

Orchestrating parallel TCP connections: cyclic and probabilistic polling policies[☆]

O. Czerniak

*Department of Statistics and Operations Research
School of Mathematical Sciences, Tel Aviv University, Israel*

E. Altman

INRIA, Sophia Antipolis, France

U. Yechiali

*Department of Statistics and Operations Research
School of Mathematical Sciences, Tel Aviv University, Israel
and
Department of Industrial Engineering & Management
Afeka College of Engineering, Tel Aviv, Israel*

Abstract

The standard Transmission Control Protocol (TCP) is based on an additive rate increase in absence of congestion, and on multiplicative decrease triggered by congestion signals. However, it does not scale well as the distances, or as the speed of the network, increase. Thus, we study some of the solutions that have been proposed to encounter this problem. These solutions include (i) splitting the transmission from a source to its destination into several

[☆]A preliminary version of this paper has appeared as [1]

Email addresses: omercz@gmail.com (O. Czerniak),
eitan.altman@sophia.inria.fr (E. Altman), uriy@post.tau.ac.il;
uriy@afeka.ac.il (U. Yechiali)

URL: <http://www-sop.inria.fr/members/Eitan.Altman/> (E. Altman),
<http://www.math.tau.ac.il/~uriy> (U. Yechiali)

parallel connections, and (ii) using Scalable TCP, which is more aggressive version of TCP. The connection whose rate decreases when a signal arrives is chosen either at random or according to a round robin policy. Our analysis concentrates on a centrally controlled TCP system having N connections. We consider both Additive Increase Multiplicative Decrease (AIMD) and Multiplicative Increase Multiplicative Decrease (MIMD) control mechanisms. The Laplace-Stieltjes Transforms (LST) of the transmission rate of each connection at a polling instant, as well as at an arbitrary moment, are derived. *Explicit* results are obtained for the mean transmission rate and (in contrast to most polling models) for its second moment. For the AIMD procedure under cyclic visit policy we show that, for both dynamic (Hamiltonian-type) and static visit order in each cycle, the connections should be visited following a simple index rule in order to achieve maximum throughput. For the probabilistic visit policy we obtain the set of optimal probabilities that maximizes mean throughput. The analysis of the probabilistic MIMD models uses transformations yielding a system's law of motion equivalent to that of an M/G/1 queue with batch service. The MIMD control mechanism with probabilistic strategy is further analyzed for the case where the transmission rate is bounded above.

Keywords: TCP, AIMD, MIMD, Cyclic Polling, Probabilistic Polling, Hamiltonian Tours, M/G/1 Batch Service, Bounded Transmission Rate, Limited Buffer

1. Introduction

Transmission Control Protocol (TCP), the widely-used transmission protocol of the Internet [14], is a reliable window-based flow control protocol where the window is increased linearly until a packet loss is detected. Upon loss detection, the window is reduced by a multiplicative factor. TCP modeling has been studied extensively in the literature (see e.g., [2, 3, 10] and references there), and many authors have been interested in the performance of several parallel TCP connections (see e.g., [5, 9, 18]).

In networks with very high speed covering long distances, the congestion avoidance phase of TCP takes a very long time to increase the window size and fully utilize the available bandwidth. Floyd writes in [11]: "for a Standard TCP connection with 1500-byte packets and a 100 ms round-trip time, achieving a steady-state throughput of 10 Gbps would require an average congestion window of 83,333 segments, and a packet drop rate of at most one congestion event every 5,000,000,000 packets (or equivalently, at most one congestion event every $1\frac{2}{3}$ hours). The average packet drop rate of at most 2×10^{-10} needed for full link utilization in this environment corresponds to a bit error rate of at most 2×10^{-14} , and this is an unrealistic requirement for current networks."

Two approaches exist to improve the performance of such connections. The first is a solution at the transport layer: new high speed protocols that are more aggressive have been proposed. They abandon the classical AIMD (Additive Increase Multiplicative Decrease) approach (according to which, the transmission rate of a source grows linearly in time in the absence of congestion, and decreases by a multiplicative factor when congestion is de-

tected). Some examples are HighSpeed TCP [11] and Scalable TCP [17]. In the latter, the transmission rate grows multiplicatively between congestion signals.

A second approach has been at the application layer. A single user would open several parallel TCP sessions, and when a loss occurs, only one of the connection decreases its transmission rate. The model we study in this paper describes situations in which several TCP connections are opened by one user between the same source and destination. Such models have been extensively studied in the literature (see e.g., [5, 12, 13]). Losses (congestions) are used as signals to reduce the window size of one of the connections. This is called the parallel TCP approach.

We shall study the parallel TCP approach. Under the parallel TCP, connections are all standard (AIMD) or scalable (MIMD).

Our model is valid when the loss instants are independent of the transmission rate. This situation is quite common in wireless channels in which radio conditions are often the bottlenecks, and not the congestions. It is also a common situation in TCP connections over long distances as was shown by experiments in [3]. In addition, when connections are subject to loss events by exogenous traffic, it has been shown that the losses are independent of the window size (as has been observed in [5]).

Consider N parallel TCP, where each of them increases its transmission rate until it gets a congestion signal. A source (connection) receiving a congestion signal reduces instantaneously its rate, and then resumes increasing it. The other sources continue in increasing their transmission rates. This continues until the next congestion signaling event. Thus, each connection

has two modes of operation: One during which the transmission rate grows, and one where it is reduced. Upon the receipt of a reduction signal at time t , the source that receives the signal reduces its sending rate X_t to βX_t , where $0 < \beta < 1$ is a constant. Such a reduction is termed *Multiplicative Decrease*. In absence of marking, each connection increases its sending rate. We distinguish between two methods of rate increase: (i) Additive Increase, such that at time $s > t$ the transmission rate is $X_s = X_t + \alpha(s-t)$, where $\alpha > 0$ is a constant, and (ii) Multiplicative Increase, where at time $s > t$, $X_s = X_t \cdot e^{\gamma(s-t)}$, where $\gamma > 0$ is a constant. We thus have two transmission methods: (i) Additive Increase Multiplicative Decrease (AIMD), and (ii) Multiplicative Increase Multiplicative Decrease (MIMD). We assume that the marking process does not depend on the transmission rates of the sources. We introduce two signaling strategies, which determine the choice of connection that has to reduce its transmission rate: (i) The cyclic strategy where the order of connections to which the signals are sent is cyclic, $1, 2, \dots, N-1, N, 1, 2, \dots$ and (ii) The probabilistic strategy where the choice of the connection to decrease its rate is done probabilistically, where after reducing connection i , the next connection to be chosen is j with probability p_j , $\sum_{j=1}^N p_j = 1$. We analyze the different TCP systems using polling systems methods. Polling systems, in which a single server visits (according to some scheduling procedure) and serves (according to some service discipline) N separate queues, have been studied extensively in the literature (see e.g., [8, 15, 19] and references there). In this paper the stationary behavior of the system is analyzed. In our model, TCP is not represented at packet level, but rather via direct fluid equations that describe the transmission rates for the set of connections.

The paper is structured as follows. In section 2, the AIMD mechanism is analyzed for both the cyclic (section 2.1) and the probabilistic (section 2.2) polling strategies. In addition, we address the problem of how to order the connections, or choose the probabilities, so as to maximize the expected throughput. We show that, for a Hamiltonian-type visit order procedure, both the dynamic and static visit orders that maximize the throughput are determined by a simple index rule. The MIMD mechanism is investigated in section 3: the probabilistic polling scheme is studied in section 3.1, and the cyclic procedure is analyzed in section 3.2. The relation of the last two polling schemes to the M/G/1 batch-service queue is exploited, and similarities between the two schemes are drawn. In section 3.3, we consider the case where each connection has a limited bandwidth.

