shown that it will yield the correct probabilities for measurements of local observables. The probabilities for nonlocal observables, however, are another matter. If, say, D [of Eq. (4)] is measured at $t=0-\epsilon$, it will certainly be confirmed that the state at $0-\epsilon$ is not the one depicted in Fig. 6 (which in fact is not an element of the Hilbert space at all), but rather the state $|\delta\rangle$ in which the particle was prepared at $t=-\infty$ (the one depicted in Fig. 5).[10]

So it seems that the reduction process *must* be instantaneous, and this, alas, puts us back where we started. If two different Lorentz observers A and B each impose this condition in their own frame, they will give conflicting accounts of the reduction process which cannot possibly be subsumed within any single covariant state history $\psi(x,t)$.

Suppose that we hope to resolve this difficulty by monitoring the entire state history of the system experimentally so as to determine where the reduction "really" occurs. The trouble with this is that the state history cannot be monitored *covariantly,* since any procedure which monitors this history as observed by A will disturb the history as observed by B; and if, on the other hand, each observer were to monitor the history in his own frame, these two procedures would disrupt one another as in Fig. 2. If A monitors the succession of states at a given time in his own frame (K), this will with certainty confirm that the reduction process occurs along $t=0$ and it will alter the state history as observed by B; and, conversely, if B monitors the state history in K', then *this* will with certainty confirm that the reduction occurs along $t'=0$, and will alter the history as observed by A.

So, that the state along α (see Fig. 7) is $|\delta\rangle$ and that the state along β is $|X_1\rangle$ are both experimental certitudes (in that if *either* were tested by a measurement, it would certainly prove true); albeit α and β intersect at P, and that

$$\psi_\alpha(P)=\frac{1}{\sqrt{2}}\neq\psi_\beta(P)=0\ . \tag{11}$$

And this is the heart of this matter: The state of this system is not a function of space-time and it cannot be subsumed [in the manner of Eq. (10)] within such a function, since the business of assigning a value to that function of P, say, is impossible and self-contradictory; but, *rather,* that it is ineluctably a *functional* on the set of spacelike hypersurfaces.

We shall require [so as to complete, together with Eq. (9), the description of the evolution of ψ from one surface to another] some covariant prescription for the collapse within this language, and such a prescription can now straightforwardly be written. The state reduction occurs separately along every spacelike hypersurface which passes through the measurement event; if one hypersurface is continuously deformed into another, the reduction occurs as the hypersurface *crosses* that event.

That this is the case, once again, is an experimental certitude (in the sense we have just described), and that it is covariant follows from the fact that it makes mention only of Lorentz-invariant objects (spacelike hypersurfaces) which have no connection with any particular frame of reference.

The fact that no value for ψ can unambiguously be associated with a given space-time point P (in Fig. 7), say,

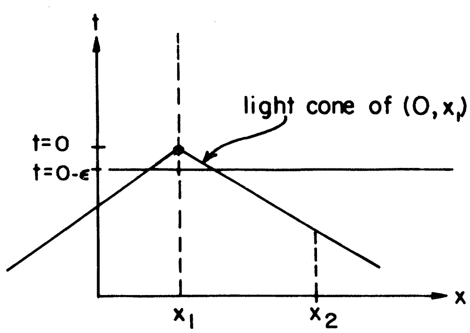

FIG. 6. The "covariant" reduction postulate of Hellwig and Kraus (Ref. 9).

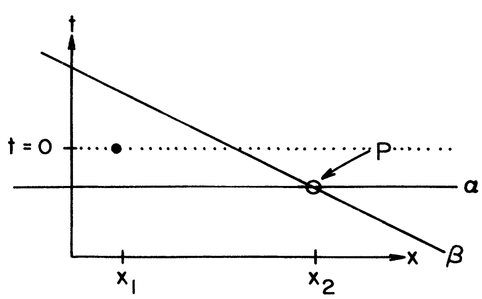

FIG. 7. Two spacelike surfaces that intersect at P.

arises because no covariant distinction can be drawn between prediction and retrodiction from one event to another, if the events are spacelike separated. For the purpose of calculating the *probability* that the particle will be found at P, P can be considered *either* to belong to α, in which case it follows by prediction from α and retrodiction from β [in the manner of Eq. (2) of part I] that this probability is zero; or to β, in which case it follows by prediction from β that this probability is zero. And this is in the nature of a general theorem; the requirement that local observables at spacelike separations commute implies that the local *probabilities* at P are independent of the global hypersurface to which P is considered to belong.[11]

