

# Lumpy Setup Costs of Investment: The Lucas Paradox Revisited

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## Abstract

A model with setup costs of new investment is developed and applied for the reconciliation of the Lucas question as to why capital does not fly from rich to poor countries so as to equate wages and relieve the desire of labor to migrate from poor to rich countries.

JEL classification: E13, F2, F4, N00, O11

## 1 Introduction

In an influential paper, Lucas (1990) poses the question: “Why doesn’t capital flow from rich to poor countries?” With standard constant-returns-to-scale production functions, when the wage (per efficiency units of labor) is higher in a rich country than in a poor country, then the return to capital must be lower in the rich country than in the poor country. Therefore, the existence of “huge” wage gap between rich and poor countries must be associated with an opposite gap in the rates of return to capital. Given that labor is not allowed to freely migrate from poor to rich countries, it follows that capital would flow in the opposite direction, thereby equating the returns to capital and concomitantly

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wages too. The equalization of wages (indirectly through international capital flows) would eliminate the need to control migration. In practice, however, this is hardly the case. Even though barriers to international capital mobility are by and large being eliminated, the wage gap is still in force, and migration quotas from poor to rich countries have to be enforced.<sup>1</sup>

Recently, Maurice Obstfeld and Alan Taylor (2003) note: " A century ago, world income and productivity levels were far less divergent than they are today, so it is all the more remarkable that so much capital was directed to countries at or below the 20 percent and 40 percent income levels (relative to the United States). Today, a much larger fraction of the world's output and population is located in such low-productivity regions, but a smaller share of global foreign investment reaches them."

Lucas reconciled the paradox (in theory and with skillful calibration) by appealing to a human capital *externality* that generates a Hicks-neutral productivity advantage for rich countries over poor countries. Nevertheless, there is no hard evidence about the magnitude of the human capital externality, its link to productivity, and its capability of fully reconciling the paradox.

In this note we offer a complementary reconciliation of the paradox by appealing to lumpy setup costs of investment. The existence of such costs is supported by both micro data [see e.g. Caballero and Engel (1999, 2000)] and macro data [e.g. Razin, Rubinstein and Sadka (2004)]. We discuss also foreign direct investment (FDI), as a key channel of international capital flows, which is expected to be closely associated with international productivity differences; more closely than foreign portfolio investment.

With setup costs of investment, it does not pay a firm to make a "small" investment, even though such an investment is called for by marginal productivity conditions (that is, the standard first-order conditions for profit maximization). Put it differently, the investment decision is two-fold now: marginal productivity conditions determine how much to invest, whereas a "total profit" condition determines whether to invest at all.

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<sup>1</sup>Note also that despite the expansion of international trade in goods, still the Stolper-Samuelson (1941) factor price equalization theorem does not manage to eliminate the wage gap.

In such a framework, it is also possible to have a coexistence of equal rates of return to capital and a wage gap.<sup>2</sup>

## 2 The Lucas Paradox

The widespread pressure of migration from poor to rich countries is undoubtedly indicative of a higher marginal productivity of labor in rich relative to poor countries (over and above the attractiveness of the rich welfare states to migrants from poor countries). However, *ceteris paribus*, a relatively lower marginal product of labor is usually associated with a relatively high marginal product of capital. In the wake of globalized capital markets, capital should flow from rich to poor countries so as to mitigate these differentials in marginal productivity of capital, and also of labor, assuming constant-returns-to-scale and identical technologies (via globalization). This is the essence of the Lucas paradox.

Lucas (1990) employs a standard constant-returns-to-scale production function:

$$Y = AF(K, L), \tag{1}$$

where  $Y$  is output,  $K$  is capital and  $L$  is **effective** labor. The latter is used in order to allow for differences in the human capital content of labor between developed and developing countries. The parameter  $A$  is a productivity index which may reflect the average level of human capital in the country, external to the firm as in Lucas (1990). In addition, the parameter  $A$  may reflect the stock of public capital (roads and other infrastructure) that is external to the firm. In per effective-labor terms, we have:

$$y \equiv Y/L = AF(K/L, 1) \equiv Af(k). \tag{2}$$

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<sup>2</sup>The international trade literature often appeals to fixed costs. The latter play a very important role in determining the extent of trade-based foreign direct investment. References include Zhang and Markusen (1999), Carr, Markusen and Keith E. Maskus (2001) and Helpman, Melitz and Yeaple (2004). See also Acemoglu and Zilibotti (1997) for an interesting application of setup costs of investment to development issues.

The return to capital is:

$$r = Af'(k), \tag{3}$$

whereas the wage per effective unit of labor is:

$$w = A[f(k) - kf'(k)]. \tag{4}$$

Let a variable subscripted by "R" stand for a rich (source) country and a variable subscripted by "P" for a poor (host) country. The function  $f$  is common to all countries. Initially (before capital is freed to flow from one country to another),  $r_R^0 < r_P^0$ . But when capital can freely move from rich to poor countries, then rates of return are equalized, so that:

$$r_R = A_R f'(k_R) = A_P f'(k_P) = r_P. \tag{5}$$

Lucas explains the paradox by appealing to a human-capital externality. This externality makes  $A_R$  larger than  $A_P$ . Hence, it follows from equation (5) that  $k_R > k_P$  (because of a diminishing marginal product of capital). Employing equation (4), it follows that  $w_R > w_P$ . Moreover, Lucas was able to simulate the observed difference between  $k_R$  and  $k_P$  and between  $w_R$  and  $w_P$  by calibrating the difference between  $A_P$  and  $A_R$ .

