A Two Ion Fluid Model with Gyrotropic pressures for the Comet Interaction with the Solar-Wind

Zameret Gan-Baruch

Department of Geophysics and Planetary Sciences, Tel Aviv University 69978 Ramat Aviv, Israel (e-mail zameret@ganymede.tau.ac.il)and Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Str.1, 85740 Garching bei München, Germany

Rudolf Wegmann

Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Str.1, 85740 Garching bei München, Germany

Aharon Eviatar

Department of Geophysics and Planetary Sciences, Tel Aviv University 69978 Ramat Aviv, Israel

Hermann U. Schmidt

Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Str.1, 85740 Garching bei München, Germany

A two ion fluid model (with gyrotropic pressure tensor) for the interaction of cometary pickup plasma with the solar wind has been developed. Whereas previous models have, in general, assumed a single fluid plasma, this generalized model follows the evolution of two fluids - solar wind protons and cometary ions. The two fluids are coupled by the magnetic field and by coupling terms which are the velocity moments of the time relaxation model of Bhatnagar-Gross-Krook. Further more, a gyrotropic pressure tensor replaces the isotropic pressure assumed in other models. Isotropization terms couple the pressures in the directions perpendicular and parallel to the magnetic field. A Godunov type upstream method which uses an approximate Riemann solver adapted to the model equations, is presented.

Keywords: Space physics; Comets; Magnetohydrodynamics; Upwind differencing; Approximate Riemann solver

1 Introduction

The physics of the pickup ions is a long standing problem in space physics. Spacecraft encounters with comets provided us with measurements in the presence of an extended and relatively intense source of pickup ions. The understanding of the evolution of the pickup ions' velocity distribution has been further improved by kinetic models. The knowledge gained elucidated the nature of the local interaction but the effect on the global phenomena has not yet been elaborated. That motivated us to develop a detailed fluid model for the global interaction of cometary plasma with the solar wind [9], [10].

The atmosphere of a comet is created by evaporation of neutral molecules from the surface of its nucleus due to solar heating. The evaporating molecules expand away from the nucleus with their thermal velocity of about 1 km/s. They are ionized with a typical time scale of $\approx 10^6$ seconds. A new ion is injected into the solar wind with the velocity $\mathbf{V}_{inj} \equiv \mathbf{W}_n - \mathbf{V}_{sw}$ where $\mathbf{W}_n$ is the neutrals velocity, and $\mathbf{V}_{sw}$ is the solar wind velocity. The motion of the ionized particle across the interplanetary magnetic field carried by the solar wind generates an electric field which forces the particle to gyrate around the magnetic field lines.

In figure 1 the pickup process appears on a solar wind magnetic field aligned (SWB) frame. In this frame, the z axis is parallel to the magnetic field $\mathbf{B}$, the y axis is parallel to the electric field $\mathbf{E} = \mathbf{V}_{inj} \times \mathbf{B}$ at the moment of the particle's injection, and the x axis is in the direction of the cross product of the fields. The velocity of the solar wind in this frame is zero. Perpendicular to the magnetic field the ion assumes the solar wind velocity, but parallel to the field it retains its initial velocity. This causes a drift of the cometary ions relative to the solar wind plasma. The gyration of the new ion around the magnetic field creates a ring distribution in velocity space with a radius $V_{inj} \sin\theta$. Since the neutral velocity is very small in comparison to the solar wind velocity, θ can be considered as the angle between the solar wind direction and the magnetic field. The angle θ is also the pitch angle of the new ions because its tangent equals the ratio of the perpendicular to the parallel velocity. The ring distribution created is unstable to the ion cyclotron instability. Alfvén waves will be excited and will interact with the new ions.

Galeev et al. (1991,[8]) applied quasilinear theory to examine the ion cyclotron instability in the pickup process. They supposed that the ion distribution evolves to an asymptotic state $f = f_0 + f_1(\theta)$ where f_0 is the isotropic part and f_1 is the anisotropic part of the distribution function, which represents the injected ions at angle θ. Although $f_0 \gg f_1$ there is sufficient anisotropy to maintain wave growth as new particles are added at the ring position. As a result, the ions are scattered in pitch angle and form to a first approximation a shell distribution with radius V_{inj}. It can be seen in figure 1 that the bulk velocity of a uniformly filled ring is $\left(0,0,V_{inj\|}\right)$, and of a simple shell is $(0,0,0)$, which is also the solar wind velocity in the SWB frame. In other words the ring distribution of new cometary plasma lags behind the solar wind. The drift velocity of the cometary plasma, relative to the solar wind is along the magnetic field, and equals the parallel component of the injection velocity in the SWB frame. The simple shell is only a first approximation while the actual distribution created is a bispherical shell. The Alfvén waves which are generated by the instabilities move mostly upstream along the field but also a small fraction moves downstream. The conservation of energy of waves and particles requires that the particles' velocities will appear in the velocity space as two spherical shells which are centered along the field at $\pm V_A$. Since pickup of new cometary particles proceeds, the ring distribution is recreated. The asymptotic distribution function is, therefore, a bispherical shell combined with a ring at the injection angle. This non spherically symmetric asymptotic distribution results in anisotropic pressure. The pitch angle scattering, and shell formation is not the last step of the evolution of the particles' distribution. Velocity diffusion as well as other energization mechanisms would cause broadening of the shell.

A. Coates (1991,[5]) analysed Giotto measurements of the velocity distribution of the water group ion distribution. His results confirm that while approaching the bow-shock the pickup ions distribution evolves from a ring to a shell distribution which then broadens in energy. The three phases of the pickup process, i.e. the formation of a ring distribution, the scattering to a bispherical shell and its approximate "thermalization" were obtained also by kinetic and hybrid simulations ([23],[15],[11]). Miller et al. (1991,[15]), showed that for injection angles larger than $40°$ the velocity distributions are bispherical because of the propagation of waves along and against the beam direction while for smaller angles, the distribution is monospherical.

The models yield the local evolution of the plasma in the aforementioned pickup process. Because of computer time and memory limitations, it is still not possible to develop a global model of the interaction by means of kinetic or hybrid simulations. This motivated us to develop a global fluid model which is a step beyond the Magneto-Hydro-Dynamics (MHD) for being able to incorporate more of the macroscopic effects that originate from the kinetic processes, that is the pressure anisotropies and the two fluid interaction of cometary plasma with the solar wind. The generalizations introduced alter the large scale phenomena significantly [10].