Notation: For a random variable X , we denote its mean by $E[X] = x$ and its second moment by $E[X^2] = x^{(2)}$. If X is continuous then its LST is denoted by $\tilde{X}(\cdot)$; and if X is discrete, its probability generating function (PGF) is denoted by $\hat{X}(\cdot)$.

2. AIMD

2.1. Cyclic Strategy

Under a cyclic strategy, system signals occur randomly in time and are directed in a cyclic manner between the connections. We call an instant where a reduction signal occurs a "polling instant", and refer to a cycle of connection i as the time interval from the moment that connection i is polled until its next polling instant. Let X_i^j denote the transmission rate at connection j ($j = 1, 2, \dots, N$) at the instant when the server decides

to reduce the transmission rate at connection i ($i = 1, 2, \dots, N$). $X_i = (X_i^1, X_i^2, \dots, X_i^N)$ is the state of the system at that instant. Let the random variable U_i denote the time between the instant of the signal that causes the server to reduce the transmission rate of connection i and the one that causes the $i + 1$ st connection to reduce its transmission rate. All U_i 's are independent, identically distributed as a generic random variable U . The transmission rate of connection i is continuously growing at a rate $\alpha_i > 0$. When the server polls connection i , the transmission rate decreases by a factor of $\beta_i > 0$. Thus, the evolution of the stationary transmission rates of the system at a "polling instant" is given by (where the indices i are read "modulo N ")

$$X_{i+1}^j = \begin{cases} X_i^j + \alpha_j U_i & \text{if } j \neq i \\ \beta_i X_i^i + \alpha_i U_i & \text{if } j = i. \end{cases} \quad (1)$$

That is, the transmission rate of connection j at a polling instant of connection $i + 1$ ($j \neq i$) is composed of: (i) the transmission rate of connection j at a polling instant of connection i , and (ii) the growth of the transmission rate at that connection during the time between the two signals. In the case where $i = j$ the transmission rate of connection i is composed as before, except that the transmission rate of connection i just after its polling instant is $\beta_i X_i^i$.

Define the multidimensional LST, $L_i(\underline{\theta})$, of the state of the system at a polling instant of connection i ($i = 1, 2, \dots, N$) as

$$L_i(\underline{\theta}) = L_i(\theta_1, \dots, \theta_{i-1}, \theta_i, \theta_{i+1}, \dots, \theta_N) = E[e^{-\sum_{j=1}^N \theta_j X_i^j}]. \quad (2)$$

Then, for $i = 1, 2, \dots, N$, using the fact that U_i and X_i^j are independent, we

obtain $L_{i+1}(\underline{\theta})$ in terms of $L_i(\cdot)$, namely, for $i = 1, 2, \dots, N$,

$$\begin{aligned}
L_{i+1}(\underline{\theta}) &= E[e^{-\sum_{j=1}^N \theta_j X_{i+1}^j}] \\
&= E[e^{-\sum_{j=1}^N \theta_j \alpha_j U_i}] E[e^{-\sum_{j=1, j \neq i}^N \theta_j X_i^j} e^{-\theta_i \beta_i X_i^i}] \\
&= L_i(\theta_1, \dots, \theta_{i-1}, \beta_i \theta_i, \theta_{i+1}, \dots, \theta_N) \cdot \tilde{U}\left(\sum_{j=1}^N \theta_j \alpha_j\right).
\end{aligned} \tag{3}$$

Equations (3) are now used to derive moments of the variables X_i^j .

Transmission Rate at Reduction Instants: Moments

The mean transmission rate, $f_i(j) \triangleq E[X_i^j]$, at connection j when the server polls connection i is given by

$$f_i(j) \triangleq E[X_i^j] = -\left. \frac{\partial L_i(\underline{\theta})}{\partial \theta_j} \right|_{\underline{\theta}=\mathbf{0}}. \tag{4}$$

This leads to the following N^2 linear equations,

$$f_{i+1}(j) = \begin{cases} f_i(j) + \alpha_j u & \text{if } j \neq i \\ \beta_i f_i(i) + \alpha_i u & \text{if } j = i. \end{cases} \tag{5}$$

Clearly, equations (5) can also be obtained directly by taking expectation over (1).

The solution of (5) is given by

$$f_i(j) = \begin{cases} \frac{\alpha_j N u}{1 - \beta_j} - \alpha_j (j - i) u & j > i \\ \frac{\alpha_i N u}{1 - \beta_i} & j = i \\ \frac{\alpha_j N u}{1 - \beta_j} - \alpha_j (N - (i - j)) u & j < i. \end{cases} \tag{6}$$

Denoting by C the cycle time of connection i (all cycle times are identically distributed), then the explanation of (6) is as follows: since in a stationary

state, $f_i(i) = \beta_i f_i(i) + \alpha_i E[C]$, where clearly $E[C] = Nu$, then

$$f_i(i) = \frac{\alpha_i Nu}{1 - \beta_i}. \quad (7)$$

Regarding the case where $j > i$, $f_i(j)$ equals $f_j(j) - (j - i)\alpha_j u$ since the mean time until the next polling of connection j is $(j - i)u$, and during that time connection j increases its rate by $\alpha_j(j - i)u$ to the value of $f_j(j)$ (the case where $j < i$ is explained in the same manner).

The second and mixed moments of the X_i^j are given by

$$f_i(j, k) \triangleq E[X_i^j X_i^k] = \frac{\partial^2 L_i(\underline{\theta})}{\partial \theta_j \partial \theta_k} \Big|_{\underline{\theta}=\underline{0}}. \quad (8)$$

Differentiating (3) with respect to θ_j and θ_k , we get the following N^3 linear equations:

$$\begin{aligned} f_{i+1}(j, k) &= \alpha_j u f_i(k) + \alpha_j \alpha_k u^{(2)} + \alpha_k u f_i(j) + f_i(j, k) \\ &k, j \neq i, \end{aligned} \quad (9)$$

$$\begin{aligned} f_{i+1}(i, j) &= \alpha_j \beta_i u f_i(i) + \alpha_j \alpha_i u^{(2)} + \alpha_i u f_i(j) + \beta_i f_i(i, j) \\ &j \neq i, \end{aligned} \quad (10)$$

$$f_{i+1}(i, i) = 2\alpha_i \beta_i u f_i(i) + \alpha_i^2 u^{(2)} + \beta_i^2 f_i(i, i). \quad (11)$$

In contrast to most gated and exhaustive polling regimes (see, e.g., [15, 8]), computing $f_i(j, k)$ ($1 \leq i, j, k \leq N$) involves solving a set of N linear equations only, and can be done analytically. However, we can find the second moment of X_i^i from the definition (since the cycle time is independent of the transmission rate at any of these connections). Define $X_i^{i(k)}$ as the transmission rate of connection i at the k th cycle, then we have

$$X_i^{i(k+1)} = \beta_i X_i^{i(k)} + \alpha_i C, \quad (12)$$

where $C = \sum_{j=1}^N U_j$, meaning that at the beginning of a cycle, $X_i^{i(k)}$ is reduced by a factor of β_i and then it grows linearly at a rate of α_i . Define $\tilde{X}_i^{(k)}(s) = E[e^{-sX_i^{i(k)}}]$. As $X_i^{i(k)}$ and C are independent

$$\begin{aligned}\tilde{X}_i^{(k+1)}(s) &= E[e^{-sX_i^{i(k+1)}}] = E[e^{-s(\beta_i X_i^{i(k)} + \alpha_i C)}] \\ &= \tilde{X}_i^{(k)}(\beta_i s) \tilde{C}(\alpha_i s).\end{aligned}\tag{13}$$