Thus, at the calibrated equilibrium, workers can earn higher wages (per effective labor) in the rich country than in the poor country, and administrative means (migration quotas) are employed to impede the flow of labor from poor to rich countries. Yet there is no pressure on capital to flow in the opposite direction because rates-of-return on capital are already equalized.

In essence, the driving force in Lucas analysis of capital flows is cross-country differences in marginal productivity of capital. We supplement this *marginal* analysis with a *total* analysis.

### 3 Lumpy Adjustment Cost of Investment

We employ a “lumpy” adjustment cost for new investment, in the form of a fixed setup cost of investment. Consider again a pair of countries, "poor" (host) and "rich" (source), in a many-country world of free capital mobility, which fixes the world rate of interest, denoted by  $r$ . We will now describe the poor country, whose economic variables will be subscripted by " $P$ ". The description of the source country is similar with a subscript " $R$ ". Variables that are not subscripted are identical for the two countries. There is a single industry whose product serves both for consumption and investment. For simplicity, suppose that existing firms will last for two periods. In the first period there exists a continuum of  $N_P$  firms which differ from each other by a productivity index  $\varepsilon$ . We denote a firm which has a productivity index of  $\varepsilon$  by an  $\varepsilon$ -firm. The cumulative distribution function of  $\varepsilon$  is denoted by  $G(\cdot)$ , with a density function  $g(\cdot)$ .

We assume for simplicity that the initial net capital stock of each firm is the same and denote it by  $(1 - \delta)K_P^0$ . This consists of the net initial stock,  $K_P^0$ , of the preceding period, multiplied by one minus the depreciation rate,  $\delta$ . If an  $\varepsilon$ -firm invests  $I$  in the first period, it augments its capital stock to  $K = (1 - \delta)K_P^0 + I$  and its gross output in the second period will be  $A_P(K, L)(1 + \varepsilon)$ , where  $L$  is the labor input (in effective units). Naturally,  $\varepsilon \geq -1$  so that  $G(-1) = 0$ .

We assume that there exists a fixed setup cost of investment,  $C_P$ , which is the same for all firms (that is, independent of  $\varepsilon$ ). In order for the firm to be able to incur such a cost, we no longer assume that  $F$  exhibits constant returns to scale. We assume instead that, due to some (suppressed) fixed factor,  $F$  exhibits diminishing returns to scale in  $K$  and  $L$ ; that is  $F(\cdot)$  is strictly concave. Thus, the average cost curve is U-shaped which is consistent with perfect competition. Consider an  $\varepsilon$ -firm which does invest in the first period an amount  $I = K - (1 - \delta)K_P^0$  in order to augment its stock of capital to  $K$ . Its present value becomes:

$$V^+(A_P, C_P, K_P^0, \varepsilon, w_P) = \underset{(K,L)}{Max} \left\{ \frac{A_P F(K, L)(1 + \varepsilon) - w_P L + (1 - \delta)K}{1 + r} - [K - (1 - \delta)K_P^0 + C_P] \right\}. \quad (6)$$

The demands of such a firm for  $K$  and  $L$  are denoted by  $K^+(A_P, \varepsilon, w_P)$  and  $L^+(A_P, \varepsilon, w_P)$ . They are given by the marginal productivity conditions:

$$A_P F_K(K, L)(1 + \varepsilon) = r + \delta, \quad (7)$$

and

$$A_P F_L(K, L)(1 + \varepsilon) = w_P. \quad (8)$$

Note, however, that an  $\varepsilon$ -firm may choose not to invest at all [that is, to stick to its existing stock of capital  $(1 - \delta)K_P^0$ ] and avoid the setup lumpy cost  $C_P$ . In this case its labor input, denoted by  $L^-(A_P, K_P^0, \varepsilon, w_P)$  is defined by:

$$A_P F_L[(1 - \delta)K_P^0, L](1 + \varepsilon) = w_P. \quad (9)$$

Note that  $L^-$  depends on the initial stock of capital. Naturally, a firm with a low  $\varepsilon$  may not find it worthwhile to incur the setup cost  $C_P$ . In this case its present value is:

$$V^-(A_P, K_P^0, \varepsilon, w_P) = \underset{L}{Max} \left\{ \frac{A_P F[(1 - \delta)K_P^0, L](1 + \varepsilon) - w_P L + (1 - \delta)^2 K_P^0}{(1 + r)} \right\}. \quad (10)$$

