Partial cross ownership and tacit collusion*

David Gilo and Yossi Spiegel†

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Abstract

This paper shows how competing firms can facilitate tacit collusion by making passive investments in rivals. When firms are identical, only multilateral partial cross ownership (PCO) facilitates tacit collusion; the incentives of firms to collude in this case depend in a comlex way on the whole set of PCO in the industry. A firm's controller can facilitate tacit collusion further by investing directly in rival firms and by diluting his stake in his own firm. In the presence of cost asymmetries, even unilateral PCO of an efficient firm in a less efficient rival can facilitate tacit collusion.

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†Gilo: Recanati Graduate School of Business Adminstration and The Buchman Faculty of Law, Tel-Aviv University; email: gilod@post.tau.ac.il. Spiegel: Recanati Graduate School of Business Adminstration, Tel Aviv University, email: spiegel@post.tau.ac.il, http://www.tau.ac.il/~spiegel

1 Introduction

There are many cases in which firms acquire their rivals' stock as passive investments which give them a share in the rivals' profits but not in the rivals' decision making. For example, Microsoft acquired in August 1997 approximately 7% of the nonvoting stock of Apple, its historic rival in the PC market, and in June 1999 it took a 10% stake in Inprise/Borland Corp. which is one of its main competitors in the market for software applications.[1] Gillette, the international and U.S. leader in the wet shaving razor blade market acquired 22.9% of the nonvoting stock and approximately 13.6% of the debt of Wilkinson Sword, one of its largest rivals.[2] Investments in rivals are often multilateral; examples of industries that feature complex webs of partial cross ownerships are the Japanese and the U.S. automobile industries (Alley, 1997), the global airlines industry (Airline Business, 1998), the Dutch Financial Sector (Dietzenbacher, Smid, and Volkerink, 2000), and the Nordic power market (Amundsen and Bergman, 2002).[3] There are also many cases in which a controller (majority or dominant shareholder) makes a passive investment in rivals. A striking example existed during the first half of the 90's in the car

[1] See "Microsoft Investments Draw Federal Scrutiny," *Pittsburgh Post-Gazette*, August 10, 1997, B-11, and "Corel Again Buys a "Victim" of Microsoft Juggernaut," *The Ottawa Citizen*, February 8, 2000, C1.

[2] *United States v. Gillette Co.*, 55 FR 28312 (1990).

[3] Multilateral investments in rivals are also common in the European automobile industry; for instance, in 1990, Renault acquired a 45% stake in Volvo Trucks, a 25% stake in Volvo Car, and a 8.2% stake in Volvo A.B., Volvo's holding company, while Volvo acquired 20% of Renault S.A. and 45% of Renault's truck-making operations (see "New Head is Selected For Renault," *N.Y Times*, May 25, 1992, p. 35). Another example is the global steel industry, where two of the worlds' largest steelmakers, Japanese Nippon Steel and Korean Pohang Iron held ownership stakes in each other that started from 0.5% in the early 90's, increased to 1% in the late 90's, and are recently planned to incease to 3%. Nippon Steel also reached an agreement in November 2002 with two of its main rivals in Japan, Sumitomo Metal Industries and Kobe Steel, according to which Nippon Steel and Sumitomo will each own about 2% of Kobe while Kobe will acquire approximately 0.3% of Nippon (see "Nippon Steel, Posco Extend Partnership; Steel World's Largest Producers Put Historical Animosities Behind Them and Increase Shareholdings," *Financial Times*, August 3, 2000, Companies & Finance: Asia-Pacific, 23; "Japanese Steelmaker to Trade Stakes," *The Daily Deal*, November 15, 2002, M&A). Likewise, Japan's second largest producer, Kawasaki Steel Company, purchased a minority stake in Korean Dongkuk Steel Company, while holding (at the time) a 40% stake in American steelmaker Armco (see "Dongkuk Enters Strategic Alliance with Kawasaki," *Financial Times*, August 6, 1999, Companies & Finance: Asia-Pacific, 26). Similar multilateral investments exist among American and Canadian steelmakers (see "Canadian Firms Split over Curbing U.S. Steel Imports: The Federal Government is Caught between an American Rock and a Foreign Hard Place," *N.Y. Times*, December 17, 2002, D10), and among European steelmakers (see "Usinor to Enter Brazilian Market," *Financial Times*, May 27, 1998, Companies & Finance: The Americas, 27; and "Uddeholm and Bohler Form Steel Alliance," *Financial Times*, April 3, 1990, International Companies & Finance, 29.) Analysts argue that one of the major motivations behind such arrangements among steelmakers is to retain "more stable prices," as excess capacity in the industry tends to cause prices to fluctuate often (see "Asia Briefs," *Asian Wall Street Journal*, May 10, 1999, A15; "Steelmakers Close to Deal on Alliance," *Financial Times*, August 1, 2000, Companies & Finance: Asia-Pacific, 25).

rental industry where National Car Rental's controller, GM, passively held a 25% stake in Avis, National's rival, while Hertz's controller, Ford, had acquired 100% of the preferred nonvoting stock of Budget Rent a Car (Purohit and Staelin, 1994 and Talley, 1990).[4]

Surely, if Microsoft were to merge with Apple, Gillette with Wilkinson Sword, National Car Rentals with Avis, or Hertz with Budget, antitrust agencies would acknowledge that competition may be substantially lessened. However, passive investments in rivals were granted a de facto exemption from antitrust liability in leading antitrust cases, and have gone unchallenged by antitrust agencies in recent antitrust cases (Gilo, 2000).[5] This lenient approach towards passive investments in rivals stems from the courts' interpretation of the exemption for stock acquisitions "solely for investment" included in Section 7 of the Clayton Act.

In this paper we study the competitive effects of passive investments in rivals. In particular, we wish to examine whether the lenient approach of courts and antitrust agencies towards such investments is justified. Like other horizontal practices (e.g., horizontal mergers), (passive) partial cross ownership (PCO) arrangements raise two main antitrust concerns: concerns about unilateral competitive effects and concerns about coordinated competitive effects. We focus on the latter and consider an infinitely repeated Bertrand oligopoly model (with and without cost asymmetries) in which firms and/or their controllers acquire some of their rivals (nonvoting) shares. A main advantage of this model is that PCO do not affect the equilibrium in the one shot case and therefore do not have any unilateral competitive effects. This allows us to focus on the effect of PCO on the ability of firms to engage in tacit collusion. We say that PCO facilitate tacit collusion if it expands the range of discount factors for which tacit collusion can

[4] See also "Will Ford Become The New Repo Man?; Financial Powerhouse Takes Aim at Bad Credit Risks," *N.Y Times*, December 15, 1996, Section 3, p. 1. For additional examples of investments by firms and their controllers in rivals, see Gilo (2000).

[5] The FTC approved TCI's 9% stake in Time Warner which at the time was TCI's main rival in the cable TV industry and even allowed TCI to raise its stake in Time Warner to 14.99% in the future, after being assured that TCI's stake would be completely passive (see *Re Time Warner Inc.,* 61 FR 50301, 1996). The FTC also agreed to a consent decree approving Medtronic Inc.'s almost 10% passive stake in SurVivaLink, one of its only two rivals in the automated External Defibriallators market (In *Re Medtronic, Inc.*, FTC File No. 981-0324, 1998). The DOJ approved Gillette's 22.9% stake in Wilkinson Sword after being assured that this stake would be passive (see *United States v. Gillette* Co. 55 Fed. Reg. at 28,312, infra Section II.C). Northwest Airline's purchase of 14% of Continental's common stock was attacked by the DOJ, but only due to the DOJ's suspicion that Northwest will influence Continental's activity (*US v. Northwest Airlines Corporation*, No. 98-74611, Amended Complaint (D. Mich. 1998), at par. 37-41). To the best of our knowledge, Microsoft's investments in the nonvoting stocks of Apple and Inprise/Borland Corp. were not challenged by antitrust agencies.

be sustained.