2 The Generalized Formulation of the Interaction

The time scales for the evolution of the particles' distribution function are the main factor to be considered when examining which of the kinetic effects will relax within the fluid time scales and which will result in macroscopic effects that will have to be taken into account in the fluid simulations. The fluid time scales are of the order of the characteristic distance divided by the characteristic propagation speeds. In the case of a comet, taking L_{mhd} to be $10^5 - 10^6$km, and $V_{mhd} = V_{Alfven}$ with $B = 5\gamma$ and $\rho = 5\mathrm{m_p}\ \mathrm{cm}^{-3}$ yields $\tau_{mhd} \approx 10^3 - 10^4$s. The kinetic time scales are the photoionization time τ_i, the pickup time τ_p which is the time needed for the creation of a ring distribution in the phase space by the cyclotron motion, pitch angle scattering time τ_s for creation of a shell, and energy diffusion time τ_d for a filled shell creation. By means of hybrid simulations, Gaffey et al. (1988,[7]), obtained time scales for the evolution of the distribution function of the pickup ions to a ring, to a shell, and finally to a filled shell for H_2O^+ in the vicinity of Halley's comet. Far upstream τ_p was 150 sec, τ_s was 260 sec, and τ_d was 1400 sec. Near the bow-shock τ_p was again 150 sec, τ_s was 153 sec, and τ_d was 142 sec. Coates et al. (1989,[4]) derived from the measurements of Giotto at Halley an upper limit of 10,250 seconds for the pitch angle scattering in the far upstream region, and energy scattering time ranging from 10^4 seconds downstream to 10^5 seconds far upstream. Their results indicate slower evolution than indicated by theory. The time scales above, lead to the conclusion that in the presence of a continuous source of pickup ions, the isotropization and the thermalization of the pickup plasma would be of the same order as the MHD time scale. The MHD approximation is, therefore, inaccurate. The pickup definitely destroys the isotropy of the pressure and might lead to non-vanishing higher moments of the distribution function. Isotropic pressure is not sufficient to describe the comet - solar wind interaction. Furthermore, the relatively slow scattering of the ring distribution to a shell distribution which moves with the solar wind velocity, and the slow energy diffusion time, imply that the one fluid approximation of MHD should be replaced by two coupled fluids, each with its own velocity field. The fast pickup time which is also the ion cyclotron time implies that in the direction perpendicular to the magnetic field there is an immediate assimilation of the velocities of the two fluids. The generalized formulation of the interaction presented here incorporates the two effects of two interacting fluids, and anisotropic pressure. Raeder 1990, [17] incorporated the two effects in his model which was solved by a hybrid numerical method called Particle In Cell (PIC). Wegmann 1995, [22] incorporated the anisotropic pressure in his one fluid model.

The first to derive transport equations in a dilute plasma without interparticle collisions for a distribution function of higher order than the Maxwell distribution function were Chew, Goldberger and Low (CGL), (1956,[2]). In order to truncate the hierarchy of the transport equations, CGL assumed that all moments higher than the thermal pressure tensor vanish. The neglect of the heat flux vector is controversial for solar system plasmas. Kulsrud (1983,[13]) pointed out that one of the consequences of the neglect of the heat flux is the inconsistency of the condition for the excitation of the mirror instability ([19,3,20]). The condition derived from the guiding center formalism,

$$P_\perp > P_\parallel + \frac{P_\parallel}{P_\perp}\frac{B^2}{2\mu}$$

requires much lower pressure anisotropy with perpendicular pressure larger than the parallel pressure as a sufficient potential for the instability growth than the condition derived from the double adiabatic formalism,

$$P_\perp > 6\left(P_\parallel + \frac{P_\parallel}{P_\perp}\frac{B^2}{2\mu}\right)$$

The Firehose instability [19,16] requires that the parallel pressure be larger than the perpendicular pressure. An identical criterion for its excitation,

$$P_\parallel > P_\perp + \frac{B^2}{\mu}$$

is obtained from both formalisms [13] since parallel heat flow plays no role. The extent of the above inconsistency is lowered in this model by the use of properly adjusted isotropization terms which limit the anisotropies.

In the present phase of the model development, the velocity moments of the Bhatnagar-Gross-Krook (BGK) approximation (1954,[1]) for the collision term

$$\left(\frac{\partial f}{\partial t}\right)_{coll.} = -\frac{1}{\tau(v)}\left(f - f_M\right)$$

where f_M is the Maxwell distribution function, and $\tau(v)$ is the relaxation time of the distribution function to the equilibrium distribution function, are used as pressure isotropization terms and as coupling terms of the two fluids in the direction parallel to the magnetic field. The BGK model is only a first approximation for the collision terms. Furthermore, the dependence of $\tau(v)$ on the particles' velocity cannot be included in fluid simulations. In the present state of the code development, the relaxation times taken are constant, and equal for all velocity moments. However, one must keep in mind that the velocity moments of the collision term yield different relaxation times for its different moments [14], and that the wave turbulence which by wave-particle interactions isotropizes the fluids and couples them, depends on time and distance from the comet. Information from measurements [4], and kinetic, or hybrid simulations [7] can yield estimates for the relaxation times. The phenomenological isotropization and coupling terms, in their present form, use these estimates.

From the discussion above we can draw two conclusions. First, the pickup process leads to anisotropic pressure. The pressure is a tensor quantity not necessarily gyrotropic. Still it is a step forward to assume in a pickup environment, a gyrotropic instead of an isotropic pressure. Hence in our model the double adiabatic energy equations of Chew-Goldberger-Low modified by isotropization terms replace the adiabatic energy equation. Second, the interaction of the cometary plasma with the solar wind plasma is a two fluid phenomena. The two fluids adjust their velocities in the direction perpendicular to the magnetic field, however, in the parallel direction their coupling is slow. The third fluid, the electron component, is assumed cold in this model and therefore degenerate. It has no dynamic effect. The equations which satisfy the requirements of this model are presented in the next section.

3 The Model Equations

The one fluid approach of MHD sums over all ion species, and electrons in order to get the total plasma variables. In this model the solar wind plasma, and the cometary plasma, are treated separately as two coupled fluids. It is not possible to consider each of these fluids in the one fluid approach, and sum over its ions and electrons for the total fluid variables, because the electron population cannot be partitioned and related to each of the ion populations. Hence, the equations derived are the ion equations for the solar wind plasma, and the ion equations for the cometary plasma.

3.1 Derivation of the model equations

The different components of the plasma will be indicated by the following indices:

(i) cometary ions (one species) - ci.
(ii) solar wind ions (one species) - si.
(iii) electrons - e.

The electrons which cannot be distinguished according to their origin, are indicated by one index.

The macroscopic variables of the plasma are:

The species mass density:

$$\rho_{si} = n_{si} m_{si}$$

$$\rho_{ci} = n_{ci} m_{ci}$$

$$\rho_{e} = n_{e} m_{e}$$

where m_{si} and m_{ci} are the average molecular masses of the solar wind ions, and of the cometary ions respectively, and m_e is the electron mass.