By iterating we have

$$\tilde{X}_i^{(k+1)}(s) = \tilde{X}_i^{(1)}(\beta_i^k s) \prod_{j=0}^{k-1} \tilde{C}(\alpha_i \beta_i^j s).\tag{14}$$

When $k \rightarrow \infty$ we have $\lim_{k \rightarrow \infty} \tilde{X}_i^{(1)}(\beta_i^k s) = 1$ then

$$\tilde{X}_i(s) = \prod_{j=0}^{\infty} \tilde{C}(\alpha_i \beta_i^j s).\tag{15}$$

Differentiating (15) at $s = 0$ and using the fact that $E[C^2] = Nu^{(2)} + N(N-1)u^2$, we get

$$E[(X_i^i)^2] = \frac{1}{1 - \beta_i^2} \left[\frac{2\beta_i \alpha_i^2 N^2 u^2}{1 - \beta_i} + \alpha_i^2 (Nu^{(2)} + N(N-1)u^2) \right],\tag{16}$$

$$\text{Var}[X_i^i] = \frac{\alpha_i^2 N (u^{(2)} - u^2)}{(1 - \beta_i)(1 + \beta_i)} = \frac{N \cdot \text{Var}[\alpha_i U]}{1 - \beta_i^2}.\tag{17}$$

Throughput of Connection i

Let L_i be the transmission rate at connection i at an arbitrary moment, and let $L_i(t)$ be the transmission rate at connection i at time t within the current cycle. The LST of L_i is calculated by dividing the expected area of the function $e^{-sL_i(t)}$ over an arbitrary cycle, by the expected cycle time. That is,

$$\tilde{L}_i(s) = E[e^{-sL_i}] = \frac{E[\int_0^C e^{-sL_i(t)} dt]}{E[C]}.\tag{18}$$

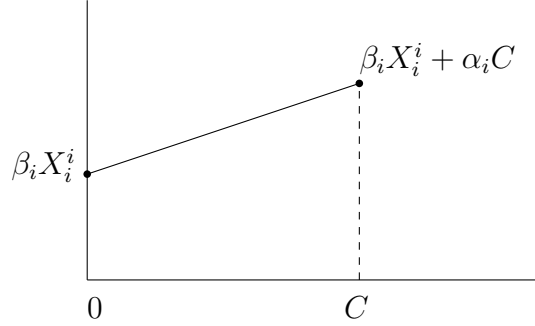


Figure 1: Transmission rate during a cycle

Figure 1 shows the transmission rate at connection i during a full cycle.

Thus,

$$\tilde{L}_i(s) = \frac{E[\int_0^C e^{-s(\beta_i X_i^i + \alpha_i t)} dt]}{E[C]} = \frac{\tilde{X}_i(\beta_i s)(1 - \tilde{C}(\alpha_i s))}{s\alpha_i E[C]}. \quad (19)$$

By differentiating (19) we get

$$E[L_i] = \beta_i E[X_i^i] + \alpha_i \frac{E[C^2]}{2E[C]} = \frac{\beta_i \alpha_i N u}{1 - \beta_i} + \frac{\alpha_i u^{(2)}}{2u} + \frac{\alpha_i (N - 1)u}{2}. \quad (20)$$

That is, the mean transmission rate equals the sum of the rate just after the polling instant ($\beta_i E[X_i^i]$) and of the accumulated rate during the mean residual time of a cycle ($\alpha_i \frac{E[C^2]}{2E[C]}$).

The total throughput of the system is given by

$$\sum_{i=1}^N E[L_i] = N u \sum_{i=1}^N \left(\frac{\beta_i \alpha_i}{1 - \beta_i} \right) + \frac{u^{(2)}}{2u} \sum_{i=1}^N \alpha_i + \frac{(N - 1)u}{2} \sum_{i=1}^N \alpha_i. \quad (21)$$

Dynamic and Static rate-reduction procedure to maximize throughput

One may wish to find a Hamiltonian type dynamic visit order of connections such that, in each cycle of visit, all connections are visited, while the dynamic feature is achieved by changing the order of visits for each new cycle (see e.g.,

[6, 19]). The objective is to visit the connection in such an order that, at the end of the cycle, the overall throughput of all connections is maximized. Consider a visit-order policy, Π_0 , such that the connection are visited in a regular order: $1, 2, 3, \dots, j-1, j, j+1, \dots, N$. Let x_i ($i = 1, 2, \dots, N$) be the actualy transmission rate at connection i at the start of the cycle, and let x'_i be the transmission rate at the end of the cycle, just before a new cycle starts. Then

$$x'_i = \beta_i(x_i + \alpha_i S_{i-1}) + \alpha_i(C - S_{i-1}), \quad (22)$$

where $S_i = \sum_{k=1}^i U_k$ ($S_0 = 0$), and $C = \sum_{k=1}^N U_k = S_n$. Thus, under Π_0 , the total throughput of the system at the end of the cycle is given by

$$\text{Throughput}(\Pi_0) = \sum_{i=1}^N x'_i. \quad (23)$$

Now consider a visit-order policy Π_1 where the visit order of connections j and $j+1$ is interchanged. Then, under Π_1 , the total throughput at the of the cycle is

$$\begin{aligned} \text{Throughput}(\Pi_1) = & \sum_{i=1}^{j-1} x'_i + \\ & \beta_{j+1}(x_{j+1} + \alpha_{j+1} S_{j-1}) + \alpha_{j+1}(C - S_{j-1}) + \\ & \beta_j(x_j + \alpha_j S_j) + \alpha_j(C - S_j) + \\ & \sum_{i=j+2}^N x'_i. \end{aligned} \quad (24)$$

It follows (after some algebra) that $\text{Throughput}(\Pi_0) > \text{Throughput}(\Pi_1)$ if and only if

$$\alpha_j(1 - \beta_j) > \alpha_{j+1}(1 - \beta_{j+1}). \quad (25)$$

The result is somewhat surprising: it does not depend on the values of x_i . Thus, in each cycle the same visit order should follow and it is determined by the index $\alpha_j(1 - \beta_j)$ where connections are ordered by a decreasing index value. Hence, our dynamic visit order is indeed a static one.

2.2. Probabilistic Strategy

Under the probabilistic strategy, when the server gets a signal it decides to reduce the transmission rate to one connection, but the choice of the connection to decrease its rate is done probabilistically. Let p_i be the probability that the signal is sent to connection i ($i = 1, \dots, N$), where $\sum_{i=1}^N p_i = 1$. Let $X_i^{(n)}$ denote the transmission rate at connection i just before the n th reduction (polling) instant. We assume that $X_i^{(n)}$ converges to X_i when $n \rightarrow \infty$. The transmission rate of connection i is continuously growing at a rate α_i . When the server polls connection i with probability p_i , the transmission rate decreases by a factor of β_i . Hence the evolution of the state of the system (transmission rate) is given by

$$X_i^{(n+1)} = \begin{cases} X_i^{(n)} + \alpha_i U & \text{w.p } 1 - p_i \\ \beta_i X_i^{(n)} + \alpha_i U & \text{w.p } p_i. \end{cases} \quad (26)$$

To calculate the LST of the transmission rate at polling instant, $L(\theta_1, \dots, \theta_N)$, we express $L_{n+1}(\theta_1, \dots, \theta_N)$ in terms of $L_n(\theta_1, \dots, \theta_N)$. This is done by conditioning on the specific connection being chosen at the n th reduction signal,

$$\begin{aligned} L_{n+1}(\theta_1, \dots, \theta_{i-1}, \theta_i, \theta_{i+1}, \dots, \theta_N | A_i) &= E[e^{-\sum_{j=1}^N \theta_j X_j^{(n)}} | A_i] \\ &= E[e^{-\sum_{j=1, j \neq i}^N \theta_j (X_j^{(n)} + \alpha_j U)} e^{-\theta_i \beta_i X_i^{(n)} - \theta_i \alpha_i U}] \\ &= L_n(\theta_1, \dots, \theta_{i-1}, \beta_i \theta_i, \theta_{i+1}, \dots, \theta_N) \cdot \tilde{U}\left(\sum_{j=1}^N \theta_j \alpha_j\right), \end{aligned} \quad (27)$$

where A_i is the event that connection i was polled at the previous (in this case, the n th) polling instant.