Our analysis reveals that in the presence of PCO arrangements, the incentive of each firm to engage in tacit collusion depends in a complex way on the whole set of PCOs in the industry and not only on the firm's own investments in rivals. This complexity arises since PCO create an infinite recursion between the profits of firms who hold each other's shares. It might be thought that since PCO allow firms to internalize part of the harm they impose on rivals when they deviate from a collusive scheme, any increase in the level of PCO in the industry will facilitate tacit collusion. We show, however, that this intuition need not be correct: there are at least three important cases in which a change in firm i's PCO will have no effect on tacit collusion. The first case arises when at least one other firm in the industry does not invest in rivals. This firm then is the *maverick firm* in the industry (the firm with the strongest incentive to deviate from a collusive agreement) and its incentives to collude are not affected by the level of PCO among its rivals.[6] The second case in which a change in firm i's PCO will have no effect on tacit collusion arises when the maverick firm has no stake in firm i either directly or indirectly (i.e., does not invest in a firm that invests in firm i and does not invest in a firm that invests in a firm that invests in firm i and so on). The third case arises when firm i increases its investment in the industry maverick.

Our analysis also shows that there are important cases in which an increase in PCO will facilitate tacit collusion. In particular, when all firms hold exactly the same ownership stakes in rivals, collusion is facilitated when the symmetric ownership stake increases and when one firm unilaterally raises its aggregate ownership stake *in more than one rival*. Such a unilateral increase in PCO is most effective in facilitating tacit collusion when it is evenly spread among all rivals. A controlling shareholder (whether a person or a parent corporation) can facilitate tacit collusion further by making a direct passive investment in rival firms. Such investment particularly facilitates collusion if the controller has a relatively small stake in his own firm.[7]

[6] The Horizontal Merger Guidelines of the US Department of Justice and FTC define maverick firms as "firms that have a greater economic incentive to deviate from the terms of coordination than do most of their rivals," see www.usdoj.gov/atr/public/guidelines/horiz_book/hmg1.html. For an excellent discussion on the role that the concept of maverick firms plays in the analysis of coordinated competitive effects, see Baker (2002).

[7] Interstingely, shortly after it had acquired a passive stake in Budget, Hertz's controller, Ford, diluted its stake in Hertz from 55% to 49%, by selling shares to Volvo (see "Chrysler Buying Thrifty Rent-A-Car," *St. Louis Post-Dispatch*, May 19, 1989, Business, 8C). Our result suggests that such dilution may have promoted collusion in the car rental indusrty.

This implies in turn that even relatively small direct passive investments by controllers in rival firms can raise considerable antitrust concern. And, when firms have asymmetric costs, even unilateral PCO by the most efficient firm in its rivals may facilitate tacit collusion. Moreover, the collusive price is higher than it would be absent PCO. The most efficient firm prefers to first invest in its most efficient rival both because this is the most effective way to promote tacit collusion and because such investment leads to a collusive price that is closer to the most efficient firm's monopoly price.

The unilateral competitive effects of PCO have been already studied in the context of static oligopoly models by Reynolds and Snapp (1986), Farrell and Shapiro (1990), Bolle and Güth (1992), Flath (1991, 1992), Reitman (1994), and Dietzenbacher, Smid, and Volkerink (2000).[8] Our paper by contrast focuses on the coordinated competitive effects of PCO and examines a repeated Bertrand model. The distinction between the unilateral and coordinated competitive effects of PCO is important. In particular, PCO which may not be optimal in static oligopoly models are shown to be optimal in our model once their coordinated effects are taken into account. For example, Flath (1991) shows that firms may be reluctant to invest in rivals even though such investments relax product market competition. The reason for this reluctance is that in a perfectly competitive capital market, the price of the rival's shares reflects their post-acquisition value, so the entire increase in the rival's value accrues to the rivals' shareholders. Consequently, the investing firm gains only if the value of its own shares increases which is the case only when product market competition involves strategic complements.[9] In our model, firms may benefit from investing in rivals even when product market competition involves strategic substitutes since such investments may facilitate tacit collusion. Reitman (1994) shows that symmetric firms may not wish to invest in rivals because such investments benefit noninvesting firms more than they benefit the investing firms. In our model, when firms

[8] See also Bresnahan and Salop (1986) and Kwoka (1992) for a related analysis of static models of horizontal joint ventures. Alley (1997) and Parker and Röller (1997) provide empirical evidence on the effect of PCO on collusion. Alley (1997) finds that failure to account for PCO leads to misleading estimates of the price-cost margins in the Japanese and U.S. automobile industries. Parker and Röller (1997) find that cellular telephone companies in the U.S. tend to collude more in one market if they have a joint venture in another market.

[9] Charléty, Fagart, and Souam (2002) study a related model but consider PCO by controllers rather than by firms. They show that although a controller's investments in rivals lower the profit of the controller's firm, they may increase the rival's profit by a larger amount and thereby benefit the controller at the expense of the minority shareholders in his own firm.

are symmetric, all of them need to invest in rivals to sustain tacit collusion (i.e., each firm is "pivotal") so there is no free-rider as in Reitman. And, Farrell and Shapiro (1990) show that in a static Cournot model, it is never optimal for a low-cost firm to acquire a passive stake in a high-cost rival. Our model shows in contrast that this kind of commonly observed phenomenon is in fact optimal: by investing in a high-cost rival, the low-cost firm facilitates tacit collusion.

We are aware of only one other paper, Malueg (1992), that studies the coordinated effects of PCO. His paper differs from ours in at least three important respects. First, Malueg considers a repeated Cournot game and finds that in general, PCO has an ambiguous effect on the ability of firms to collude. The ambiguity arises because, although PCO weaken the incentive of firms to deviate from a collusive scheme (firms internalize part of the losses that they inflict on rivals when they deviate), in a Cournot model, they also soften product market competition following a breakdown of the collusive scheme; the latter effect strengthens the incentive to deviate. However, we believe that in specific cases, the first positive effect is likely to dominate the second negative effect, otherwise firms will have no incentive to invest in rivals. The Bertrand framework that we use allows us to neutralize the negative effect of PCO and focus attention on first positive effect. Second, Malueg considers a symmetric duopoly in which the firms hold identical stakes in one another, while we consider an n firms oligopoly in which firms may have asymmetric costs and need not invest similar amounts in one another. Third, unlike us, Malueg does not consider passive investments in rivals by controllers.

The rest of the paper is organized as follows: Section 2 examines the effect of PCO on the ability of firms to achieve the fully collusive outcome in the context of an infinitely repeated Bertrand model with symmetric firms. Section 3 shows that PCO by firms' controllers may further facilitate tacit collusion. Section 4 considers an infinitely repeated Bertrand model in which firms have asymmetric costs. We conclude in Section 5. The Appendix contains two technical proofs.

2 Partial cross ownership (PCO) with symmetric firms

In this section we examine the coordinated competitive effects of PCO in the context of the familiar infinitely repeated Bertrand oligopoly model with $n \geq 2$ identical firms. As mentioned in the Introduction, this simple setting allows us to focus on complex issues, such as the chain-effects of multilateral PCO and the effect of PCO on tacit collusion under cost asymmetries.

Specifically, we assume that the n firms produce a homogenous product at a constant marginal cost c and that in every period they simultaneously choose prices and the lowest price firm captures the entire market. In case of a tie, the set of lowest price firms get equal shares of the total sales. As is well-known (e.g., Tirole, 1988, Ch. 6.3.2.1), the fully collusive outcome in which all firms charge the monopoly price and each firm gets an equal share in the monopoly profit can be sustained as a subgame perfect equilibrium of the infinitely repeated game provided that the intertemporal discount factor, δ, is such that

$$\delta \geq \widehat{\delta} \equiv 1 - \frac{1}{n}. \tag{1}$$

That is, the fully collusive outcome can be sustained provided that the firms are sufficiently patient (i.e., care sufficiently about their long run profits).