The species velocity:

$$\mathbf{U_{si}}, \quad \mathbf{U_{ci}}, \quad \mathbf{U_{e}}$$

The total current density:

$$\mathbf{J} = n_{si} e \mathbf{U_{si}} + n_{ci} e \mathbf{U_{ci}} - n_e e \mathbf{U_{e}}$$

The species pressure dyads:

$$\hat{\mathbf{P}}_{\mathbf{si}}, \quad \hat{\mathbf{P}}_{\mathbf{ci}}, \quad \hat{\mathbf{P}}_{\mathbf{e}}$$

where the gyrotropic non viscous pressure dyad is

$$\hat{\mathbf{P}} = P_{\perp} \hat{\mathbf{I}} + (P_{\|} - P_{\perp}) \mathbf{bb}$$

Here $P_{\|}$ is the parallel pressure and $P_{\perp}$ is the perpendicular pressure with respect to the magnetic field. Also $\hat{\mathbf{I}}$ is the unit dyad, and $\mathbf{bb}$ is the dyad formed from the unit vector $\mathbf{b} = \frac{\mathbf{B}}{|B|}$ where $\mathbf{B}$ is the magnetic field vector.

To derive the model equations without source terms, the transport equations of mass, momentum, and the double adiabatic energy equations of CGL are applied to the solar wind ions with q=si, and to the cometary ions with q=ci (equ. (3.1)-(3.4)). For the electrons, which are assumed a cold component, a momentum equation is applied (equ. (3.5)). In addition, Faraday's magnetic induction law (equ. (3.6)) after neglect of the displacement current, and Ampère's law (equ. (3.7)) for the current density are used. Source terms and coupling terms for the transport equations are presented in §3.2 as well as isotropization terms which modify the double adiabatic energy equations.

$$\frac{\partial \rho_q}{\partial t} + \nabla \cdot \rho_q \mathbf{U}_q = 0 \tag{3.1}$$

$$\rho_q \left(\frac{\partial \mathbf{U}_q}{\partial t} + \mathbf{U}_q \cdot \nabla \mathbf{U}_q \right) - e n_q \left(\mathbf{E} + \mathbf{U}_q \times \mathbf{B} \right) + \nabla \cdot \hat{\mathbf{P}}_q = 0 \tag{3.2}$$

$$\frac{\partial \left(P_{\perp q} \right)}{\partial t} + \mathbf{U}_q \cdot \nabla (P_{\perp q}) + 2 P_{\perp q} \nabla \cdot \mathbf{U}_q - \left(P_{\perp q} \mathbf{bb} \cdot \nabla \right) \cdot \mathbf{U}_q = 0 \tag{3.3}$$

$$\frac{\partial \left(\frac{P_{\parallel q}}{2} \right)}{\partial t} + \mathbf{U}_q \cdot \nabla \left(\frac{P_{\parallel q}}{2} \right) + \left(\frac{P_{\parallel q}}{2} \right) \nabla \cdot \mathbf{U}_q + \left(P_{\parallel q} \mathbf{bb} \cdot \nabla \right) \cdot \mathbf{U}_q = 0 \tag{3.4}$$

$$\rho_e \left(\frac{\partial \mathbf{U_e}}{\partial t} + \mathbf{U_e} \cdot \nabla \mathbf{U_e} \right) + e n_e \left(\mathbf{E} + \mathbf{U_e} \times \mathbf{B} \right) + \nabla \cdot \hat{\mathbf{P}}_\mathbf{e} = 0 \tag{3.5}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \tag{3.6}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \tag{3.7}$$

The assumptions made for the electron momentum equation (3.5) are:

The electron inertia vanishes,

$$\rho_e \frac{d\mathbf{U_e}}{dt} = \rho_e \left(\frac{\partial \mathbf{U_e}}{\partial t} + \mathbf{U_e} \cdot \nabla \mathbf{U_e} \right) = 0$$

A second assumption

$$\nabla \cdot \hat{\mathbf{P}}_\mathbf{e} = 0$$

which is more crude, considers the electrons as a cold negative background which does not influence the dynamics but is required for charge neutrality. The relation obtained for the electric field under the two assumptions is:

$$\mathbf{E} = -\mathbf{U_e} \times \mathbf{B} \tag{3.8}$$

The total current density, defined above, can be substituted into equation (3.7), together with the quasi neutrality condition for the total plasma

$$n_e = n_{si} + n_{ci}$$

From the relation obtained

$$\nabla \times \mathbf{B} = \mu_0 \; e \left(n_{si} \mathbf{U_{si}} + n_{ci} \mathbf{U_{ci}} - (n_{si} + n_{ci}) \mathbf{U_e} \right)$$

the electron velocity

$$\mathbf{U_e} = \frac{n_{si} \mathbf{U_{si}} + n_{ci} \mathbf{U_{ci}}}{n_{si} + n_{ci}} - \frac{\nabla \times \mathbf{B}}{\mu_0 e (n_{si} + n_{ci})} \tag{3.9}$$

is extracted. It is then substituted into equation (3.8) for the electric field, which is substituted into the ion momentum equations (3.2). The resulting equations are

$$\rho_{si} \left(\frac{\partial \mathbf{U_{si}}}{\partial t} + \mathbf{U_{si}} \cdot \nabla \mathbf{U_{si}} \right) - e \frac{n_{si} n_{ci}}{n_{si} + n_{ci}} \left(\mathbf{U}_{si} - \mathbf{U}_{ci} \right) \times \mathbf{B}$$

$$- \frac{n_{si}}{\mu_0 (n_{si} + n_{ci})} (\nabla \times \mathbf{B}) \times \mathbf{B} + \nabla \cdot \hat{\mathbf{P}}_{\mathbf{si}} = 0 \tag{3.10}$$

$$\rho_{ci} \left(\frac{\partial \mathbf{U_{ci}}}{\partial t} + \mathbf{U_{ci}} \cdot \nabla \mathbf{U_{ci}} \right) - e \frac{n_{si} n_{ci}}{n_{si} + n_{ci}} \left(\mathbf{U}_{ci} - \mathbf{U}_{si} \right) \times \mathbf{B}$$

$$- \frac{n_{ci}}{\mu_0 (n_{si} + n_{ci})} (\nabla \times \mathbf{B}) \times \mathbf{B} + \nabla \cdot \hat{\mathbf{P}}_{\mathbf{ci}} = 0 \tag{3.11}$$

In the second term in the momentum equations there is a cross product of a velocity difference of solar wind and cometary ions with the magnetic field. Since this difference is cross multiplied with the magnetic field, its component parallel to the field does not contribute. The velocities' difference perpendicular to the magnetic field gives a small contribution within one cyclotron period which is short relative to the time scales possible in fluid simulations. This consideration justifies the neglect of the second term in the momentum equations.

The electric field in Faraday induction law, (3.6), is also replaced by relation (3.8) with the electron velocity extracted from the total current. The equation derived is known as the Magnetic flux equation.

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\frac{n_{si} \mathbf{U_{si}} + n_{ci} \mathbf{U_{ci}}}{n_{si} + n_{ci}} \times \mathbf{B} - \frac{1}{(n_{si} + n_{ci}) e} \mathbf{J} \times \mathbf{B} \right) \tag{3.12}$$

The second term on the right hand side of equation (3.12) is the Hall term which relative to the other terms can be neglected for the long time scales considered.