By unconditioning (27) and letting n approach infinity we obtain

$$\begin{aligned}
L(\theta_1, \dots, \theta_N) &= \tilde{U}\left(\sum_{j=1}^N \theta_j \alpha_j\right) \\
&\cdot \left(p_1 L(\beta_1 \theta_1, \dots, \theta_N) + \dots + p_i L(\theta_1, \dots, \theta_{i-1}, \beta_i \theta_i, \theta_{i+1}, \dots, \theta_N) \right. \\
&\left. + \dots + p_N L(\theta_1, \dots, \beta_N \theta_N) \right). \tag{28}
\end{aligned}$$

Transmission Rate at Reduction Instants: Moments

The moments of X_i are derived from (28) (or directly from (26)),

$$E[X_i] = -\left. \frac{\partial L(\theta_1, \dots, \theta_N)}{\partial \theta_i} \right|_{\theta=0} = \frac{\alpha_i u}{p_i(1 - \beta_i)}. \tag{29}$$

For the special case where $p_i = \frac{1}{N}$, we find that (29) is equal to the equivalent expression for $f_i(i)$ under the cyclic strategy system (see equation (6)).

Unlike many other polling systems, in this model we can derive explicit expressions for the second (and mixed) moments in a non-identical connections case:

$$E[X_i^2] = \frac{\alpha_i^2(u^{(2)} - 2u^2)}{p_i(1 - \beta_i)(1 + \beta_i)} + \frac{2\alpha_i^2 u^2}{p_i^2(1 - \beta_i)^2(1 + \beta_i)}, \tag{30}$$

$$Var[X_i] = \frac{\alpha_i^2(u^{(2)} - u^2)}{p_i(1 - \beta_i)(1 + \beta_i)} + \frac{(1 - p_i)\alpha_i^2 u^2}{p_i^2(1 - \beta_i)(1 + \beta_i)}, \tag{31}$$

$$\begin{aligned}
E[X_i X_j] &= \alpha_i \alpha_j \left(\frac{(u^{(2)} - 2u^2)}{p_i(1 - \beta_i) + p_j(1 - \beta_j)} \right. \\
&\quad \left. + \frac{u^2}{p_i(1 - \beta_i)p_j(1 - \beta_j)} \right) \quad j \neq i, \tag{32}
\end{aligned}$$

and

$$Cov(X_i, X_j) = \frac{\alpha_i \alpha_j (u^{(2)} - 2u^2)}{p_i(1 - \beta_i) + p_j(1 - \beta_j)} \quad j \neq i. \tag{33}$$

Throughput of Connection i

Let C_i denote the time between two successive polling instants of connection i . Then,

$$C_i = \sum_{j=1}^{T_i} U_j, \quad (34)$$

where all U_j 's are distributed as U , and T_i is the number of polling instants between two successive polling of connection i , and is distributed geometrically with parameter p_i . Hence,

$$E[C_i] = E[T_i]u = \frac{u}{p_i}, \quad (35)$$

$$\begin{aligned} E[C_i^2] &= E\left[\left(\sum_{i=1}^{T_i} U_i\right)^2\right] = E_{T_i}\left[E\left[\left(\sum_{i=1}^{T_i} U_i\right)^2 \mid T_i\right]\right] \\ &= E_{T_i}\left[E\left[(U_1 + \dots + U_{T_i})^2\right]\right] \\ &= E\left[T_i U_1^2 + T_i(T_i - 1)U_1 U_2\right] \\ &= E[T_i]u^{(2)} + (E[T_i^2] - E[T_i])u^2 \\ &= \frac{u^{(2)}}{p_i} + \left(\frac{2}{p_i^2} - \frac{2}{p_i}\right)u^2 = \frac{u^{(2)}}{p_i} + \frac{2(1 - p_i)u^2}{p_i^2}. \end{aligned} \quad (36)$$

Let L_i be a random variable denoting the transmission rate at connection i at arbitrary times. Using the same analysis as in the previous section we get

$$\tilde{L}_i(s) = \frac{E\left[\int_0^{C_i} e^{-s(\beta_i X_i + \alpha_i t)} dt\right]}{E[C_i]} = \frac{\tilde{X}_i(\beta_i s)(1 - \tilde{C}_i(\alpha_i s))}{s\alpha_i E[C_i]}, \quad (37)$$

where $\tilde{X}_i(s) = L(0, \dots, 0, s, 0, \dots, 0)$. Hence,

$$E[L_i] = \beta_i E[X_i] + \alpha_i \frac{E[C_i^2]}{2E[C_i]} = \frac{\alpha_i \beta_i u}{p_i(1 - \beta_i)} + \alpha_i \left(\frac{u^{(2)}}{2u} + \frac{(1 - p_i)u}{p_i}\right). \quad (38)$$

When $p_i = \frac{1}{N}$

$$\begin{aligned} E[L_i|\text{prob}] &= \frac{\alpha_i \beta_i N u}{1 - \beta_i} + \alpha_i \left(\frac{u^{(2)}}{2u} + (N - 1)u \right) \\ &= E[L_i|\text{cyclic}] + \frac{\alpha_i (N - 1)u}{2}, \end{aligned} \quad (39)$$

we get that for connection i , the difference between the mean transmission rate of the probabilistic strategy and that of the cycle strategy is $\frac{\alpha_i (N - 1)u}{2}$. This phenomenon can be better understood when looking at Figure 1: if the time intervals between rate reductions are less regular (i.e., probabilistic vs. cyclic), then the area under the graph (between two consecutive reduction instants) increases.

Summing (38) for all i gives the mean total throughput of the system,

$$\sum_{i=1}^N E[L_i] = \sum_{i=1}^N \left(\frac{\alpha_i \beta_i u}{p_i (1 - \beta_i)} + \alpha_i \left(\frac{u^{(2)}}{2u} + \frac{(1 - p_i)u}{p_i} \right) \right). \quad (40)$$

In the case where for all i , $p_i = \frac{1}{N}$, the mean overall throughput under the probabilistic strategy is larger than that of the cycle strategy by the amount $\frac{(N-1)u}{2} \sum_{i=1}^N \alpha_i$.

Optimal values of p_i

By using Lagrange multipliers we get the optimal values of p_i that maximize equation (40), denoted p_i^* , as

$$p_i^* = \frac{\sqrt{\frac{1 - \beta_i}{\alpha_i}}}{\sum_{j=1}^N \sqrt{\frac{1 - \beta_j}{\alpha_j}}}. \quad (41)$$

3. MIMD

3.1. Probabilistic Strategy

Our approach will be based on showing that a logarithmic transformation applied to the transmission rate process results in a process that has the same evolution as the queue size in an M/G/1 batch service queue. The LST of the equivalent queuing process thus obtained provides the moments of the transmission rate of the connections. The transmission rate of connection i grows continuously, exponentially by e^{γ_i} , and when the server decides to reduce the rate of connection i , it is decreased by a factor of β_i ($0 < \beta_i < 1$). We consider a probabilistic polling strategy. As in the previous section, $X_i^{(n)}$ denotes the transmission rate at connection i just before the n th polling instant, and $U^{(n)}$ is the time between the n th and the $n + 1$ st polling instants (all $U^{(n)}$ are identically distributed as a general random variable U). Altman et al. [4] analyzed a similar model where a connection is multiplicative increased by a constant factor, i.e., $X_i^{(n+1)} = \alpha_i X_i^{(n)}$ ($\alpha_i > 1$), whereas in our model $X_i^{(n+1)}$ is increased by a function of $U^{(n)}$. The evolution of the state of the system is given by

$$X_i^{(n+1)} = \begin{cases} X_i^{(n)} e^{\gamma_i U^{(n)}} & \text{w.p } 1 - p_i \\ \beta_i X_i^{(n)} e^{\gamma_i U^{(n)}} & \text{w.p } p_i. \end{cases} \quad (42)$$