Taking condition (1) as a benchmark, we shall examine the competitive effects of PCO by looking at its effect on the critical discount factor, $\widehat{\delta}$, above which the fully collusive outcome can be sustained. In other words, the value of $\widehat{\delta}$ will be our measure of the ease of collusion.[10] If PCO lowers $\widehat{\delta}$, then tacit collusion becomes sustainable for a wider set of discount factors. Hence, we will say that it facilitates tacit collusion. Conversely, if PCO raises $\widehat{\delta}$, we will say that it hinders tacit collusion.

To examine the impact of PCO on $\widehat{\delta}$, let $Q(p)$ be the downward sloping demand function in the industry, and let

$$\pi^m \equiv \max_p \quad Q(p)(p-c)$$

be the associated monopoly profit. Moreover, let α_i^j be firm i's ownership stake in firm j. We assume that the pricing decisions of each firm are effectively made by its controller (i.e., a controlling shareholder) whose ownership stake is β_i. Now, suppose that all controllers adopt the same trigger strategy whereby they set the monopoly price in every period unless at least

[10] Of course, the repeated game admits multiple equilibria. We focus on the fully collusive outcome and on $\widehat{\delta}$ because this is a standard way to measure the notion of "ease of collusion."

one firm has charged a different price in any previous period; then all firms set a price equal to c forever after. To write the condition that ensures that this trigger strategy can support the fully collusive outcome as a subgame perfect equilibrium, we first need to express the profit of each firm under collusion and following a deviation from the fully collusive scheme.

To this end, note that if all firms charge the monopoly price, then each firm earns $\frac{\pi^m}{n}$, and on top of that it also gets a share in its rivals' profits due to its ownership stake in these firms. Hence, the vector of profits in the industry, $(\pi_1, \pi_2, ..., \pi_n)$ is given by the solution to the following system of n equations:

$$\begin{aligned}
\pi_1 &= \frac{\pi^m}{n} + \alpha_1^2 \pi_2 + \alpha_1^3 \pi_3 + \cdots + \alpha_1^n \pi_n, \\
\pi_2 &= \frac{\pi^m}{n} + \alpha_2^1 \pi_1 + \alpha_2^3 \pi_3 + \cdots + \alpha_2^n \pi_n, \\
&\vdots \\
\pi_n &= \frac{\pi^m}{n} + \alpha_n^1 \pi_1 + \alpha_n^2 \pi_2 + \cdots + \alpha_n^{n-1} \pi_{n-1}.
\end{aligned} \tag{2}$$

System (2) reveals that the profit of each firm depends in a complex way on the profits of all other firms and on the structure of PCO in the industry. For instance, firm 1 may get a share α_1^2 of firm 2's profit which may reflect firm 2's share, α_2^3, in the profit of firm 3, which in turn may reflect firm 3's share, α_3^5, in the profit of firm 5. The fact that each firm's profit depends on the whole PCO matrix is striking. It implies for instance that a firm's profit and incentive to engage in tacit collusion may be affected by a change in PCO levels among rivals even if this change does not affect the firm directly (i.e., even if the firm's PCO levels in rivals or the rivals' PCO in that firm remain unchanged).

To solve system (2) for the vector $(\pi_1, \pi_2, ..., \pi_n)$, it is useful to rewrite the system compactly as

$$(I - A)\,\pi = k, \tag{3}$$

where I is an $n \times n$ identity matrix, $\pi = (\pi_1, ..., \pi_n)'$ and $k = (\frac{\pi^m}{n}, ..., \frac{\pi^m}{n})'$ are $n \times 1$ vectors, and

$$A=\begin{pmatrix} 0 & \alpha_1^2 & \cdots & \alpha_1^n \\ \alpha_2^1 & 0 & \cdots & \alpha_2^n \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_n^1 & \alpha_n^2 & \cdots & 0 \end{pmatrix},$$

is the PCO matrix. System (3) is a Leontief system (see e.g., Berck and Sydsæter, Ch. 21.21, p. 111). Since the sum of the ownership stakes that firm i's controller and rival firms hold in each firm i is less or equal to 1 (it is equal to 1 only if the only minority shareholders in firm i are rival firms), $\beta_i + \sum_{j=1}^{n} \alpha_j^i \leq 1$ for all $i = 1, ..., n$, implying that $\sum_{j=1}^{n} \alpha_j^i < 1$ for all $i = 1, ..., n$. That is, the sum of the rivals' ownership stakes in each firm i is less than 1. Consequently, system (3) has a unique solution $\pi \geq 0$ (see Berck and Sydsæter, Ch. 21.22, p. 111). This solution is defined by

$$\pi = (I - A)^{-1} k. \tag{4}$$

If firm i's controller deviates from the fully collusive scheme, his firm can capture the entire market by slightly undercuting the rivals' prices (the deviating firm's profit then is arbitrarily close to π^m; to simplify matters we therefore write it as π^m). Given the PCO matrix, the vector of firms' profits in the period in which firm i's controller deviates is defined by

$$\pi^{d_i} = (I - A)^{-1} k_i, \tag{5}$$

where $k_i = (0, ..., 0, \pi^m, 0, ..., 0)'$ is an $n \times 1$ vector with π^m in the i'th entry and 0's in all other entries. In all subsequent periods, the profits of all firms in the industry are 0 as all firms charge a price equal to their marginal cost, c.

Before proceeding it is worth noting that the accounting profits, π_i and $\pi_i^{d_i}$, overstate the cash flow of each firm i. This overstatement arises because the accounting profits of firm i take into account not only the cash flow of firm i and its share in its rivals' cash flows, but also its indirect share in these cash flows via its stake in rivals that have stakes in rivals (see

Dietzenbacher, Smid, and Volkerink (2000) and Ritzberger and Shorish (2003) for additional discussion of this effect of PCO). In particular, the aggregate (accounting) profits of all firms will exceed the monopoly profit, π^m. Nonetheless, the payoffs of the controllers and outside investors (i.e., equityholders that are not rival firms) do sum up to π^m and therefore are not overstated. To see that, note that if we sum up the n equations in system (2) and rearrange terms, we get

$$\left(1-\sum_{j\neq 1}\alpha_j^1\right)\pi_1+\left(1-\sum_{j\neq 2}\alpha_j^2\right)\pi_2+\cdots+\left(1-\sum_{j\neq n}\alpha_j^n\right)\pi_n=\pi^m,$$

where $\left(1-\sum_{j\neq i}\alpha_j^i\right)$ is the aggregate ownership stake held by firm i's controller and the firm's outside equityholders ($\sum_{j\neq i}\alpha_j^i$ is the aggregate stake that rival firm have in firm i). Therefore, under collusion, the aggregate payoff of controllers and outside investors is exactly equal to the aggregate cash flow, π^m. A similar computation shows that this is also case when one of the controllers deviates from the fully collusive scheme. To illustrate this point further, suppose that there are only 2 firms that hold ownership stakes of 25% in each other; the rest of the 75% ownership stakes in firms 1 and 2 are held by controllers 1 and 2, respectively. Assuming further that $\pi^m=100$, the profits of the two firms when they collude are $\pi_1=\frac{100}{2}+0.25\pi_2$ and $\pi_2=\frac{100}{2}+0.25\pi_1$. Solving this system, we get $\pi_1=\pi_2=66.66$. Hence, the collusive payoff of each controller is $66.66\times 0.75=50$. This calculation shows that the controllers' payoffs sum up to 100 (the real cash flow) despite the fact that the accounting profits of the two firms sum up to 133.33. If firm 1's controller, say, deviates, the profits of the two firms are $\pi_1=100+0.25\pi_2$ and $\pi_2=0+0.25\pi_1$, so $\pi_1=106.66$ and $\pi_2=26.66$. Now, the payoff of firm 1's controller is 80 while that of firm 2's controller is 20. Again, the controllers' payoffs sum up to 100 despite the fact that the firms' profits sum up to 133.33.