The model equations ((3.13)-(3.21)) with source, coupling, and isotropization terms (presented in detail in §3.2) are:

$$\frac{\partial \rho_{si}}{\partial t} + \nabla \cdot \rho_{si} \mathbf{U_{si}} = S_{si1} \tag{3.13}$$

$$\frac{\partial \rho_{si} \mathbf{U_{si}}}{\partial t} + \nabla \cdot (\rho_{si} \mathbf{U_{si}} \mathbf{U_{si}}) - \frac{n_{si}}{\mu_0 (n_{si} + n_{ci})} (\nabla \times \mathbf{B}) \times \mathbf{B} + \nabla \cdot \hat{\mathbf{P}}_{\mathbf{si}} = \mathbf{S_{si2}} \tag{3.14}$$

$$\frac{\partial \left(P_{\perp si}\right)}{\partial t} + \mathbf{U_{si}} \cdot \nabla (P_{\perp si}) + 2 P_{\perp si} \nabla \cdot \mathbf{U_{si}} - (P_{\perp si} \mathbf{bb} \cdot \nabla) \cdot \mathbf{U_{si}} = S^{\perp}{}_{si3} \tag{3.15}$$

$$\frac{\partial \left(P_{\parallel si}\right)}{\partial t} + \mathbf{U_{si}} \cdot \nabla \left(P_{\parallel si}\right) + \left(P_{\parallel si}\right) \nabla \cdot \mathbf{U_{si}} + \left(2 P_{\parallel si} \mathbf{bb} \cdot \nabla\right) \cdot \mathbf{U_{si}} = S^{\parallel}{}_{si3} \tag{3.16}$$

$$\frac{\partial \rho_{ci}}{\partial t} + \nabla \cdot \rho_{ci} \mathbf{U_{ci}} = S_{ci1} \tag{3.17}$$

$$\frac{\partial \rho_{ci} \mathbf{U_{ci}}}{\partial t} + \nabla \cdot (\rho_{ci} \mathbf{U_{ci}} \mathbf{U_{ci}}) - \frac{n_{ci}}{\mu_0 (n_{si} + n_{ci})} (\nabla \times \mathbf{B}) \times \mathbf{B} + \nabla \cdot \hat{\mathbf{P}}_{\mathbf{ci}} = \mathbf{S_{ci2}} \tag{3.18}$$

$$\frac{\partial \left(P_{\perp ci}\right)}{\partial t} + \mathbf{U_{ci}} \cdot \nabla (P_{\perp ci}) + 2 P_{\perp ci} \nabla \cdot \mathbf{U_{ci}} - (P_{\perp ci} \mathbf{bb} \cdot \nabla) \cdot \mathbf{U_{ci}} = S^{\perp}{}_{ci3} \tag{3.19}$$

$$\frac{\partial \left(P_{\parallel ci}\right)}{\partial t} + \mathbf{U_{ci}} \cdot \nabla \left(P_{\parallel ci}\right) + \left(P_{\parallel ci}\right) \nabla \cdot \mathbf{U_{ci}} + \left(2 P_{\parallel ci} \mathbf{bb} \cdot \nabla\right) \cdot \mathbf{U_{ci}} = S^{\parallel}{}_{ci3} \tag{3.20}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\frac{n_{si} \mathbf{U_{si}} + n_{ci} \mathbf{U_{ci}}}{n_{si} + n_{ci}} \times \mathbf{B} \right) \tag{3.21}$$

3.2 Source terms, Coupling terms, and Isotropization terms

a. *Source terms*

The comet is an extended source of neutral particles which are ionized and then picked up by the solar wind. The ionization mechanisms are charge exchange with the solar wind, photoionization by solar radiation, and electron impact ionization. In this model, the chemical processes will not be treated, and only photoionization will be considered.

The number density production rate at a distance R from the nucleus under the assumption of spherical symmetry, and neutral velocity W_n, lifetime against ionization τ_i, and total production rate of cometary neutrals, $G(s^{-1})$, is

$$A = \frac{G}{\tau_i 4\pi R^2 W_n} \exp\left(-\frac{R}{\tau_i W_n}\right) \tag{3.22}$$

This model describes the R^{-2} dependence of the density in spherical symmetry modified by exponential exhaustion of the neutrals because of ionization. In this two fluid model, ionization of cometary neutrals gives the source for the transport equations of the cometary ions.

b. *Perpendicular velocity assimilation*

The acceleration of a new ion during the pickup process can be well described by kinetic models, in which the cyclotron motion can maintain the drift velocity perpendicular to the magnetic field equal for both types of ions. In a two fluid model, the assimilation of the perpendicular velocities of the two fluids can be maintained by the following averaging procedure:

(a) The bulk velocity of the total plasma is calculated,

$$< \mathbf{U} > = \frac{\rho_{si}\mathbf{U_{si}} + \rho_{ci}\mathbf{U_{ci}}}{\rho_{si} + \rho_{ci}}$$

(b) The perpendicular component of the bulk velocity $< \mathbf{U} >$ replaces the perpendicular component of the velocity of each of the fluids,

$$< \mathbf{U} >_{\|} = (< \mathbf{U} > \cdot \mathbf{B})\frac{\mathbf{B}}{B^2}$$

$$< \mathbf{U} >_{\perp} = < \mathbf{U} > - < \mathbf{U} >_{\|}$$

$$\mathbf{U_{si}}^{new} = \mathbf{U}_{\| si} + < \mathbf{U} >_{\perp}$$

$$\mathbf{U_{ci}}^{new} = \mathbf{U}_{\| ci} + < \mathbf{U} >_{\perp}$$

(c) The change in the kinetic energy because of the momentum averaging in the perpendicular direction transforms to perpendicular thermal energy density.

$$P_{\perp}{}_{si}^{new} = P_{\perp si} + \frac{1}{2}\rho_{si}(\mathbf{U}_{\perp si} - < \mathbf{U} >_{\perp})^2$$

$$P_{\perp}{}_{ci}^{new} = P_{\perp ci} + \frac{1}{2}\rho_{ci}(\mathbf{U}_{\perp ci} - < \mathbf{U} >_{\perp})^2$$

In the numerical calculation this procedure is applied in the end of each time step. $\mathbf{U}_q$ and $P_{\perp q}$ are replaced by $\mathbf{U}_q{}^{new}$ and $P_{\perp}{}_q^{new}$ such that the constraint $\mathbf{U}_{\perp si} = \mathbf{U}_{\perp ci}$ is enforced and momentum and energy are conserved.

c. *Parallel velocity coupling*

The parallel velocity components of the two fluids are coupled by relaxation-time model collision terms. This phenomenological coupling has its origin in the kinetic process that scatters the bimodal distribution of cometary plasma and solar-wind to a unimodal distribution.

d. *Isotropization terms*
The parametrized isotropization terms are also based on a relaxation-time model. They should describe the kinetic process of the pitch angle scattering. The extent to which the anisotropy grows is limited by the fitted isotropization rates.