Finally, we would like to remark, by means of an example, upon the richness of this language. Suppose that a particle is prepared at $t=-\infty$ in $|\delta\rangle$, and suppose that at $t=0$ (in some frame K) the particle is found to be in the state $|X_1\rangle$. Now, this latter measurement can be accomplished in two different ways: either by measuring locally at X_1 that the particle is at X_1, or by measuring locally at X_2 that the particle is *not* at X_2, and these two, so far as the state function in K is concerned, are entirely indistinguishable from one another or from a combination of both (any of these will, within that language, simply effect a collapse at $t=0$ from $|\delta\rangle$ to $|X_1\rangle$). Within the language of functionals, on the other hand, this distinction can be very precisely drawn, to wit: if the measurement occurs at L (in Fig. 8), the state along γ will be $|\delta\rangle$ and that along η will be $|X_1\rangle$; if it occurs at Q, the state along γ will be $|X_1\rangle$ and that along η will be $|\delta\rangle$; and if it occurs at both L and Q, the states along both γ and η will be $|X_1\rangle$.

IV. SYSTEM SUBJECTED TO A SEQUENCE OF STATE REDUCTIONS

Now all that remains to discuss are circumstances in which more than one measurement (involving a collapse) is carried out within a given interval, and this will no longer present any difficulty.

It happens, if *several* noncommuting measurements are carried out on a system at mutually spacelike separated points, not only that no covariant time ordering exists between these measurements and some other spacelike separated point P, but, in addition, that none exists between the various measurement events themselves. In such circumstances, then, ambiguities will arise within the language of state functions not only as to the position in space-time of some individual collapse (such as we have encountered in the previous section), but also as to what new state each collapse produces, and as to which states are present in the history of some given system at all (such as we have encountered in the Introduction).

Suppose that the state $|\delta\rangle$ of Eq. (1) is prepared at $t=-\infty$, and that the measurements described thereafter are subsequently carried out. Along the surface t in Fig. 9 (which represents an equal-time hypersurface in K, say) the apparatus of prediction and retrodiction described in Ref. 3 will incorporate, respectively, the states $|\beta\rangle$ and $|X_3\rangle$, whereas along the surface t' (an equal-time surface in K') this apparatus will incorporate, respectively, $|\delta\rangle$ and $|X_3\rangle$, albeit $|\beta\rangle$ and $|\gamma\rangle$ do *not* transform into one another under Lorentz transformations. So this history can likewise not be subsumed within a covariant function, and it requires to be represented in such a way that the states along t and t' need not be related by such transformations; that is, it requires it to be represented as a functional.

Once this is said, there ceases to be anything perplexing or apparently lacking covariance in this example. The laws governing the evolution of ψ_σ with respect to arbitrary (continuous) deformations of σ are precisely those which we have described heretofore: ψ_σ collapses as σ crosses a local measurement event, and ψ_σ otherwise evolves according to (9); and the reader can very straightforwardly persuade himself that these alone are sufficient for the description of every physical situation, and that they are sufficient, in particular, to account for the coexistence of $|\beta\rangle$ along t and $|\gamma\rangle$ along t' in Fig. 9.

One further example will elucidate how this independence of states along crossing hypersurfaces may extend even to these states being orthogonal: Suppose that a particle is prepared initially in $|\delta\rangle$, that a verification experiment for $|\delta\rangle$ is carried out involving local interactions at A and B (in Fig. 10), and that a verification experiment for the orthogonal state

$$|\delta\rangle_- \equiv |X_1\rangle - |X_2\rangle \tag{12}$$

is carried out involving local interactions at the points C and D. Now this latter experiment will disrupt the first (as in Fig. 2), so that, notwithstanding the initial preparation of the system, the results of *neither* will be certain. Nonetheless it may be the case that both experiments will turn out positively, and in this event the state along the entire family of hypersurfaces $\{f\}$ will be $|\delta\rangle$, and that along the entire intersecting family $\{g\}$ will be $|\delta\rangle_-$, and *both* of these will be verifiable on the *same* system, without mutual disruption.