Given the profits of the n firms on the equilibrium path and following a deviation from the fully collusive scheme, the condition that ensures that the fully collusive outcome can be sustained as a subgame perfect equilibrium can be written as

$$\frac{\beta_i\pi_i}{1-\delta}\geq\beta_i\pi_i^{d_i},\qquad i=1,...,n, \tag{6}$$

where $\pi_i^{d_i}$ is the i'th entry in the vector π^{d_i}. The left side of (6) is the infinite discounted payoff of firm i's controller which consists of his share in firm i's collusive profit. The right side of (6) is the controller's share in the one time profit that firm i earns in the period in which it undercuts its rivals slightly. If (6) holds, no controller wishes to unilaterally deviate from the fully collusive scheme.[11] Condition (6) gives rise to the following result:

Proposition 1: *Let* $\widehat{\delta}_i \equiv 1 - \frac{\pi_i}{\pi_i^{d_i}}$. *Then, with PCO, the fully collusive outcome can be sustained as a subgame perfect equilibrium of the infinitely repeated game provided that the intertemporal discount factor,* δ, *is such that*

$$\delta \geq \widehat{\delta}^{po} \equiv \max\left\{\widehat{\delta}_1, ..., \widehat{\delta}_n\right\}. \tag{7}$$

The intuition for Proposition 1 is straightforward. Although firms here are symmetric in the sense that they produce a homogenous product and have the same marginal cost, their incentives to collude are not necessarily identical due to their possibly different levels of ownership stakes in rivals. Proposition 1 shows that whether or not the fully collusive scheme can be sustained depends entirely on the firm with the minimal ratio between the equilibrium profit, π_i, and the profit following a deviation, $\pi_i^{d_i}$. In what follows we shall therefore refer to this firm as the *industry maverick*.

From (7) it is clear that in order to study the effect of PCO on tacit collusion, we must find out how it affects the vector of collusive profits, $(\pi_1, \pi_2, ..., \pi_n)$, and the vectors of profits following a deviation by the controller of each firm i, $(\pi_1^{d_i}, \pi_2^{d_i}, ..., \pi_n^{d_i})$. In particular, let b_{ij} be the entry in the i'th row and j'th column of the inverse Leontief matrix $(I-A)^{-1}$. Then, by (4) and (5), firm i's per-period collusive profit is $\pi_i = \frac{\pi^m}{n}\sum_{i=1}^{n} b_{ij}$ and its one time profit following a deviation by its controller is $\pi_i^{d_i} = b_{ii}\pi^m$. Therefore,

$$\widehat{\delta}_i \equiv 1 - \frac{\pi_i}{\pi_i^{d_i}} = 1 - \frac{\frac{1}{n}\sum_{i=1}^{n} b_{ij}}{b_{ii}}. \tag{8}$$

[11] We study here the case of "pure" price coordination: firms collude by fixing a price and consumers randomize between them thus giving all firms equal market shares. There could be more elaborate collusive schemes in which firms will also divide the market between them in which case their market shares need not be equal. Such schemes however will require some firms to ration their sales and will therefore be harder to enforce and easier to detect.

In order to assess the effect of PCO on tacit collusion, we need to examine how the highest $\widehat{\delta}_i$ in the industry is affected by PCO (of course, if a change in the PCO matrix changes the identity of the industry maverick, we would need to compare the highest $\widehat{\delta}_i$ in the industry before and after the change). Unfortunately, we are not aware of any general comparative static results that establish how $\max\{\widehat{\delta}_1, ..., \widehat{\delta}_n\}$ changes following an arbitrary change in one (or more) of the entries in A.[12] We will therefore consider here several special cases which are corollaries of Proposition 1.

Corollary 1: *Suppose that at least one firm in the industry does not invest in rivals. Then, $\widehat{\delta}^{po} = \widehat{\delta}$, implying that PCO has no effect on the ability of firms to engage in tacit collusion.*

Proof: Suppose that firm i does not invest in rivals. Then (2) implies that $\pi_i = \frac{\pi^m}{n}$. If firm i's controller deviates, then π^m replaces $\frac{\pi^m}{n}$ in the i'th row of (2) while 0 replaces $\frac{\pi^m}{n}$ in all other lines. Hence, $\pi_i^{d_i} = \pi^m$. Consequently, $\widehat{\delta}_i = 1 - \frac{1}{n}$. Now consider firm i that does invest in rivals. Then, $\widehat{\delta}_i$ is given by (8). Since all entries in the PCO matrix, A, are nonnegative and $\sum_{j=1}^n \alpha_j^i < 1$ for all $i = 1, ..., n$, the inverse Leontief matrix is such that $(I - A)^{-1} = \sum_{r=0}^{\infty} A^r$ (see Berck and Sydsæter, Ch. 21.22, p. 111). But since the entries of A are all nonnegative, this also implies that all entries in the matrix $(I - A)^{-1}$ are nonnegative. That is, $b_{ij} \geq 0$ for all $i, j = 1, ..., n$. Consequently, $\frac{1}{n}\frac{\sum_{i=1}^n b_{ij}}{b_{ii}} > \frac{1}{n}$, so $\widehat{\delta}^{po} \equiv \max\{\widehat{\delta}_1, ..., \widehat{\delta}_n\} = 1 - \frac{1}{n} = \widehat{\delta}$. ■

Corollary 1 shows that PCO facilitates tacit collusion only if *every* firm in the industry has a stake in at least one rival. Otherwise, $\widehat{\delta}^{po} = \widehat{\delta}$, exactly as in the case without PCO. From a policy perspective, this implies that in industries with similar firms, antitrust authorities should not be too concerned with unilateral PCO since only multilateral PCOs facilitate tacit collusion.

Given Corollary 1, we will assume in the rest of this section that every firm in the industry invests in at least one rival. Intuitively, it seems that in this case any increase in the level of PCO in the industry would facilitate collusion while any decrease in the level of PCO would hinder it. The following two results show however that this is not so: there are cases in which an increase in PCO has no effect on tacit collusion.

Corollary 2: *Suppose that the PCO matrix A is decomposable and can be expressed as $A =$*

[12] For a comprehensive analysis of the effects of perturbations in Leontief systems, see Dietznbacher (1991).

$\begin{pmatrix} A_{11} & 0 \\ A_{12} & A_{22} \end{pmatrix}$, *where* A_{11}, A_{12}, *and* A_{22}, *respectively, are* $\ell \times \ell$, $(n-\ell) \times \ell$, *and* $(n-\ell) \times (n-\ell)$ *submatrices. That is, firms* $1, ..., \ell$ *invest only in each other but none of them has an ownership stake in firms* $\ell+1, ..., n$. *Then, if firm* $i \in \{1, ..., \ell\}$ *is the industry maverick, a changes in the ownership stakes that firms* $\ell+1, ..., n$ *hold in rivals do not facilitate tacit collusion.*

Proof: If A is decomposable as in the corollary, the profits of firms $1, ..., \ell$ both under collusion and following a deviation from the fully collusive scheme are independent of the ownership stakes that firms $\ell+1, ..., n$ hold in rivals. Hence, changes in these ownership stakes have no effect on the profits of firms $1, ..., \ell$ and hence on $\widehat{\delta}_1, ..., \widehat{\delta}_\ell$. Therefore, if $\widehat{\delta}^{po} \in \{\widehat{\delta}_1, ..., \widehat{\delta}_\ell\}$, the change in ownership structure will have no effect on collusion. If the change turns firm $j \in \{\ell+1, ..., n\}$ into the industry maverick (i.e., $\widehat{\delta}_j > \max\{\widehat{\delta}_1, ..., \widehat{\delta}_\ell\}$, then tacit collusion is hindered ■

Corollary 2 says that a change in firm j's PCO cannot facilitate tacit collusion if the industry maverick has no stake in firm j either directly or indirectly.[13] To illustrate, suppose that there are 10 firms in the industry and firms $1-4$ invest only in each other. That is, firms $1-4$ do not have direct or indirect stakes in firms $5-10$. Then, if the industry maverick is either firm 1, 2, 3, or 4, then any changes in the ownership stakes that firms $5-10$ hold in rivals, including changes in their ownership stakes in firms $1-4$, will not facilitate tacit collusion.