Below, the full source terms, coupling terms, and isotropization terms of this model are presented. The relaxation rates which were discussed in §2 are designated $\nu_{\|}$ for the parallel velocity relaxation rate, and $\nu_{isot.}$ for the isotropization rate.

$$S_{si1} = 0$$

$$\mathbf{S_{si2}} = -\nu_{\|}\rho_{si}(\mathbf{U_{si}} - <\mathbf{U}>)_{\|}$$

$$S^{\perp}{}_{si3} = \frac{1}{3}\nu_{isot.}(P_{\| si} - P_{\perp si})$$

$$S^{\|}{}_{si3} = 2\nu_{\|}\rho_{si}(\mathbf{U_{si}} - <\mathbf{U}>)^2_{\|} + \frac{2}{3}\nu_{isot.}(P_{\perp si} - P_{\| si})$$

$$S_{ci1} = Am_{ci}$$

$$\mathbf{S_{ci2}} = Am_{ci}\mathbf{W_n} - \nu_{\|}\rho_{ci}(\mathbf{U_{ci}} - <\mathbf{U}>)_{\|}$$

$$S^{\perp}{}_{ci3} = \frac{1}{2}Am_{ci}\left((\mathbf{W_n} - \mathbf{U_{ci}})^2 - (\mathbf{W_n} - \mathbf{U_{ci}})^2_{\|}\right) + \frac{1}{3}\nu_{isot.}(P_{\| ci} - P_{\perp ci})$$

$$S^{\|}{}_{ci3} = Am_{ci}(\mathbf{W_n} - \mathbf{U_{ci}})^2_{\|} + 2\nu_{\|}\rho_{ci}(\mathbf{U_{ci}} - <\mathbf{U}>)^2_{\|} + \frac{2}{3}\nu_{isot.}(P_{\perp ci} - P_{\| ci})$$

4 The numerical method

The numerical method used is a Godunov type upwind difference scheme [12]. It uses an approximate Riemann solver which is based on the linearized equations. The method was chosen because of its good stability properties which make it suitable for treating discontinuities and shocks. The method has been applied for the MHD equations by [21], and for the single fluid anisotropic plasma in [22].

The model equations (3.13) - (3.21) in one space variable, x, have the form

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{U})}{\partial x} + \mathbf{M}(\mathbf{U})\frac{\partial \mathbf{U}}{\partial x} = 0 \tag{4.1}$$

where $\mathbf{U}$ is the state variable, or the conserved quantity, and $\mathbf{F}$ is the flux which transports the conserved quantity. Equation (4.1) has a conservative term $\frac{\partial \mathbf{F}}{\partial x}$ and a non-conservative term $\mathbf{M}\frac{\partial \mathbf{U}}{\partial x}$.

The quasilinear form of the system is,

$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{C_x}\frac{\partial \mathbf{U}}{\partial x} = 0 \tag{4.2}$$

with the coefficients matrix

$$\mathbf{C_x} = \mathbf{C_x}(\mathbf{U}) = \frac{\partial \mathbf{F}(\mathbf{U})}{\partial \mathbf{U}} + \mathbf{M}(\mathbf{U})$$

where

$$\left(\frac{\partial \mathbf{F}(\mathbf{U})}{\partial \mathbf{U}}\right)_{\mathrm{i,j}} = \frac{\partial F_{\mathrm{i}}}{\partial U_{\mathrm{j}}}$$

is the Jacobian of transformation from the states to the conservative fluxes. Assuming that $\mathbf{C_x}$ is a locally constant matrix designated $\bar{\mathbf{C}}_\mathbf{x}$, (4.2) becomes a linear system of equations,

$$\frac{\partial \bar{\mathbf{U}}}{\partial t} + \bar{\mathbf{C}}_\mathbf{x}\frac{\partial \bar{\mathbf{U}}}{\partial x} = 0 \tag{4.3}$$

for the state variable $\bar{\mathbf{U}}$. It can be integrated to obtain the Rankine-Hugoniot jump condition

$$\bar{\mathbf{C}}_\mathbf{x} \cdot [\bar{\mathbf{U}}] = \bar{\lambda}[\bar{\mathbf{U}}]$$

according to which the flux difference on both sides of a discontinuity equals the states difference multiplied by the speed of the discontinuity designated $\bar{\lambda}$. The equation obtained is a characteristic equation. The jump $[\bar{\mathbf{U}}]$, therefore, must be an eigenvector of the matrix $\bar{\mathbf{C}}_\mathbf{x}$, and the speed $\bar{\lambda}$ is its eigenvalue. The Riemann problem for equation (4.3) can be solved explicitly in terms of the eigensystem of the matrix $\bar{\mathbf{C}}_\mathbf{x}$. This solution is an approximate solution of the Riemann problem for the original equation (4.1) and it is a building block for a Godunov-type difference method. In a numerical simulation, each two adjacent calculation cells can be treated as a left state $\mathbf{U_l}$, and a right state $\mathbf{U_r}$, for which an approximate Riemann problem is solved in the following way. With the matrix

$$\bar{\mathbf{C}}_\mathbf{x} = \mathbf{C_x}(\frac{\mathbf{U_l} + \mathbf{U_r}}{2})$$

the coefficients c_p of the expansion of the jump

$$[\bar{\mathbf{U}}] = \mathbf{U_r} - \mathbf{U_l} = \sum_{p=1}^{n_p} \bar{c}_p \bar{\mathbf{r}}_p$$

in terms of the right eigenvectors $\bar{\mathbf{r}}_p$ of $\bar{\mathbf{C}}_\mathbf{x}$ for the eigenvalues $\bar{\lambda}_p$ can be calculated. Then, the flux across the surface of the discontinuity can be taken as

$$\mathbf{F_s} = \frac{\mathbf{F}(\mathbf{U_l}) + \mathbf{F}(\mathbf{U_r})}{2} - \frac{1}{2}\sum_{p=1}^{n_p} |\bar{\lambda}_p| \bar{c}_p \bar{\mathbf{r}}_p$$

where the term added on the right hand side of the above equation is called upstream (upwind) correction [18]. If not all of the eigenvalues of $\bar{\mathbf{C}}_\mathbf{x}$ are real, then the upstream correction must be modified to

$$\frac{1}{2}\sum_{p=1}^{n_p} |Re\{\bar{\lambda}_p\}| Re\{\bar{c}_p \bar{\mathbf{r}}_p\}$$

This scheme allows the evolution of physical instabilities associated with imaginary eigenvalues. Wegmann (1997,[22]) performed a one-dimensional test calculation which demonstrates that the modified upstream correction works also in the presence of physical instabilities. The modified upstream correction is used in this scheme because the model equations are hyperbolic, i.e. the eigensystem is real, only in the range of parameters where instabilities like the Mirror instability, and the Firehose instability have not been developed. When pressure anisotropies obey the conditions (presented in §2) which are necessary for the growth of these two physical instabilities, the system becomes elliptic, and the eigensystem is complex. The effect of these instabilities is to isotropize the pressure. Since the resulting isotropization is less than obtained in reality by micro instabilities, phenomenological isotropization terms which limit the anisotropies, and therefore stabilize the flow were included in the model equations.

