

## Homework problem 7

### Equivalence of selecting “reference class” in multinomial logistic regression

In class we defined the logistic model as:

$$\begin{aligned}\frac{P(G = 1|X)}{P(G = K|X)} &= X^T \beta_1 \\ &\vdots \\ \frac{P(G = K - 1|X)}{P(G = K|X)} &= X^T \beta_{K-1},\end{aligned}$$

with resulting probabilities:

$$\begin{aligned}P(G = k|X) &= \frac{\exp\{X^T \beta_k\}}{1 + \sum_{l < K} \exp\{X^T \beta_l\}}, \quad k < K \\ P(G = K|X) &= \frac{1}{1 + \sum_{l < K} \exp\{X^T \beta_l\}}.\end{aligned}$$

Show that if we choose a different class in the denominator, we can obtain the same set of probabilities by a different set of linear models (i.e., values of  $\beta$ ). Hence the two representations are equivalent in the probabilities they yield.