The next corollary to Proposition 1 shows another situation in which an increase in the level of PCO will not facilitate tacit collusion.

Corollary 3: *Suppose that firm* i *is the industry maverick. Then, changes in the investment levels of rivals in firm* i *do not facilitate tacit collusion.*

Proof: If firm i is the industry maverick, then $\widehat{\delta}^{po} = \widehat{\delta}_i \equiv 1 - \frac{\pi_i}{\pi_i^{d_i}}$. Using Cramer's rule, it follows from (3) that

$$\pi_i = \frac{\det L_1}{\det (I-A)},$$

[13] By indirect stake we mean that the industry maverick does not have a stake in a firm that has a stake in firm j, and it does not have a stake in a firm that has a stake in a firm that has a stake in firm j, and so on.

where L_1 is the matrix $I - A$ with the vector $k = (\frac{\pi^m}{n}, ..., \frac{\pi^m}{n})'$ replacing the i'th column. Analogously,

$$\pi_i^{d_i} = \frac{\det L_2}{\det (I - A)},$$

where L_2 is the matrix $I - A$ with the vector $k_i = (0, ..., 0, \pi^m, 0, ..., 0)'$ replacing the i'th column. Using the last two equations,

$$\widehat{\delta}_i = 1 - \frac{\det L_1}{\det L_2}.$$

Since the i'th column in $I - A$ contains the rivals' investments in firm i, $(\alpha_1^i, \alpha_2^i, ..., \alpha_n^i)$, and since this column is missing from both L_1 and L_2, $\widehat{\delta}_i$ is independent of $(\alpha_1^i, \alpha_2^i, ..., \alpha_n^i)$. Hence, changes in the rivals' investments in firm i do not affect $\widehat{\delta}_i$. This implies in turn that if firm i remains the industry maverick, then tacit collusion is not affected by the change. If the change turns another firm into the industry maverick then as in the proof of Corollary 2, tacit collusion will be hindered. ■

Corollary 3 reveals that there is an important difference between the type of passive investments in rivals that we study and horizontal mergers in which firms obtain control over their rivals. Specifically, the Horizontal Merger Guidelines of the US Department of Justice and FTC state that the "acquisition of a maverick firm is one way in which a merger may make coordinated interaction more likely."[14] This concern is in stark contrast to Corollary 3 since the corollary shows that increase in the level of passive investments in the maverick firm can never facilitate tacit collusion. Intuitively, although a (passive) investment by a rival firm in the industry maverick boosts the collusive profits of the maverick's controller, it also boosts the controller's profit from deviating from the fully collusive scheme. Since the controller's profits in both cases increase by exactly the same magnitude, the controller's incentives to engage in tacit collusion remain unchanged. In the Appendix we show, using an example with 4 firms, that investments in the industry maverick is the *only* case in which passive investments in rivals have no effect on tacit collusion: increases in the passive investments of rivals in all other firms

[14] See www.usdoj.gov/atr/public/guidelines/horiz_book/hmg1.html

do facilitate tacit collusion. In other words, increase in the level of passive investments of rivals in any firm *but the industry maverick* will facilitate tacit collusion.

To obtain further insights about the effect of PCO on tacit collusion, we now turn to the symmetric case in which all firms hold exactly the same ownership stakes in rivals, i.e., $\alpha_i^j = \overline{\alpha}$ for all $i = 1, ..., n$ and all $j \neq i$. Consequently, system (2) has a symmetric solution

$$\pi_i = \frac{\pi^m}{n\left(1-(n-1)\overline{\alpha}\right)}, \qquad i = 1, ..., n, \tag{9}$$

where the denominator is positive since the sum of the ownership stakes held by rivals in each firm j, $(n-1)\overline{\alpha}$, is less than 1. Note that the collusive profit of each firm depends only on its aggregate ownership stake in rivals and not on the way this aggregate stake is allocated among the different rivals. That is, so long as each firm holds the same aggregate stake in rivals, system (2) has a symmetric solution even though some firms may invest in only one rival while others may invest in all rivals. If firm i's controller deviates from the fully collusive scheme, then system (2) can be written as

$$\begin{aligned} \pi_i^{d_i} &= \pi^m + (n-1)\,\overline{\alpha}\pi_j^{d_i}, \\ \pi_j^{d_i} &= \overline{\alpha}\pi_i^{d_i} + (n-2)\,\overline{\alpha}\pi_j^{d_i}, \qquad j = 1, ..., n, \quad j \neq i. \end{aligned} \tag{10}$$

Solving this system for $\pi_i^{d_i}$ yields,

$$\pi_i^{d_i} = \frac{\left(1-(n-2)\overline{\alpha}\right)\pi^m}{\left(1-(n-1)\overline{\alpha}\right)\left(1+\overline{\alpha}\right)}. \tag{11}$$

Substituting from (9) and (11) into (7), it follows that the fully collusive outcome can be sustained as a subgame perfect equilibrium of the infinitely repeated game provided that

$$\delta \geq \widehat{\delta}^{po} = \widehat{\delta} - \frac{(n-1)\,\overline{\alpha}}{n\left(1-(n-2)\overline{\alpha}\right)}. \tag{12}$$

This expression gives rise to the following result:

Corollary 4: *Suppose that* $\alpha_i^j = \overline{\alpha}$ *for all* $i = 1, ..., n$ *and all* $j \neq i$. *Then:*

(i) *PCO facilitates collusion in the sense that* $\widehat{\delta}^{po} < \widehat{\delta}$.

(ii) *Holding* n *fixed,* $\widehat{\delta}^{po}$ *is decreasing with* $\overline{\alpha}$, *implying that the larger* $\overline{\alpha}$ *is, the greater is the ease of collusion.*

(iii) *Holding* $\overline{\alpha}$ *fixed,* $\widehat{\delta}^{po}$ *is increasing with* n *for* $n < 1 + \frac{1}{2\overline{\alpha}}$ *but is decreasing with* n *otherwise.*

Parts (i) and (ii) of Corollary 4 indicate that symmetric PCO facilitates tacit collusion (relative to the case where there are no PCOs) and that its effect on tacit collusion is larger, the larger are the ownership stakes that firms hold in each other. To illustrate this result, consider again the example where there are only 2 firms that hold ownership stakes of 25% in each other and $\pi^m = 100$. As we showed above, the collusive payoff of each controller is 50 while the one time gain from deviation is 80. Hence, collusion can be sustained provided that $\frac{50}{1-\delta} \geq 80$, or $\delta \geq 0.375$. Absent PCO, collusion can be sustained if $\delta \geq 1 - \frac{1}{2} = 0.5$; hence, PCO facilitates tacit collusion. Similar calculations reveal that if the ownership stakes of the two firms in each other increase to 50%, collusion can be sustained provided that $\delta \geq 0.25$.

Part (iii) of Corollary 4 shows that in stark contrast with the case absent PCO, with PCO, an increase in the number of firms in the industry may facilitate collusion rather than hinder it. The reason for this surprising result is that holding $\overline{\alpha}$ fixed, an increase in n implies that each firm receives a larger fraction of its profits from rivals. Hence, deviation from the fully collusive scheme which hurts rival firms may become unattractive. When n is sufficiently large, this positive effect of n on the ease of tacit collusion outweighs the usual negative effect of n which renders tacit collusion harder to sustain as n increases.