With the fluxes calculated on interfaces which separate the cells, the state vector, assumed constant in each cell, can be advanced to its value in the next time step,

$$\mathbf{U_i}^{n+1} = \mathbf{U_i}^n - \frac{\Delta t_n}{\Delta x}\Big(\mathbf{F_s}^n_{i+\frac{1}{2}} - \mathbf{F_s}^n_{i-\frac{1}{2}} + \mathbf{M}(\mathbf{U}_i{}^n)\frac{\mathbf{U}^n_{i+1} - \mathbf{U}^n_{i-1}}{2}\Big)$$

In this formula, $\mathbf{U}_i^n$ are the values of $\mathbf{U}$ at cell centers $x_i = i\Delta x, i = 1, \cdots, N_x$ at time $t_n = \sum_{\nu=1}^{n-1} \Delta t_\nu$ where Δt_ν is the time step, and Δx is the distance between two grid points. The grid points are the location of interfaces which separate calculation cells. The discretization of the conservative part of the equations is done in terms of the fluxes $\mathbf{F}$ as is usual for the conservative equations while the non-conservative part of the equations is added in form of central differences of the state variable multiplied by the matrix $\mathbf{M}$ evaluated at the state $\mathbf{U}_i^n$.

The maximal time step Δt_n is limited by Courant, Friedrichs, Lewy (CFL) limit, [6]

$$\Delta t_n = \frac{\Delta x}{\max_p |\lambda_p|}$$

which ensures that the time-space domain of the discretized calculation is within the true domain of the equations. Extension of (4.1) to three dimensions yields

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{U})}{\partial x} + \frac{\partial \mathbf{G}(\mathbf{U})}{\partial y} + \frac{\partial \mathbf{H}(\mathbf{U})}{\partial z} + \mathbf{M}_x(\mathbf{U})\frac{\partial \mathbf{U}}{\partial x} + \mathbf{M}_y(\mathbf{U})\frac{\partial \mathbf{U}}{\partial y} + \mathbf{M}_z(\mathbf{U})\frac{\partial \mathbf{U}}{\partial z} = 0$$

where $\mathbf{U}$ is again the state vector, $\mathbf{F}$, $\mathbf{G}$, $\mathbf{H}$ are the conservative flux vectors and $\mathbf{M}_x$, $\mathbf{M}_y$, and $\mathbf{M}_z$ are the coefficients matrices of the non-conservative terms, in the directions x, y, and z, respectively. This can be brought into the quasi-linear form

$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{C_x}\frac{\partial \mathbf{U}}{\partial x} + \mathbf{C_y}\frac{\partial \mathbf{U}}{\partial y} + \mathbf{C_z}\frac{\partial \mathbf{U}}{\partial z} = 0$$

where

$$\mathbf{C_x} = \frac{\partial \mathbf{F}}{\partial \mathbf{U}} + \mathbf{M_x}$$

$$\mathbf{C_y} = \frac{\partial \mathbf{G}}{\partial \mathbf{U}} + \mathbf{M_y}$$

$$\mathbf{C_z} = \frac{\partial \mathbf{H}}{\partial \mathbf{U}} + \mathbf{M_z}$$

In the three dimensional problem, the procedure above should be repeated for each of the coefficients matrices $\mathbf{C_x}$, $\mathbf{C_y}$, and $\mathbf{C_z}$ to get the upstream correction for the fluxes through each of the interfaces which separate calculation cells.

The discretization of the three dimensional problem is

$$
\begin{aligned}
\mathbf{U}^{n+1}_{i,j,k} = \mathbf{U}^{n}_{i,j,k} \\
&- \frac{\Delta t_n}{\Delta x}\left(\mathbf{F_s}{}^n_{i+\frac{1}{2},j,k} - \mathbf{F_s}{}^n_{i-\frac{1}{2},j,k} + \mathbf{M}_x(\mathbf{U}^n_{i,j,k})\frac{\mathbf{U}^n_{i+1,j,k} - \mathbf{U}^n_{i-1,j,k}}{2}\right) \\
&- \frac{\Delta t_n}{\Delta y}\left(\mathbf{G_s}{}^n_{i,j+\frac{1}{2},k} - \mathbf{G_s}{}^n_{i,j-\frac{1}{2},k} + \mathbf{M}_y(\mathbf{U}^n_{i,j,k})\frac{\mathbf{U}^n_{i,j+1,k} - \mathbf{U}^n_{i,j-1,k}}{2}\right) \\
&- \frac{\Delta t_n}{\Delta z}\left(\mathbf{H_s}{}^n_{i,j,k+\frac{1}{2}} - \mathbf{H_s}{}^n_{i,j,k-\frac{1}{2}} + \mathbf{M}_z(\mathbf{U}^n_{i,j,k})\frac{\mathbf{U}^n_{i,j,k+1} - \mathbf{U}^n_{i,j,k-1}}{2}\right) \qquad (4.4)
\end{aligned}
$$

It is easier to derive the coefficients matrix $\tilde{\mathbf{C}}_{\mathrm{k}}$ for the model equations with respect to the state vector

$$
\tilde{\mathbf{U}} = \left(\rho_{si}, P_{\perp si}, P_{\| si}, \mathbf{U}_{si}, \mathbf{B}, \mathbf{U}_{ci}, \rho_{ci}, P_{\perp ci}, P_{\| ci}\right)
$$

Derivation with respect to the above state vector, requires that the momentum equations (3.14), (3.18) are written for the velocity instead of the momentum as the time evolving parameter. The matrix $\tilde{\mathbf{C}}_{\mathrm{k}}$ can be derived quite generally for an arbitrary direction $\mathbf{k}$ of differentiation. It has a block structure

$$
\tilde{\mathbf{C}}_{\mathrm{k}} = \begin{pmatrix}
\tilde{\mathbf{C}}_{\mathrm{k}11} & \tilde{\mathbf{C}}_{\mathrm{k}12} & 0 & 0 & 0 \\
\tilde{\mathbf{C}}_{\mathrm{k}21} & \tilde{\mathbf{C}}_{\mathrm{k}22} & \tilde{\mathbf{C}}_{\mathrm{k}23} & 0 & 0 \\
\tilde{\mathbf{C}}_{\mathrm{k}31} & \tilde{\mathbf{C}}_{\mathrm{k}32} & \tilde{\mathbf{C}}_{\mathrm{k}33} & \tilde{\mathbf{C}}_{\mathrm{k}34} & \tilde{\mathbf{C}}_{\mathrm{k}35} \\
0 & 0 & \tilde{\mathbf{C}}_{\mathrm{k}43} & \tilde{\mathbf{C}}_{\mathrm{k}44} & \tilde{\mathbf{C}}_{\mathrm{k}45} \\
0 & 0 & 0 & \tilde{\mathbf{C}}_{\mathrm{k}54} & \tilde{\mathbf{C}}_{\mathrm{k}55}
\end{pmatrix}
$$

with 3×3 matrices $\tilde{\mathbf{C}}_{\mathrm{k}11}, \tilde{\mathbf{C}}_{\mathrm{k}12}, \ldots, \tilde{\mathbf{C}}_{\mathrm{k}55}$. The blocks can explicitly be represented in terms of the unit vectors $\mathbf{k}$ and $\mathbf{b}$ in the following way with $b_{\mathrm{k}} = \mathbf{b}\cdot\mathbf{k}$, $B_{\mathrm{k}} = \mathbf{B}\cdot\mathbf{k}$, $U_{\mathrm{k}_{si}} = \mathbf{U}_{si}\cdot\mathbf{k}$ and $U_{\mathrm{k}_{ci}} = \mathbf{U}_{ci}\cdot\mathbf{k}$.