We assume that the transmission rate is bounded below by a value of 1. Altman et al. [4] showed the importance of the bounded value, hence (42) turns into

$$X_i^{(n+1)} = \begin{cases} X_i^{(n)} e^{\gamma_i U^{(n)}} & \text{w.p } 1 - p_i \\ \max(\beta_i X_i^{(n)}, 1) e^{\gamma_i U^{(n)}} & \text{w.p } p_i. \end{cases} \quad (43)$$

In order to evaluate the moments of X_i , we take the logarithm of equation (43) and get

$$\log X_i^{(n+1)} = \begin{cases} \log X_i^{(n)} + \gamma_i U^{(n)} & \text{w.p } 1 - p_i \\ \max(\log X_i^{(n)} + \log \beta_i, 0) + \gamma_i U^{(n)} & \text{w.p } p_i. \end{cases} \quad (44)$$

Dividing equation (44) by $-\log \beta_i > 0$ and using the substitution $Y_i = \frac{\log X_i}{-\log \beta_i}$, we obtain

$$Y_i^{(n+1)} = \begin{cases} Y_i^{(n)} + \frac{\gamma_i}{-\log \beta_i} U^{(n)} & \text{w.p } 1 - p_i \\ \max(Y_i^{(n)} - 1, 0) + \frac{\gamma_i}{-\log \beta_i} U^{(n)} & \text{w.p } p_i. \end{cases} \quad (45)$$

Defining $T_i^{(n)} = \frac{\gamma_i}{-\log \beta_i} U^{(n)}$, $T_i^{(n)}$ is a non-negative random variable. Further, $T_i^{(n)}$ and $Y_i^{(n)}$ are independent random variables (since $U^{(n)}$ and $Y_i^{(n)}$ are independent). Then from equation (45) we obtain

$$Y_i^{(n+1)} = \begin{cases} Y_i^{(n)} + T_i^{(n)} & \text{w.p } 1 - p_i \\ \max(Y_i^{(n)} - 1, 0) + T_i^{(n)} & \text{w.p } p_i. \end{cases} \quad (46)$$

If $T_i^{(n)}$ is an integer (as well as $Y_i^{(0)}$) then equation (46) has the same form as the equation describing the number of customers in an M/G/1 queue just after the n th service (of length U), or a vacation period, where when the server finishes a service (or a vacation) period it serves the next customer with probability p_i , or takes a vacation of length U with probability $1 - p_i$. $T_i^{(n)}$ is the number of new arrivals during the length of time U . Solving equation (46) for the continuous case might be difficult, but by approximation we can solve it for the rational case. Let's assume that $T_i^{(n)}$ is a fraction of an integer w . Hence, $T_i^{(n)}$ can have the following values $(0, \frac{1}{w}, \frac{2}{w}, \dots, \frac{w-1}{w}, 1, \frac{w+1}{w}, \dots, \infty)$.

Define $Q_i^{(n)} = w \cdot Y_i^{(n)}$ and $M_i^{(n)} = w \cdot T_i^{(n)}$. Then,

$$Q_i^{(n+1)} = \begin{cases} Q_i^{(n)} + M_i^{(n)} & \text{w.p } 1 - p_i \\ \max(Q_i^{(n)} - w, 0) + M_i^{(n)} & \text{w.p } p_i. \end{cases} \quad (47)$$

$Q_i^{(n)}$ is an integer, and thus $Q_i^{(n)}$ can be modeled as a discrete state space Markov chain. The last equation is actually the law of motion for the M/G/1 queue with batch service of size w (see [7]) where upon finishing a service the server chooses whether to serve the next batch or take a vacation. The PGF of Q_i is obtained from the law of motion (47) using the following:

$$\begin{aligned} E[z^{Q_i^{(n+1)}}] &= (1 - p_i)E[z^{Q_i^{(n)} + M_i^{(n)}}] \\ &\quad + p_i \left(E[z^{Q_i^{(n)} + M_i^{(n)}} | Q_i^{(n)} \geq w] P(Q_i^{(n)} \geq w) \right. \\ &\quad \left. + E[z^{M_i^{(n)}} | Q_i^{(n)} < w] P(Q_i^{(n)} < w) \right). \end{aligned} \quad (48)$$

Recall that $Q_i^{(n)}$ and $M_i^{(n)}$ are independent random variables. Then, from (48) we obtain, when $Q_i^{(n)} \rightarrow Q_i$ (and $M_i^{(n)}$ is distributed like M_i for all n),

$$\hat{Q}_i(z) = \frac{p_i \hat{M}_i(z) \sum_{j=0}^{w-1} \pi_i^{(j)} (z^w - z^j)}{z^w - \hat{M}_i(z) ((1 - p_i)z^w + p_i)}, \quad (49)$$

where $\pi_i^{(j)}$ is the probability that $Q_i = j$. The expression for $\hat{Q}_i(z)$ contains w unknown parameters $\pi_i^{(0)}, \pi_i^{(1)}, \dots, \pi_i^{(w-1)}$. To determine these we use the following equality

$$\sum_{j=0}^{w-1} \pi_i^{(j)} (z^w - z^j) = (z - 1) \sum_{j=0}^{w-1} v_i^{(j)} z^j, \quad (50)$$

where $v_i^{(j)} = \sum_{k=0}^j \pi_i^{(k)}$ (see pp. 33 in [16]). Hence, we write

$$\hat{Q}_i(z) = \frac{p_i \hat{M}_i(z) (z - 1) \sum_{j=0}^{w-1} v_i^{(j)} z^j}{z^w - \hat{M}_i(z) ((1 - p_i)z^w + p_i)}. \quad (51)$$

Then, $\hat{Q}_i(1) = 1$ implies that

$$\sum_{j=0}^{w-1} v_i^{(j)} = w - \frac{m_i}{p_i}, \quad (52)$$

which is meaningful if and only if $\frac{m_i}{p_i} < w$. That is, the mean number of arrivals between two consecutive visits to queue i , namely $\frac{m_i}{p_i}$, must be smaller than the batch service amount w .

Assuming equation (51) to be an analytic function in the disk $z : |z| \leq 1 + \delta$ implies that the numerator is zero whenever the denominator vanishes in $z : |z| \leq 1 + \delta$. That is, the numerator and the denominator of (51) have exactly the same number of roots in the above disk. Let's state Rouché's theorem [16].