Therefore, there exists a cutoff level of  $\varepsilon$ , denoted by  $\varepsilon_0$ , such that an  $\varepsilon$ -firm will make a new investment if and only if  $\varepsilon > \varepsilon_0$ . This cutoff level of  $\varepsilon$  depends on  $A_P, C_P, K_P^0$  and  $w_P$ . We write it as  $\varepsilon_0(A_P, C_P, K_P^0, w_P)$  and define it implicitly by:

$$\begin{aligned} & V^+(A_P, C_P, K_P^0, \varepsilon_0, w_P) \\ &= V^-(A_P, K_P^0, \varepsilon_0, w_P). \end{aligned} \quad (11)$$

Note that as the setup cost rises, fewer firms will chose to make new investments. That is  $\varepsilon_0(\cdot)$  is increasing in  $C_P$ . We continue to assume that labor is confined within national borders. Denoting the country's endowment of labor in effective units by  $\tilde{L}_P^0$ , we have the following labor market clearance equation:

$$\int_{-1}^{\varepsilon_0(A_P, C_P, K_P^0, w_P)} L^-(A_P, K_P^0, \varepsilon, w_P) g(\varepsilon) d\varepsilon + \int_{\varepsilon_0(A_P, C_P, K_P^0, w_P)}^{\infty} L^+(A_P, \varepsilon, w_P) g(\varepsilon) d\varepsilon = L_P^0, \quad (12)$$

where  $L_P^0 \equiv \tilde{L}_P^0 / N_P$  is the effective labor per firm.

Note that no similar market clearance equation is specified for capital, as we continue to assume that capital is freely mobile internationally and its rate of return is equalized internationally. The same description, with the subscript "R" replacing "P", holds for the rich (source) country.

Note that the differences in labor abundance between the two countries are manifested in the wage differences. To see this, suppose that the countries are identical, except that effective labor per firm is more abundant in the poor-host country than in the rich-source country, that is:  $L_P^0 > L_R^0$ . If wages were equal in the two countries, then effective labour demand per firm were equal and the market clearing condition [equation (12)] could not hold for both countries. Because of the diminishing marginal product of labor, it follows that the wage in the relatively labor-abundant country is lower than in the relatively labor-scarce country, that is:  $w_P < w_R$ .<sup>3, 4</sup> Thus, equal returns to capital (through capital mobility) coexist with unequal wages, as in Lucas (1990).

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<sup>3</sup>The equilibrium wage gap implies that the poor country employes more workers per firm than the rich country. Thus, even though the productivity distribution across firms is assumed equal, the rich country in equilibrium is effectively more productive.

<sup>4</sup>Evidently, income per capita is also lower in country relative to country P.

## 4 Extension: Dynamics and FDI

Consider now the possibility of an entrepreneur in one country establishing a new firm (that is, a greenfield investment where  $K^0 = 0$ ) in her own country or the other country. Suppose that the newcomer entrepreneur does not know in advance the productivity factor ( $\varepsilon$ ) of the potential firm. She therefore takes  $G(\cdot)$  as the cumulative probability distribution of the productivity factor of the new firm. The expected value of the new firm is therefore

$$V(A, C, w, r) \equiv \int_{-1}^{\infty} \text{Max}\{V^+(A, C, \varepsilon, w, r), 0\}g(\varepsilon)d\varepsilon. \quad (13)$$

Note when  $K^0$  is equal to zero, only the firms with an  $\varepsilon$  high enough to justify a greenfield investment have a positive value. This explains the max operator in equation (13).

Now, suppose that greenfield entrepreneurship is in limited supply and capacity. An entrepreneur in a rich-source country (and there is a limited number of them) may have to decide whether to establish a new firm at home or abroad (the poor-host country). Suppose that the source country entrepreneurs are endowed with some "intangible" capital, or know-how, stemming from their specialization or expertise in the industry at hand. We model this comparative advantage by assuming that the lumpy setup cost of investment in the poor country, when investment is done by the rich country entrepreneurs (FDI investors) is only  $C_P^*$  which is below  $C_P$ , the lumpy setup cost of investment when carried out by the host country direct investors. This means that the foreign direct investors can bid up the direct investors of the poor country in the purchase of the investing firms in the poor country. Her decision is determined by where  $V(\cdot)$ , as defined in equation (13), is higher. She will invest in the poor country rather than in the rich country if, and only if,

$$V(A_P, C_P^*, w_P) > V(A_R, C_R, w_R) \quad (14)$$



Naturally, the lower wage rate in the poor country is a pull factor for that country, that is, it works in the direction of satisfying condition (14). Thus, the lower wage rate in the host country attracts greenfield FDI. On the other hand, the total factor productivity in the rich country (namely,  $A_R$ ), is expected to be higher than its counterpart in the poor country (namely,  $A_P$ ) and this discourages FDI. Assuming that the wage differential dominates the total factor productivity differential, the poor-host country attracts greenfield FDI from the rich-source country.

Assuming that newcomer entrepreneurs evolve gradually over time and that technology spillover equates total factor productivity, eventually this process may end up with full factor price equalization. Naturally, the capital-labor ratios and  $L \equiv \tilde{L}/N$  are equalized in this long-run steady state. All this happens even though labor is not internationally mobile. The establishment of new firms in the global economy may be an engine for FDI flows by multinationals.

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