$$
\tilde{\mathbf{C}}_{\mathrm{k}11} = U_{\mathrm{k}_{si}}\mathbf{I}, \qquad \tilde{\mathbf{C}}_{\mathrm{k}22} = U_{\mathrm{k}_{si}}\mathbf{I}
$$

$$
\tilde{\mathbf{C}}_{\mathrm{k}12} = \begin{pmatrix} \rho_{si} \\ 2P_{\perp si} \\ P_{\| si} \end{pmatrix} \mathbf{k}' + b_{\mathrm{k}} \begin{pmatrix} 0 \\ -P_{\perp si} \\ 2P_{\| si} \end{pmatrix} \mathbf{b}',
$$

$$
\tilde{\mathbf{C}}_{\mathrm{k}21} = \frac{1}{\rho_{si}}\Big(\mathbf{k}(0,1,0) + b_{\mathrm{k}}\mathbf{b}(0,-1,1)\Big),
$$

$$
\tilde{\mathbf{C}}_{\mathrm{k}23} = \frac{1}{\rho_{si}}\left(\frac{(P_{\| si} - P_{\perp si})b_{\mathrm{k}}}{B}(\mathbf{I} - 2\mathbf{b}\mathbf{b}') + \frac{\alpha_{si}}{\mu}\left(\mathbf{k}\mathbf{B}' - B_{\mathrm{k}}\mathbf{I}\right)\right),
$$

$$\tilde{\mathbf{C}}_{\mathrm{k}43} = \frac{1}{\rho_{ci}} \left(\frac{(P_{\| ci} - P_{\perp ci}) b_{\mathrm{k}}}{B} (\mathbf{I} - 2\mathbf{b}\mathbf{b}') + \frac{\alpha_{ci}}{\mu} (\mathbf{k}\mathbf{B}' - B_{\mathrm{k}}\mathbf{I}) \right),$$

$$\tilde{\mathbf{C}}_{\mathrm{k}32} = \alpha_{si}(\mathbf{B}\mathbf{k}' - B_{\mathrm{k}}\mathbf{I}), \qquad \tilde{\mathbf{C}}_{\mathrm{k}34} = \alpha_{ci}(\mathbf{B}\mathbf{k}' - B_{\mathrm{k}}\mathbf{I}),$$

$$\tilde{\mathbf{C}}_{\mathrm{k}33} = (\alpha_{si} U_{\mathrm{k}_{si}} + \alpha_{ci} U_{\mathrm{k}_{ci}})\mathbf{I}, \qquad \tilde{\mathbf{C}}_{\mathrm{k}44} = U_{\mathrm{k}_{ci}}\mathbf{I}, \qquad \tilde{\mathbf{C}}_{\mathrm{k}55} = U_{\mathrm{k}_{ci}}\mathbf{I},$$

$$\tilde{\mathbf{C}}_{\mathrm{k}45} = \frac{1}{\rho_{ci}} \Big(\mathbf{k}(0,1,0) + b_{\mathrm{k}} \mathbf{b}(0,-1,1) \Big),$$

$$\tilde{\mathbf{C}}_{\mathrm{k}54} = \begin{pmatrix} \rho_{ci} \\ 2P_{\perp ci} \\ P_{\| ci} \end{pmatrix} \mathbf{k}' + b_{\mathrm{k}} \begin{pmatrix} 0 \\ -P_{\perp ci} \\ 2P_{\| ci} \end{pmatrix} \mathbf{b}',$$

$$\tilde{\mathbf{C}}_{\mathrm{k}31} = -\frac{\alpha_{ci}}{m_{si}(n_{si} + n_{ci})} \mathbf{k} \times \Big((\mathbf{U}_{si} - \mathbf{U}_{ci}) \times \mathbf{B} \Big)(1,0,0),$$

$$\tilde{\mathbf{C}}_{\mathrm{k}35} = \frac{\alpha_{si}}{m_{ci}(n_{si} + n_{ci})} \mathbf{k} \times \Big((\mathbf{U}_{si} - \mathbf{U}_{ci}) \times \mathbf{B} \Big)(1,0,0),$$

where

$$\alpha_{si} = \frac{\rho_{si}}{(\rho_{si} + \frac{m_{si}}{m_{ci}}\rho_{ci})} \qquad \text{and} \qquad \alpha_{ci} = \frac{\rho_{ci}}{(\frac{m_{ci}}{m_{si}}\rho_{si} + \rho_{ci})}$$

One splits the left and right eigenvectors $\mathbf{l}$ and $\mathbf{r}$ in parts $\mathbf{l}_1, \cdots, \mathbf{l}_5$ and $\mathbf{r}_1, \cdots, \mathbf{r}_5$ of 3 dimensional vectors adapted to the block structure of $\tilde{\mathbf{C}}_{\mathrm{k}}$. One can find a few eigenvalues explicitly:

(i) $\lambda = U_{\mathrm{k}_{si}}$ gives an entropy wave in the solar wind. Then $\mathbf{l}_1$ is perpendicular to the two vectors $(\rho_{si}, 2P_{\perp si}, P_{\| si})$, and $(0, -P_{\perp si}, 2P_{\| si})$, and $\mathbf{l}_2 = \mathbf{l}_3 = \mathbf{l}_4 = \mathbf{l}_5 = 0$

(ii) $\lambda = U_{\mathrm{k}_{ci}}$ entropy wave in the cometary plasma in a similar way.

(iii) $\lambda = \alpha_{si} U_{\mathrm{k}_{si}} + \alpha_{ci} U_{\mathrm{k}_{ci}}$, transport of magnetic field, $\mathbf{l}_1 = \mathbf{l}_2 = \mathbf{l}_4 = \mathbf{l}_5 = 0$, $\mathbf{l_3} = \mathbf{k}$.