Theorem 1. *If $f(z)$ and $g(z)$ are analytic functions of z inside and on a closed contour D , and also if $|g(z)| < |f(z)|$ on D , then $f(z)$ and $f(z) + g(z)$ have the same number of zeros inside D .*

Define $g(z) = \hat{M}_i(z)((1-p_i)z^w + p_i)$, $f(z) = z^w$. Because $g(1) = f(1) = 1$ and $g'(1) = m_i + w(1-p_i) < w = f'(1)$, we have for sufficiently small $\delta > 0$, $g(1 + \delta) < f(1 + \delta)$. Consider all z with $|z| = 1 + \delta$, then

$$\begin{aligned} |g(z)| &= |\hat{M}_i(z)| \cdot |(1-p_i)z^w + p_i| \\ &\leq \sum_{j=0}^{\infty} P(M_i = j) |z|^j \cdot ((1-p_i)|z|^w + p_i) = g(1 + \delta) \\ &< f(1 + \delta) = |f(z)|, \end{aligned} \quad (53)$$

where the first inequality is due to the triangle inequality. Hence $|g(z)| < |f(z)|$, and by Rouché's theorem we know that $z^w - \hat{M}_i(z)((1-p_i)z^w + p_i)$ has the same number of zeros as z^w , i.e., w roots in the disk $z : |z| \leq 1 + \delta$, for

every sufficiently small $\delta > 0$. Let these roots be denoted by z_1, z_2, \dots, z_{w-1} and $z_w = 1$. Since the PGF $\hat{Q}_i(z)$ is analytic within the region $|z| \leq 1$, the numerator of (51) should vanish at each of the roots. It follows that $\sum_{j=0}^{w-1} v_i^{(j)} z^j$ should vanish at z_1, z_2, \dots, z_{w-1} . We thus have the following $w - 1$ equations

$$\sum_{j=0}^{w-1} v_i^{(j)} z_k^j = 0 \quad (k = 1, 2, \dots, w - 1). \quad (54)$$

The $w - 1$ equations of (54) together with (52) are linearly independent if their determinant $\Delta \neq 0$, where $\Delta = |\prod_{i=1}^{w-1} (z_i - 1) \prod_{j=i+1}^{w-1} (z_j - z_i)|$. Since $|z_i| < 1$ and $z_i \neq z_j$, then $\Delta \neq 0$, implying that the equations are linearly independent. Using the w independent equations we get

$$\sum_{j=0}^{w-1} v_i^{(j)} z^j = (w - \frac{m_i}{p_i}) \prod_{j=1}^{w-1} \frac{z - z_j}{1 - z_j}. \quad (55)$$

Hence,

$$\hat{Q}_i(z) = \frac{\hat{M}_i(z)(z - 1)(p_i w - m_i)}{z^w - \hat{M}_i(z)((1 - p_i)z^w + p_i)} \prod_{j=1}^{w-1} \frac{z - z_j}{1 - z_j}. \quad (56)$$

Finally, the moments of X_i can be obtained using $\hat{Q}_i(z)$. At steady state we have

$$X_i = \beta_i^{-Y_i}. \quad (57)$$

Therefore, the k th moment of X_i can be obtained as follows:

$$E[X_i^k] = E[\beta_i^{-kY_i}] = E[\beta_i^{-\frac{k}{w}Q_i}] = \hat{Q}_i(\beta_i^{-\frac{k}{w}}). \quad (58)$$

The k th moment of X_i is finite as long as $\beta_i^{-\frac{k}{w}}$ is smaller than the smallest root of the denominator of (51) which is larger than 1.

3.2. Cyclic Strategy

The analysis of the cyclic strategy follows a direction similar to that of the probabilistic strategy. Let X_i^j and $X_i^{i(n)}$ be defined as in section 2.1. Hence, the evolution of the state of the system is given by

$$X_{i+1}^j = \begin{cases} X_i^j \cdot e^{\gamma_j U_i} & \text{if } j \neq i \\ \max(\beta_i X_i^i, 1) \cdot e^{\gamma_i U_i} & \text{if } j = i, \end{cases} \quad (59)$$

or

$$X_i^{i(n+1)} = \max(\beta_i X_i^{i(n)}, 1) e^{\gamma_i C_i^{(n)}}, \quad (60)$$

where $C_i^{(n)}$ is the cycle time between the n th polling instant of channel i to the $n + 1$ st polling instant of that channel. Clearly $C_i^{(n)} = \sum_{j=1}^N U_j$. Using the substitution $Y_i^{(n+1)} = \frac{\log X_i^{i(n+1)}}{-\log \beta_i}$ as in the previous section, we get

$$Y_i^{(n+1)} = \max(Y_i^{(n)} - 1, 0) + \frac{\gamma_i}{-\log \beta_i} C_i^{(n)}. \quad (61)$$

Defining $T_i^{(n)} = \frac{\gamma_i}{-\log \beta_i} C_i^{(n)}$, and assuming that $T_i^{(n)}$ is a fraction of an integer w , we can transform the evolution equation (61) to the same evolution of an M/G/1 batch service queue, with bulk w

$$Q_i^{(n+1)} = \begin{cases} Q_i^{(n)} - w + M_i & \text{if } Q_i^{(n)} > w \\ M_i & \text{if } Q_i^{(n)} \leq w, \end{cases} \quad (62)$$

where $Q_i^{(n)} = w \cdot Y_i^{(n)}$ and $M_i^{(n)} = w \cdot T_i^{(n)}$ are integers random variables. Then, from (62), the PGF of Q_i is

$$\hat{Q}_i(z) = \frac{\hat{M}_i(z) \sum_{j=0}^{w-1} \pi_i^{(j)} (z^w - z^j)}{z^w - \hat{M}_i(z)}, \quad (63)$$

and by using Rouché's theorem as in previous section, we obtain (see [7])

$$\hat{Q}_i(z) = \frac{\hat{M}_i(z)(z-1)(w-m_i)}{z^w - \hat{M}_i(z)} \prod_{j=1}^{w-1} \frac{z-z_j}{1-z_j}. \quad (64)$$

Notice the similarity between equations (56) and (64) (substituting $p_i = 1$ in (56) yields (64)) except for M_i which is defined differently in both schemes. The k th moment of X_i^i is obtained, as in equation (58), by

$$E[X_i^{i^k}] = \hat{Q}_i(\beta_i^{-\frac{k}{w}}). \quad (65)$$

3.3. Probabilistic Strategy with Upper Bound on the Transmission Rate

Suppose an upper bound c_i ($c_i > 1$) is imposed on the transmission rate of connection i . Then, the law of evolution (43) is modified by applying the minimum operator, namely,

$$X_i^{(n+1)} = \begin{cases} \min(X_i^{(n)} e^{\gamma_i U^{(n)}}, c_i) & \text{w.p } 1 - p_i \\ \min(\max(\beta_i X_i^{(n)}, 1) e^{\gamma_i U^{(n)}}, c_i) & \text{w.p } p_i. \end{cases} \quad (66)$$

As before, using the substitution $Y_i^{(n)} = \frac{\log X_i^{(n)}}{-\log \beta_i}$, we get

$$Y_i^{(n+1)} = \begin{cases} \min(Y_i^{(n)} + T_i^{(n)}, d_i) & \text{w.p } 1 - p_i \\ \min(\max(Y_i^{(n)} - 1, 0) + T_i^{(n)}, d_i) & \text{w.p } p_i, \end{cases} \quad (67)$$

where $T_i^{(n)} = \frac{\gamma_i}{-\log \beta_i} U^{(n)}$ and $d_i = \frac{\log c_i}{-\log \beta_i}$. Assuming $T_i^{(n)}$, $Y_i^{(0)}$ and d_i are fractions of an integer w , then by multiplying (67) by w , we get

$$Q_i^{(n+1)} = \begin{cases} \min(Q_i^{(n)} + M_i^{(n)}, f_i) & \text{w.p } 1 - p_i \\ \min(\max(Q_i^{(n)} - w, 0) + M_i^{(n)}, f_i) & \text{w.p } p_i, \end{cases} \quad (68)$$

where $Q_i^{(n)} = w \cdot Y_i^{(n)}$ and $M_i^{(n)} = w \cdot T_i^{(n)}$ are integers random variables, and $f_i = w d_i$ is an integer constant. Equation (68) can be explained like equation (47), with the exception that the customers' waiting room (buffer) is limited to f_i . Without loss of generality we can assume that $M_i^{(n)}$ is bounded by f_i .