Next, we ask how a deviation from the symmetric case considered in Corollary 2 affects tacit collusion. To this end, suppose that one firm, say firm 1, raises its aggregate ownership stake in rivals by Δ so that $\alpha_1^2 + \alpha_1^3 + \cdots + \alpha_1^n = (n-1)\overline{\alpha} + \Delta$. To ensure that the ownership stakes that rivals hold in each firm j are less than 1, we will assume that $(n-1)\overline{\alpha} + \Delta < 1$. All firms other than i continue to hold an ownership stake of $\overline{\alpha}$ in each of their rivals.

Corollary 5: *Starting from the symmetric case in which* $\alpha_i^j = \overline{\alpha}$ *for all* $i = 1, ..., n$ *and all* $j \neq i$, *suppose that firm* 1 *increases its aggregate ownership stake in rivals by* Δ. *This change* *in PCO facilitates tacit collusion in the sense that* $\widehat{\delta}^{po} < \widehat{\delta}$ *provided that* Δ *is spread over at least two of firm* 1*'s rivals and is most effective in facilitating tacit collusion when* Δ *is spread equally among all of firm* 1*'s rivals. If* Δ *is concentrated in only one rival, it has no effect on tacit collusion.*

Proof: *See the Appendix.*

Corollary 5 indicates that if we start from a symmetric PCO configuration in which all firms hold the exact same stakes in their rivals, a unilateral increase in PCO by one firm will facilitate tacit collusion provided this firm increases its investments in more than one rival. Moreover, the increase in PCO raises more antitrust concerns the more evenly it is spread among the rival firms. Intuitively, the firm in which firm 1 has invested the most becomes the industry maverick since its controller gains the most from deviation as a larger fraction of its profit from deviation flows back to the firm via its stake in firm 1. Obviously, an even spread of Δ among all rivals minimizes firm 1's stake in the industry maverick and therefore minimizes the incentive of the maverick's controller to deviate from the fully collusive scheme. Interestingly, when Δ is concentrated in only one firm, the change in PCO has no effect on the ease of collusion because the industry maverick's collusive profit and its profit from deviation increase by the same magnitude. This result in consistent with Corollary 3: when firm 1 invests in only one rival, this firm becomes the industry maverick. Since by Corollary 3 investments in the industry maverick do not affect tacit collusion, and since investments in all other firms do not change, the change has no effect on tacit collusion.

If firm 1 decreases its aggregate ownership stake in rivals by Δ, then Corollary 5 is reversed: Starting from the symmetric case in which $\alpha_i^j = \overline{\alpha}$ for all $i = 1, ..., n$ and all $j \neq i$, a decrease of firm 1's aggregate ownership stake in rivals by Δ hinders tacit collusion in the sense that $\widehat{\delta}^{po}$ increases (there is a smaller range of discount factors for which the collusive scheme can be sustained).[15] However, unlike Corollary 5, now only the aggregate decrease in firm 1's ownership stake in rivals matters and not how this decrease is spread among the different rivals.

Corollary 5 assumes implicitly that when firm i increases its ownership stake in rival firms, it buys additional shares from outside investors. The next corollary examines what happens

[15] See the Appendix for a proof of this claim.

when there is a transfer of ownership from one rival firm to another. A recent example of such a transfer occurred in the steel industry, where Luxemburg based Arcelor's, the world largest steelmaker, increased its stake in Brazilian CST, one of the world's largest steel makers, from 18.6% to 27.95% by buying shares from Acesita, another Brazilian steelmaker.[16] To this end, suppose that we start from the symmetric case in which each firm holds an ownership stake of $\overline{\alpha}$ in each of its rivals, and now, firm 1 say, buys a fraction Δ of firm 3 from firm 2. This exercise differs from the one considered in Corollary 5 in that now, the increase in firm 1's ownership stake comes at the expense of firm 2's stake since firm 1 buys firm 3's shares from firm 2 and not from outside shareholders.

Corollary 6: *Starting from the symmetric case in which $\alpha_i^j = \overline{\alpha}$ for all $i = 1, ..., n$ and all $j \neq i$, suppose that firm 1 buys an ownership stake $\Delta \leq \overline{\alpha}$ in firm 3 from firm 2, so after the transaction, firm 1's stake in firm 3 increases to $\overline{\alpha} + \Delta$ while firm 2's stake in firm 3 falls to $\overline{\alpha} - \Delta$. This change in PCO configuration hinders tacit collusion and more so the larger Δ is.*

Proof: *See the Appendix.*

Corollary 6 shows that, holding the aggregate amount of shares held by rival firms in each other constant, a deviation from symmetric PCO configuration hinders collusion. Intuitively, starting from a symmetric configuration, if firm 1 buys some of firm 3's shares from firm 2, firm 2 becomes the industry maverick since it now has the smallest stake in rivals in the industry. Consequently, firm 2's controller has a stronger incentive to deviate from the fully collusive scheme than he had before and hence tacit collusion is hindered. It should be noted that Corollary 6 does not contradict Corollary 5 because there, the aggregate amount of shares held by rivals in each other has increased (firm 1 bought additional shares in firm 3 from outside shareholders), whereas here it is kept constant (firm 1 buys the shares from firm 2). Together, Corollaries 5 and 6 suggest that with identical firms, symmetric PCO configurations are the most conducive to tacit collusion and should therefore raise particular anticompetitive concerns.

[16] Prior to the sale, Acesita held a 18.7% stake in CST but sold its entire stake in CST to Arcelor and to CVRD which is a large Brazilian miner of iron and ore. In addition to its stake in CST, Arcelor also owns stakes in Acesita and in Belgo-Mineira which is another Brazilian steel maker (see "CVRD, Arcelor Team up for CST," *The Daily Deal*, December 28, 2002, M&A; "Minister: Steel Duties Still Under Study - Brazil," *Business News Americas*, April 8, 2002.)

3 PCO by controllers

In this section we consider the possibility that controllers will directly acquire ownership stakes in rival firms. Let β_i^j be the stake that firm i's controller obtains in firm $j \neq i$, in addition to his controlling stake in firm i, β_i. To avoid triviality, we assume that β_i^j represents a completely passive investment (e.g., non-voting shares) that gives the controller a share β_i^j of firm j's profit but no control over its actions. We show that such completely passive investments by controllers can nonetheless facilitate tacit collusion especially if the controllers' stakes in their own firms are relatively small.

To facilitate the analysis, we shall focus on the fully symmetric case in which $\alpha_i^j = \overline{\alpha}$ for all $i = 1, ..., n$ and all $j \neq i$. Then, the per-period profit of each firm under a fully collusive scheme is given by (9). Given the controllers' direct investments in rival firms, the per period payoff of firm i's controller under a fully collusive scheme is

$$\left(\beta_i + \sum_{j \neq i} \beta_i^j\right) \frac{\pi^m}{n\left(1 - (n-1)\overline{\alpha}\right)}.$$

This payoff represents the controller's combined share in firm i's profit and in the profits of all other firms.