(iv) For Alfvén waves we make the Ansatz $\mathbf{r}_1 = \mathbf{r_5} = 0$, and $\mathbf{r}_{\mathrm{j}} = \sigma_{\mathrm{j}} \mathbf{k} \times \mathbf{b}$, with $\mathrm{j} = 2,3,4$. This reduces the eigenproblem to

$$\begin{pmatrix} U_{k_{si}} & a_{23} & 0 \\ a_{32} & \alpha_{si} U_{\mathrm{k}_{si}} + \alpha_{ci} U_{\mathrm{k}_{ci}} & a_{34} \\ 0 & a_{43} & U_{k_{ci}} \end{pmatrix} \begin{pmatrix} \sigma_2 \\ \sigma_3 \\ \sigma_4 \end{pmatrix} = \lambda \begin{pmatrix} \sigma_2 \\ \sigma_3 \\ \sigma_4 \end{pmatrix}$$

The numbers a_{ij} are defined by $\tilde{\mathbf{C}}_{\mathrm{k}_{\mathrm{ij}}}(\mathbf{k} \times \mathbf{b}) = a_{\mathrm{ij}} \mathbf{k} \times \mathbf{b}$. This gives the values

$$a_{23} = \frac{1}{\rho_{si}} \Big(\frac{(P_{\| si} - P_{\perp si}) b_{\mathrm{k}}}{B} - \frac{\alpha_{si} B_{\mathrm{k}}}{\mu} \Big),$$

$$a_{32} = -\alpha_{si} B_{\mathrm{k}}, \qquad a_{34} = -\alpha_{ci} B_{\mathrm{k}},$$

$$a_{43} = \frac{1}{\rho_{ci}} \Big(\frac{(P_{\parallel ci} - P_{\perp ci}) b_{\mathrm{k}}}{B} - \frac{\alpha_{ci} B_{\mathrm{k}}}{\mu} \Big).$$

It is not possible to present explicit analytical expressions for the full eigensystem of the coefficient matrices which are real general matrices with dimension 15 by 15. The eigensystems are, therefore, calculated numerically in each time step on each cell interface. In order to reduce the number of non-conservative terms in the model equations, the numerical calculation uses the momentum instead of the velocity. Hence, the state vector is

$$\mathbf{U} = \Big(\rho_{si}, P_{\perp si}, P_{\parallel si}, \rho_{si} \mathbf{U}_{si}, \mathbf{B}, \rho_{ci} \mathbf{U}_{ci}, \rho_{ci}, P_{\perp ci}, P_{\parallel ci} \Big)$$

The upstream correction has to be calculated from the matrix

$$\mathbf{C}_{\mathrm{k}} = \mathbf{J} \tilde{\mathbf{C}}_{\mathrm{k}} \mathbf{J}^{-1}$$

where

$$\mathbf{J} = \left(\frac{\partial \mathbf{U}}{\partial \tilde{\mathbf{U}}} \right)$$

This matrix, $\mathbf{C}_{\mathrm{k}}$, has the same eigenvalues as $\tilde{\mathbf{C}}_{\mathrm{k}}$. The right eigenvectors $\mathbf{r}_p$ must be changed to $\mathbf{J}\mathbf{r}_p$. To obtain the coefficients matrices $\mathbf{C}_x$, $\mathbf{C}_y$ and $\mathbf{C}_z$, one has to put $\mathbf{k} = \mathbf{e_x}, \mathbf{e_y}$, or $\mathbf{e_z}$, the unit vector in $\mathbf{x}$, $\mathbf{y}$, or $\mathbf{z}$ direction. Finally, the numerical eigensystems of $\mathbf{C}_x$, $\mathbf{C}_y$ and $\mathbf{C}_z$ can be used for the calculation of the upstream corrections to obtain the surface fluxes $\mathbf{F}_s$, $\mathbf{G}_s$, and $\mathbf{H}_s$, respectively, for equation (4.4).

The model was used to iterate two cases to steady state. In the first case, it was reduced to one fluid with isotropic pressure by taking high velocity coupling rates, and high isotropization rates. The isotropization rate taken was $\nu_{isot.} = 10^{-2} s^{-1}$, and the parallel velocity coupling generated immediate assimilation. In the second case, the full generalized model with two fluids and gyrotropic pressures was applied. The isotropization rate was reduced to $\nu_{isot.} = 10^{-3} s^{-1}$ for both of the fluids, and the parallel velocity coupling rate taken was $\nu_{\parallel} = 10^{-3} s^{-1}$. The results obtained [10] show that the stand-off distance of the shock is significantly larger for the lower isotropization rate. When the low isotropization rate is taken, the pressure anisotropy reaches values of 2 upstream the comet nucleus. When slow parallel velocity coupling rate is taken, the large cometary plasma pressure forces at the shock accelerate the cometary plasma along the shock to create a relative drift of the solar wind behind the cometary plasma parallel to the magnetic field. This drift is about

20% of the solar wind velocity. Acceleration of cometary plasma is obtained to a low extent also in the inner tail. Close to the nucleus of the comet, in the far tail lobes, and in the far inner tail, the cometary plasma lagged behind the solar wind. Up to now, we have not encountered any physical instabilities. As a result of numerical error, the numerically calculated eigenvalues acquire at some places small imaginary parts, but this does not influence the calculation based on our version of the approximate Riemann solver.

5 Summary

A detailed global model for the comet interaction with the solar wind has been developed. The long time scales that cometary pickup plasma needs in order to isotropize and assimilate with the ambient plasma are beyond the time scales for advection of an MHD fluid. In this environment, pressure anisotropies may evolve. Although the plasma is not necessarily gyrotropic, it is still a step forward to introduce a gyrotropic pressure tensor instead of the isotropic pressure used in MHD models. Hence, the double adiabatic energy equations of Chew-Goldberger-Low, modified by isotropization terms, replace the adiabatic energy equation. In addition, the long time needed for the assimilation of the pickup plasma with the ambient plasma indicates that the interaction should be formulated as a two fluid interaction, i.e. solar wind ions, and cometry ions, instead of the one fluid MHD approach. The two ion fluids interact on longer time scales by coupling terms based on the velocity moments of the BGK relaxation time model [1]. Presently, the electron component in our model, is assumed cold and therefore degenerate and has no dynamic effect. A numerical scheme for the model equations is presented. It is a Godunov [12] type upwind method which uses an approximate Riemann solver.

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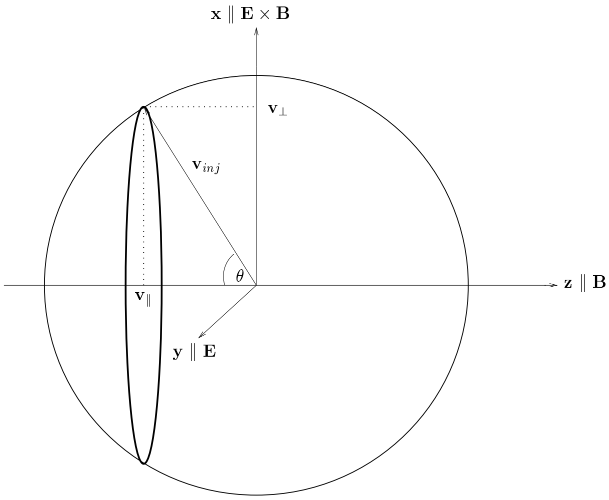

Fig. 1. The pickup process in a solar wind magnetic field aligned frame.