Then, when $Q_i^{(n)} \rightarrow Q_i$ we obtain, after tedious calculations (see equation (A.10) in Appendix A),

$$\begin{aligned} \hat{Q}_i(z) &= \frac{p_i \hat{M}_i(z) \sum_{j=0}^{w-1} \pi_i^{(j)} (z^w - z^j)}{z^w - \hat{M}_i(z) ((1-p_i)z^w + p_i)} \\ &+ \frac{p_i z^{f_i+w} \sum_{j=w}^{f_i} \sum_{k=f_i+w-j+1}^{f_i} \pi_i^{(j)} m_i^{(k)}}{z^w - \hat{M}_i(z) ((1-p_i)z^w + p_i)} - \frac{p_i \sum_{j=w}^{f_i} \sum_{k=f_i+w-j+1}^{f_i} \pi_i^{(j)} z^j m_i^{(k)} z^k}{z^w - \hat{M}_i(z) ((1-p_i)z^w + p_i)} \\ &+ \frac{(1-p_i)z^w \left(z^{f_i} \sum_{j=0}^{f_i} \sum_{k=f_i-j+1}^{f_i} \pi_i^{(j)} m_i^{(k)} - \sum_{j=0}^{f_i} \sum_{k=f_i-j+1}^{f_i} \pi_i^{(j)} z^j m_i^{(k)} z^k \right)}{z^w - \hat{M}_i(z) ((1-p_i)z^w + p_i)}, \end{aligned} \quad (69)$$

where $\mathbb{P}(Q_i = j) = \pi_i^{(j)}$ ($0 \leq j \leq f_i$) and $\mathbb{P}(M_i = k) = m_i^{(k)}$ ($0 \leq k \leq f_i$). Note that the first part of the PGF in the RHS in (69) is identical to (49). By using the same manipulation as in section 3.1, we get the following $w-1$ equations,

$$\sum_{j=0}^{w-1} v_i^{(j)} z_k^j = 0 \quad (k = 1, 2, \dots, w-1), \quad (70)$$

where $v_i^{(j)} = \sum_{k=0}^j \pi_i^{(k)}$. In addition, $\hat{Q}_i(1) = 1$ implies

$$\begin{aligned} p_i w - E[M_i] &= p_i \sum_{j=0}^{w-1} v_i^{(j)} + p_i (f_i + w) \sum_{j=w}^{f_i} \sum_{k=f_i+w-j+1}^{f_i} \pi_i^{(j)} m_i^{(k)} - p_i \sum_{j=w}^{f_i} \sum_{k=f_i+w-j+1}^{f_i} (j+k) \pi_i^{(j)} m_i^{(k)} \\ &+ (1-p_i) \left((f_i + w) \sum_{j=0}^{f_i} \sum_{k=f_i-j+1}^{f_i} \pi_i^{(j)} m_i^{(k)} - \sum_{j=0}^{f_i} \sum_{k=f_i-j+1}^{f_i} m_i^{(k)} \pi_i^{(j)} (j+k+w) \right). \end{aligned} \quad (71)$$

The above can be written as

$$\sum_{j=0}^{w-1} v_i^{(j)} = K_i, \quad (72)$$

where K_i is a function of the f_i unknown parameters $\pi_i^{(j)}$. Using the $w - 1$ equations of (70) together with (72) we find $\pi_i^{(j)}$ ($0 \leq j \leq w - 1$) in terms of K_i . In order to find the unknown parameters K_i and $\pi_i^{(j)}$ (for $w \leq j \leq f_i$), we use the transition matrix of the process $\mathbb{P}_i = \{P_i^{(k,l)} = \mathbb{P}(Q_i^{(n+1)} = l | Q_i^{(n)} = k)\}$, see Table 1 (where the index i is omitted and $q = 1 - p$). We see that

Table 1: Transition Matrix \mathbb{P} (omitting the index i)

k \ l	0	1	...	w	$w + 1$...	f
0	m_0	m_1	...	m_w	m_{w+1}	...	m_f
1	pm_0	$pm_1 + qm_0$...	$pm_w + qm_{w-1}$	$pm_{w+1} + qm_w$...	$pm_f + q \sum_{j=f-1}^f m_j$
2	pm_0	pm_1	...	$pm_w + qm_{w-2}$	$pm_{w+1} + qm_{w-1}$...	$pm_f + q \sum_{j=f-2}^f m_j$
\vdots	\vdots	\vdots	...	\vdots	\vdots	...	\vdots
$w - 1$	pm_0	pm_1	...	$pm_w + qm_1$	$pm_{w+1} + qm_2$...	$pm_f + q \sum_{j=f-w+1}^f m_j$
w	pm_0	pm_1	...	$pm_w + qm_0$	$pm_{w+1} + qm_1$...	$pm_f + q \sum_{j=f-w}^f m_j$
$w + 1$	0	pm_0	...	m_{w-1}	$pm_w + qm_0$...	$p \sum_{j=f-1}^f m_j + q \sum_{j=f-w}^f m_j$
\vdots	\vdots	\vdots	...	\vdots	\vdots	...	\vdots
f	0	0	...	0	0	...	$p \sum_{j=w}^f m_j + q \sum_{j=0}^f m_j$

$\pi_i^{(0)} = \sum_{k=0}^w \pi_i^{(k)} P_i^{(k,0)}$, i.e. $\pi_i^{(w)}$ is a function of $\pi_i^{(0)}, \pi_i^{(1)}, \dots, \pi_i^{(w-1)}$. Then we can express $\pi_i^{(w)}$ in terms of K_i . From the equation $\pi_i^{(1)} = \sum_{k=0}^{w+1} \pi_i^{(k)} P_i^{(k,1)}$ we get that $\pi_i^{(w+1)}$ is a function of $\pi_i^{(0)}, \pi_i^{(1)}, \dots, \pi_i^{(w)}$, and so on. By iterating ($w - f_i + 1$ iterations in total), we get expressions for $\pi_i^{(w)}, \pi_i^{(w+1)}, \dots, \pi_i^{(f_i)}$ in terms of K_i . Finally, to find K_i , we use the normalization equation $\sum_{k=0}^{f_i} \pi_i^{(k)} = 1$. Now $\hat{Q}_i(z)$ is fully determined, and again the k th moment of X_i is obtained,

as in equation (58), by

$$E[X_i^k] = \hat{Q}_i(\beta_i^{-\frac{k}{w}}). \quad (73)$$

4. Conclusions

This paper analyzes polling-type procedures of a TCP mechanism under the parallel TCP approach. Both the AIMD and MIMD schemes are studied for the cyclic and the probabilistic polling policies. LST, mean, and explicit value for the second moment of the transmission rate of each connection are derived, and overall mean throughput is calculated. In order to maximize throughput under the AIMD procedure we show that, for a cyclic case, the optimal visit order of connections is determined by a simple index rule, while for the probabilistic case we find the set of optimal probabilities. For the analysis of the MIMD scheme, an analogy to M/G/1 queue with batch service is utilized and enables the complete analysis of the system. The case where each connection has a limited bandwidth is further studied and analyzed.