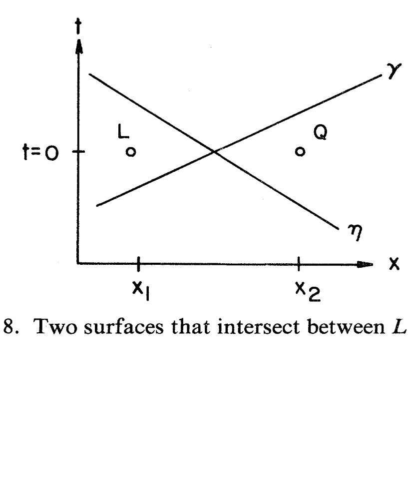

FIG. 8. Two surfaces that intersect between L and Q.

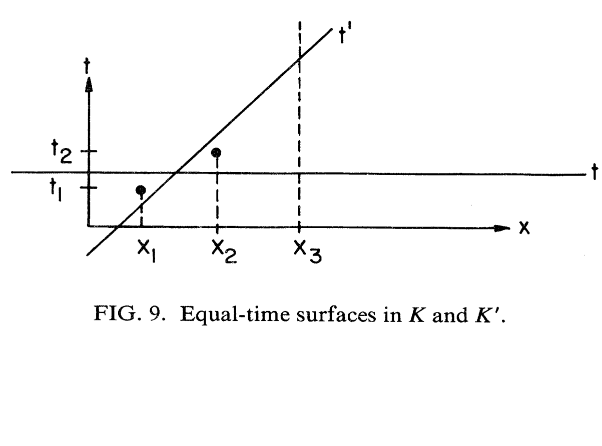

FIG. 9. Equal-time surfaces in K and K'.

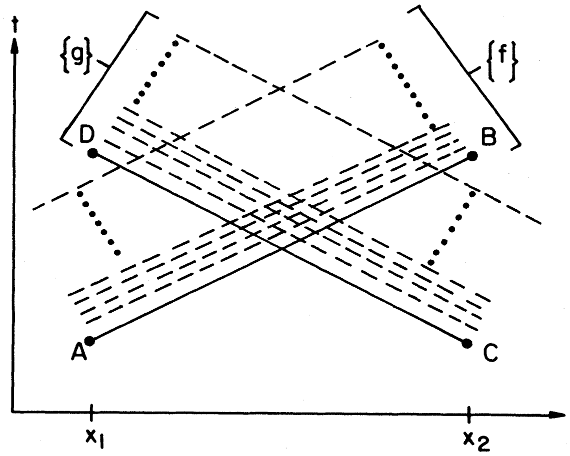

FIG. 10. Intersecting verification experiments.

V. CONCLUSION

What emerges from all this is that any covariant description of the measurement process in terms of time evolution is self-contradictory. If we are to describe such processes consistently and covariantly our description must concern itself with evolution not in a single parameter (time), but sometimes, (if, say, our system is confined to two boxes) in two parameters and sometimes in several, and in general it must concern itself with evolution in the full infinity of local times which together describe all of the possible undulations of a spacelike hypersurface; and, consequently, our description must take the form not of a function of space-time, but rather of a *functional* on the set of spacelike hypersurfaces such as was envisioned (albeit for different, and less categorical, reasons) by Dirac and Tomonaga and Schwinger many years ago.

APPENDIX

Heretofore we have been concerned exclusively with states of a single particle, both for reasons of simplicity and because the case of many particles is not different in any of its essentials. Nonetheless there are technical differences between these two cases, and we should like here to comment briefly upon these.

There are well-known difficulties connected with the construction of a covariant first-quantized Schrödinger-picture description of many-particle systems (these difficulties, indeed, were historically among the important motivations for the development of field-theoretic descriptions), to wit: The Lorentz-transformation properties of a many-particle state of the form

$$\psi(t;X_1,X_2,X_3,\ldots)\ , \qquad \text{(A1)}$$

cannot (on the face of it) be defined, since ψ depends upon unequal numbers of space and time parameters. We have emphasized in the present work that systems subjected to collapses cannot be described in terms of covariant functions of space-time; and this shall require some clarification in the case of many-particle systems, since such systems, whether or not they are subjected to collapses, are not normally described in such terms.