If the controller of firm i deviates from the collusive scheme, the profits of firm i in the period in which the deviation occurs is given by (11). System (10) implies that the profits of all other firms in that period are given by

$$\pi_j^{d_i} = \frac{\overline{\alpha}\pi^m}{\left(1 - (n-1)\overline{\alpha}\right)\left(1 + \overline{\alpha}\right)}, \qquad j \neq i.$$

Since the profits of all firms (including firm i) are 0 in all subsequent periods, it follows that the fully collusive outcome is sustainable provided that the following condition holds:

$$\left(\beta_i + \sum_{j \neq i} \beta_i^j\right) \frac{\frac{\pi^m}{n(1-(n-1)\overline{\alpha})}}{1-\delta} \geq \beta_i \frac{\left(1 - (n-2)\overline{\alpha}\right)\pi^m}{\left(1 - (n-1)\overline{\alpha}\right)\left(1 + \overline{\alpha}\right)} \tag{13}$$

$$+ \sum_{j \neq i} \beta_i^j \frac{\overline{\alpha}\pi^m}{\left(1 - (n-1)\overline{\alpha}\right)\left(1 + \overline{\alpha}\right)}, \qquad i = 1, ..., n.$$

Using this condition, we establish the following result:

Proposition 2: *Suppose that $\alpha_i^j = \overline{\alpha}$ for all $i = 1, ..., n$ and all $j \neq i$, that controllers invest in rival firms, and let $k = \left\{ i \;\middle|\; \sum_{j \neq i} \frac{\beta_i^j}{\beta_i} \leq \sum_{j \neq \ell} \frac{\beta_\ell^j}{\beta_\ell} \text{ for all } \ell = 1, ..., n \right\}$ be the firm whose controller has the lowest aggregate ownership stake in rivals relative to his ownership stake in his own firm. Then, the fully collusive outcome can be sustained as a subgame perfect equilibrium of the infinitely repeated game provided that*

$$\delta \geq \widehat{\delta}^c \equiv \widehat{\delta} - \frac{(n-1)\overline{\alpha} + \sum_{j \neq i} \frac{\beta_k^j}{\beta_k}}{n\left(1 - (n-2)\overline{\alpha} + \overline{\alpha} \sum_{j \neq i} \frac{\beta_k^j}{\beta_k}\right)}. \tag{14}$$

Proof: Condition (13) implies that the fully collusive outcome can be sustained as a subgame perfect equilibrium of the infinitely repeated game provided that

$$\delta \geq \widehat{\delta} - \frac{(n-1)\overline{\alpha} + \sum_{j \neq i} \frac{\beta_i^j}{\beta_i}}{n\left(1 - (n-2)\overline{\alpha} + \overline{\alpha} \sum_{j \neq i} \frac{\beta_i^j}{\beta_i}\right)}, \qquad i = 1, ..., n. \tag{15}$$

Since the right side of (15) is decreasing with $\sum_{j \neq i} \frac{\beta_i^j}{\beta_i}$, condition (14) ensures that condition (15) is satisfied. ■

Condition (14) generalizes condition (12). The two conditions coincide only when the controller of at least one firm does not invest in rivals in which case, $\sum_{j \neq i} \frac{\beta_k^j}{\beta_k} = 0$. Otherwise, the right side of (14) is lower than the right side of (12), implying that investments by controllers in rival firms lower $\widehat{\delta}^c$ and therefore facilitate tacit collusion. Intuitively, such investments facilitate tacit collusion because they allow each controller to internalize part of the losses that rivals bear when the controller deviates from the collusive scheme.

It should be noted that the critical discount factor above which the fully collusive scheme is sustainable does not depend on the entire matrix of controllers' private ownership stakes in rivals. Rather, given that all firms have similar PCO's in rivals, it depends only on the lowest $\sum_{j \neq i} \frac{\beta_i^j}{\beta_i}$ in the industry. Thus, while the PCO of firm i's controller in rivals unambiguously strengthens the controller's incentive to engage in tacit collusion, such investments affect $\widehat{\delta}^c$ only if $\sum_{j \neq i} \frac{\beta_i^j}{\beta_i}$ is the lowest in the industry. Moreover, $\widehat{\delta}^c$ does not depend on the controller's *absolute* stake in rival firms; rather it depends on the controller's stake in rivals *relative* to his stake in his own firm. In particular, a controller can lower $\widehat{\delta}^c$ either by raising his aggregate ownership stake in rivals or by diluting his ownership stake in his own firm (subject of course to retaining control over the firm). Such dilution effectively raises the weight that the controller assigns to rivals' profits and therefore weakens the controller's incentive to deviate from the collusive scheme. This suggests that if controllers are reluctant to invest large amounts in rivals (say due to the fear of antitrust scrutiny), they can dilute their stakes in their own firms and thereby achieve the same effect as in the case where their stakes in rivals are large.

Proposition 2 has important policy implications which have been overlooked in antitrust cases involving PCO by controllers (see Gilo 2000). It implies that in the presence of PCO by controllers, antitrust agencies need to be concerned not only with a controller's stakes in rivals, but also with his stake (current or future) in his own firm, especially when this stake is relatively small. This suggests in turn that consent decrees approving passive investment by controllers should stipulate that the controllers will abstain from further diluting their stakes in their own firms as such dilution promotes tacit collusion.[17]

Interestingly, the ability of firms to collude is greatly diminished when a firm's controller internalizes the interests of the minority shareholders in his firm and acts to maximize the total value of his firm rather than only the value of his own stake. This is because such behavior has the exact opposite effect of dilution of the controller's stake: a controller who acts to maximize total firm value acts as if $\beta_i = 1$ in which case $\widehat{\delta}^c$ is maximized. In this sense, minority shareholders would prefer the controller to disregard their interests when choosing the firm's pricing decisions. Thus, contrary to conventional wisdom that sees the disregard of minority shareholders as a value decreasing "agency cost," here such disregard is actually beneficial to all shareholders as it facilitates tacit collusion in the industry.

[17] In firms that are controlled by managers, compensation that is linked to the profits of rivals may play the same role as investments in rivals. This suggests that in these cases, executive compensation should receive similar antitrust scrutiny as investments of controllers in rival firms.

4 PCO when firms have asymmetric costs

We now turn to the case where firms have different marginal costs. We show that unlike in the identical marginal costs case, here even unilateral investment by one firm may facilitate collusion. We begin this section by considering the case in which there are no PCO. Using this result as a benchmark, we will then examine how unilateral PCO can facilitate tacit collusion.

4.1 Tacit collusion absent PCO

In order to consider cost asymmetries, let c_i be the marginal cost of firm i and assume $c_1 < c_2 < ... < c_n$. That is, higher indices represent higher cost firms. Let

$$\pi_i(p) = Q(p)(p - c_i),$$

be firm i's profit when it serves the entire market at a price p. We shall now make the following assumptions on $\pi_i(p)$:

Assumption 1: $\pi_i(p)$ has a unique (local and global) maximizer, p_i^m.

Assumption 2: $p_1^m > c_n$ and $\pi_1(c_2) > \frac{\pi_1(c_j)}{j-1}$ for all $j = 3, ..., n$.

Assumption 1 is standard and holds whenever the demand function is either concave or not too convex. Note that since $c_1 < c_2 < ... < c_n$, then $p_1^m < p_2^m < ... < p_n^m$.[18] That is, higher cost firms prefer higher monopoly prices. The first part of Assumption 2 ensures that all firms are effective competitors as it states that the monopoly price of the most efficient firm exceeds the marginal cost of the least efficient firm. The second part of Assumption 2 implies that in a static Bertrand game without PCO, firm 1 will prefer to set a price slightly below c_2 and capture the entire market than share the market with firm 2 at a price slightly below c_3, or share the market with firms 2 and 3 and a price slightly below c_4, and so on. Given this assumption, it

[18]To see why, note by revealed preferences that since $\pi_i(\cdot)$ has a unique maximizer, $Q(p_i^m)(p_i^m - c_i) > Q(p_j^m)(p_j^m - c_i)$, and $Q(p_j^m)(p_j^m - c_j) > Q(p_i^m)(p_i^m - c_j)$. Summing up these inequalities and simplifying, yields $Q(p_i^m)(c_j - c_i) > Q(p_j^m)(c_j - c_i)$. Assuming without a loss of generality that $j > i$, and noting that $Q'(\cdot) < 0$, it follows that $p_j^m > p_i^m$.

is clear that absent collusion, firm 1 will prefer to monopolize the market by charging a price slightly below c_2.

When the stage game is infinitely repeated, firms may be able to engage in tacit collusion. Unlike in Sections 2 and 3 where all firms had the same monopoly price, here different firms have different monopoly prices. This raises the obvious question of which price would firms coordinate on in a collusive equilibrium. If side payments were possible, firms would clearly let firm 1, which is the most efficient firm, serve the entire market at a price p_1^m (e.g., firms $2, ..., n$ would all set prices above p_1^m and would make no sales) and would then use side payments to share the monopoly profit $\pi_1(p_1^m)$. We rule out this possibility by assuming that side payments are not feasible, say due to the fear of antitrust prosecution.