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Appendix A. Finding the PGF of Q_i - MIMD with Upper Bound

For simplicity we omit the subscript i . Then,

$$\hat{Q}(z) = E[Z^Q] = (1-p) \cdot E[Z^{\min(Q+M,f)}] + pE[Z^{\min(\max(Q-w,0)+M,f)}]. \quad (\text{A.1})$$

We first calculate $E[Z^{\min(Q+M,f)}]$:

$$\begin{aligned} E[Z^{\min(Q+M,f)}] &= \mathbb{P}(Q+M > f)E[Z^{\min(Q+M,f)}|Q+M > f] \\ &\quad + \mathbb{P}(Q+M \leq f)E[Z^{\min(Q+M,f)}|Q+M \leq f] \\ &= \mathbb{P}(Q+M > f)Z^f + \mathbb{P}(Q+M \leq f)E[Z^{Q+M}|Q+M \leq f] \\ &= z^f \sum_{j=0}^f \pi_j \sum_{k=f+1-j}^f m_k + \sum_{j=0}^f \pi_j z^j \sum_{k=0}^{f-j} m_k z^k \\ &= z^f \sum_{j=0}^f \pi_j \sum_{k=f+1-j}^f m_k + \hat{Q}(z)\hat{M}(z) - \sum_{j=0}^f \pi_j z^j \sum_{k=f-j+1}^f m_k z^k, \end{aligned} \quad (\text{A.2})$$

where the last equality is due to $\sum_{k=0}^{f-j} m_k z^k = \hat{M}(z) - \sum_{k=f-j+1}^f m_k z^k$.

Next,

$$\begin{aligned} E[Z^{\min(\max(Q-w,0)+M,f)}] &= \mathbb{P}(\max(Q-w,0) + M > f)Z^f \\ &\quad + \mathbb{P}(\max(Q-w,0) + M \leq f)E[Z^{\max(Q-w,0)+M}|\max(Q-w,0) + M \leq f]. \end{aligned} \quad (\text{A.3})$$

Since (note that $\mathbb{P}(M > f) = 0$),

$$\begin{aligned}
\mathbb{P}(\max(Q - w, 0) + M > f) &= \mathbb{P}(\max(Q - w, 0) + M > f | Q \geq w) \mathbb{P}(Q \geq w) \\
&\quad + \mathbb{P}(\max(Q - w, 0) + M > f | Q < w) \mathbb{P}(Q < w) \\
&= \mathbb{P}(Q + M > f + w | Q \geq w) \mathbb{P}(Q \geq w) + \mathbb{P}(M > f) \mathbb{P}(Q < w) \\
&= \mathbb{P}(Q + M > f + w, Q \geq w) \\
&= \sum_{j=w}^f \sum_{k=f+w+1-j}^f \pi_j m_k,
\end{aligned} \tag{A.4}$$

and

$$\begin{aligned}
\mathbb{P}(\max(Q - w, 0) + M \leq f) &= \mathbb{P}(\max(Q - w, 0) + M \leq f | Q \geq w) \mathbb{P}(Q \geq w) \\
&\quad + \mathbb{P}(\max(Q - w, 0) + M \leq f | Q < w) \mathbb{P}(Q < w) \\
&= \mathbb{P}(Q + M \leq f + w | Q \geq w) \mathbb{P}(Q \geq w) + \mathbb{P}(M \leq f) \mathbb{P}(Q < w) \\
&= \mathbb{P}(Q + M \leq f + w, Q \geq w) + \mathbb{P}(M \leq f) \mathbb{P}(Q < w) \\
&= \sum_{j=w}^f \sum_{k=0}^{f+w-j} \pi_j m_k + \left(\sum_{j=0}^{w-1} \pi_j \right) \sum_{k=0}^f m_k,
\end{aligned} \tag{A.5}$$

then,

$$\begin{aligned}
&\mathbb{P}(\max(Q - w, 0) + M \leq f) E[Z^{\max(Q-w,0)+M} | \max(Q - w, 0) + M \leq f] = \\
&\sum_{j=w}^f \sum_{k=0}^{f+w-j} \pi_j z^{j-w} m_k z^k + \left(\sum_{j=0}^{w-1} \pi_j \right) \sum_{k=0}^f m_k z^k.
\end{aligned} \tag{A.6}$$

By substitution (A.4) and (A.6) in (A.3) we have,

$$\begin{aligned}
E[Z^{\min(\max(Q-w,0)+M,f)}] &= z^f \sum_{j=w}^f \sum_{k=f+w+1-j}^f \pi_j m_k \\
&+ \sum_{j=w}^f \sum_{k=0}^{f+w-j} \pi_j z^{j-w} m_k z^k + \left(\sum_{j=0}^{w-1} \pi_j \right) \sum_{k=0}^f m_k z^k \\
&= z^f \sum_{j=w}^f \sum_{k=f+w+1-j}^f \pi_j m_k \\
&+ z^{-w} (\hat{Q}(z) - \sum_{j=0}^{w-1} \pi_j z^j) \hat{M}(z) - z^{-w} \sum_{j=w}^f \pi_j z^j \sum_{k=f+w-j+1}^f m_k z^k \\
&+ \hat{M}(z) \sum_{j=0}^{w-1} \pi_j.
\end{aligned} \tag{A.7}$$

Finally, by substitution (A.2) and (A.7) in (A.1) we have,

$$\begin{aligned}
\hat{Q}(z) &= p \left(z^{-w} (\hat{Q}(z) - \sum_{j=0}^{w-1} \pi_j z^j) \hat{M}(z) - z^{-w} \sum_{j=w}^f \pi_j z^j \sum_{k=f+w-j+1}^f m_k z^k \right. \\
&+ \hat{M}(z) \sum_{j=0}^{w-1} \pi_j + z^f \sum_{j=w}^f \sum_{k=f+w+1-j}^f \pi_j m_k \left. \right) \\
&+ (1-p) \left(\hat{Q}(z) \hat{M}(z) - \sum_{j=0}^f \pi_j z^j \sum_{k=f-j+1}^f m_k z^k + z^f \sum_{j=0}^f \pi_j \sum_{k=f+1-j}^f m_k \right).
\end{aligned} \tag{A.8}$$

Then, after simple algebraic manipulations,

$$\begin{aligned}
\hat{Q}(z) \cdot \frac{z^w - \hat{M}(z)((1-p)z^w + p)}{z^w} &= p \left(\frac{\hat{M}(z)(z^w \sum_{j=0}^{w-1} \pi_j - \sum_{j=0}^{w-1} \pi_j z^j)}{z^w} \right. \\
&- z^{-w} \sum_{j=w}^f \pi_j z^j \sum_{k=f+w-j+1}^f m_k z^k + z^f \sum_{j=w}^f \sum_{k=f+w+1-j}^f \pi_j m_k \Big) \\
&+ (1-p) \left(- \sum_{j=0}^f \pi_j z^j \sum_{k=f-j+1}^f m_k z^k + z^f \sum_{j=0}^f \pi_j \sum_{k=f+1-j}^f m_k \right),
\end{aligned} \tag{A.9}$$

and we finally get (for Q_i),

$$\begin{aligned}
\hat{Q}_i(z) &= \frac{p_i \hat{M}_i(z) \sum_{j=0}^{w-1} \pi_i^{(j)} (z^w - z^j)}{z^w - \hat{M}_i(z)((1-p_i)z^w + p_i)} \\
&+ \frac{p_i z^{f_i+w} \sum_{j=w}^{f_i} \sum_{k=f_i+w-j+1}^{f_i} \pi_i^{(j)} m_i^{(k)}}{z^w - \hat{M}_i(z)((1-p_i)z^w + p_i)} - \frac{p_i \sum_{j=w}^{f_i} \sum_{k=f_i+w-j+1}^{f_i} \pi_i^{(j)} z^j m_i^{(k)} z^k}{z^w - \hat{M}_i(z)((1-p_i)z^w + p_i)} \\
&+ \frac{(1-p_i) z^w \left(z^{f_i} \sum_{j=0}^{f_i} \sum_{k=f_i-j+1}^{f_i} \pi_i^{(j)} m_i^{(k)} - \sum_{j=0}^{f_i} \sum_{k=f_i-j+1}^{f_i} \pi_i^{(j)} z^j m_i^{(k)} z^k \right)}{z^w - \hat{M}_i(z)((1-p_i)z^w + p_i)}.
\end{aligned} \tag{A.10}$$