Let us, therefore, be somewhat more precise. Given any multiparticle state $|\psi\rangle_\sigma$ defined along a spacelike hypersurface σ, we may define for every local observable $O(y)$ (where y is a parameter of σ) an associated function

$$\psi_{\sigma_0}(y)\equiv {}_\sigma\langle\psi|O(y)|\psi\rangle_\sigma\ . \qquad \text{(A2)}$$

The complete set of such functions will suffice to define ψ unambiguously along σ, and it will be the case that if no measurement whatsoever is carried out on the system, the various $|\psi\rangle_\sigma$ along different σ will thus conspire together:

$$\psi_{\sigma_0}(y)=\psi_{\chi_0}(z)=\psi_0(x,t)\equiv\langle\psi|O(x,t)|\psi\rangle\ , \qquad \text{(A3)}$$

wherein $(y)_\sigma=(z)_\chi=(x,t)$ is a point at which σ and χ intersect, for *every* local observable O.

Equation (A3), then, is the multiparticle version of (10), and all of the subsequent discussions of single-particle wave functions can now be understood to apply as well to each ψ_0 (for every local observable O) separately. In the event that (A3) is satisfied for all points (x,t), we shall say (by analogy with the one-particle case) that $|\psi\rangle_\sigma\forall\sigma$ can be subsumed within a collection of covariant functions of space-time [the $\psi_0(x,t)\forall 0$]; and otherwise that it cannot, that it can only be covariantly described in terms of a collection of *functionals* on the set $\{\sigma\}$ of spacelike hypersurfaces.

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[1]Throughout this work, particles are taken to be localized only to within regions *larger* than their Compton wavelengths; the phenomenon of pair creation then can, to any desired accuracy, be ignored, and we will not need to have recourse to the various mathematical devices (Newton-Wigner operators and the like) which become indispensable to any discussion of the localization of particles to within regions smaller than that. The length scales which are of interest for our present considerations (the separations between the boxes in Sec. I, say) can always be made as large as we like, and we shall take them here to be sufficiently large that, on such scales, the Compton wavelengths of the particles involved can be neglected altogether. On such scales, the full field theory can always accommodate the notion of a single, relativistic, quantum-mechanical particle, such as occupies us in this investigation, and wherein considerations of relativistic covariance, as the reader shall discover, continue to play an essential and problematical role.

[2]Y. Aharonov and D. Albert, Phys. Rev. D 21, 2235 (1980).

[3]Y. Aharonov and D. Albert, Phys. Rev. D 24, 3359 (1981).

[4]We shall always understand "spacelike" here in a dynamical, rather than a kinematical, sense. That is, A and B are spacelike separated if and only if all local observables at A commute with those at B. Such circumstances may well arise, in two impenetrable boxes, say, at perhaps unequal times, in the nonrelativistic theory as well.

[5]The language of functionals, in such circumstances, is then extremely redundant, and needlessly more complex than that of

functions.

[6]Suppose that the state in question dictates that certain local properties of the system at a set of points $(Y_i)_\sigma$ along σ are correlated in a nonlocal way (i.e., in such a way that none of these properties, nor any combination of them, is defined on any proper subset of these points). The state of such a system can be verified both along σ and along χ, without these two verification procedures disrupting one another, if and only if χ intersects either the future light cones of *all* of the $(Y_i)_\sigma$ or the past light cones of *all* the $(Y_i)_\sigma$; in the event that χ intersects the future light cones of some of the $(Y_i)_\sigma$ and the past light cones of others, then the verification of states along both will be impossible.

[7]The covariance of *this* description will follow simply from that of the equation of motion.

[8]Perhaps it will not be amiss here to remind the reader that, in speaking of the localization of the particle to X_1, we are working within an approximation wherein the Compton wavelength of the particle can be neglected. The question of the description of such systems on smaller scales (scales, that is, of the order of the Compton wavelengths), which shall not occupy us here, is no doubt an interesting one. Such an investigation would require a fully field-theoretic treatment: What passes here for a localized measurement event would there have to be described as an averaging (in the manner of Bohr and Rosenfeld) over a finite, and vaguely bounded, volume of space-time j. Conceivably the spacelike hypersurface of Secs. II, III, and IV might have to be taken to be finitely thick and of course the phenomenon of pair creation will come dramatically into play. We hope to undertake such an investigation in the future; for the present all that can be said is that whatever happens on these smaller scales must necessarily reduce, as the scales are enlarged, to what we are describing here, and must necessarily partake of the same difficulties of covariance.

[9]K.-E. Hellwig and K. Kraus, Phys. Rev. D 1, 566 (1970).

[10]This, as we have shown, could in principle have been verified by experiment at $t=0-\epsilon$.

[11]A proof follows easily from the canonical commutation relations for observables; see, for example, Ref. 9.