Instead, we consider a collusive scheme led by firm 1. According to this scheme, firm 1 sets a price $\widehat{p}$ which all firms in the industry adopt. Consumers then randomize between the n firms thus giving all firms equal shares in the aggregate demand, $Q(\widehat{p})$.[19] The price $\widehat{p}$ is some compromise between the monopoly prices of the various firms, $p_1^m \leq \widehat{p} \leq p_n^m$, i.e., it lies between the lowest and highest monopoly prices. Although $\widehat{p}$ can exceed firm 1's monopoly price, p_1^m, it cannot exceed it by too much. This is because firm 1 can always ensure itself a profit of $\pi_1(c_2)$ by setting a price slightly below c_2 and capturing the entire market. Hence, it must be the case that $\pi_1(\widehat{p}) \geq \pi_1(c_2)$. Noting from Assumption 2 that $c_2 < p_1^m \leq \widehat{p}$, it follows that c_2 lies on the increasing segment of $\pi_1(\cdot)$ while $\widehat{p}$ lies on the decreasing segment. Hence, $\widehat{p}$ is bounded from above by $\overline{p}$, where $\overline{p}$ is implicitly defined by $\pi_1(\overline{p}) \equiv \pi_1(c_2)$.

In order to proceed, we add the following assumption which is illustrated in Figure 1:

Assumption 3: $\overline{p} < p_2^m$, where $\overline{p}$ is implicitly defined by $\pi_1(\overline{p}) \equiv \pi_1(c_2)$.

Recalling that $p_1^m < p_2^m < ... < p_n^m$, Assumption 3 implies that $\overline{p} < p_j^m$ for all $j = 2, ..., n$.[20]

[19] There could be other collusive schemes of course. For instance, a market-sharing scheme whereby firm 1 also offers its rivals market shares $s_2, ..., s_n$ and each firm committs not to sell more than its share in the aggregate demand, $Q(\widehat{p})$. Or, firm 1 could divide the market among the n firms and allow each firm to set any price its wants in its own market segment. We focus here on the pure price fixing scheme without any market division because market division schemes are likely to attract more antitrust scrutiny and because they are harder to enforce.

[20] To illustrate, suppose that $Q(p) = A - p$. Then, $p_i^m = \frac{A+c_i}{2}$ and $\overline{p} = A + c_1 - c_2$, so Assumption 3 is satisfied if $A < 3c_2 - 2c_1$ (this ensures that $\overline{p} < p_2^m$). Note however that A cannot be too low since Assumption 2 requires that $A > 2c_n - c_1$.

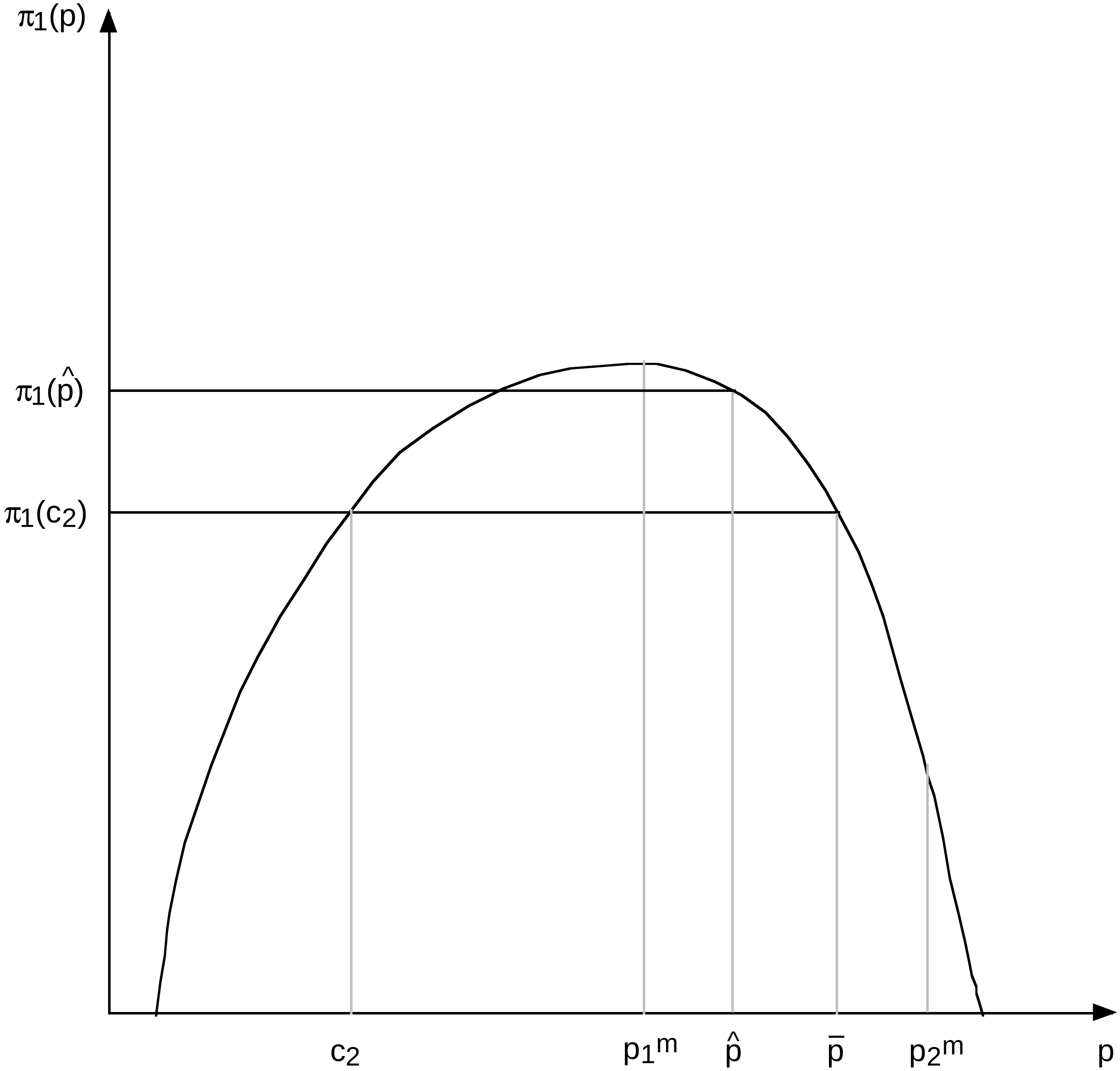

Figure 1: illustrating Assumption 3

Since $\widehat{p} \leq \overline{p}$, it follows that $\widehat{p} < p_j^m$ for all $j = 2, ..., n$: the collusive price is below the monopoly prices of all firms but 1. This implies in turn that if the controller of firm $j = 2, ..., n$ deviates from the collusive scheme, the controller will charge a price slightly below $\widehat{p}$ and will then make a one time profit of $\pi_j(\widehat{p})$.

If all firms accept $\widehat{p}$, the per period profit of each firm i is $\frac{\pi_i(\widehat{p})}{n}$. However, if any controller (including firm 1's controller) deviates and sets a price below $\widehat{p}$, then the scheme breaks down. In that case, firm 1 charges a price slightly below c_2 forever after, so its per period profit is $\pi_1(c_2)$ while the per period profit of all other firms is 0. The condition that ensures that the controllers of firms $2, ..., n$ do not wish to deviate from the collusive scheme is given by

$$\beta_j \frac{\pi_j(\widehat{p})}{n(1-\delta)} \geq \beta_j \pi_j(\widehat{p}), \qquad j = 1, ..., n, \tag{16}$$

where the left hand side of the inequality represents the one time profit of firm j's controller from undercutting $\widehat{p}$ slightly. This condition is equivalent to condition (1). Hence, firms $2, ..., n$ do not wish to deviate provided that $\delta \geq \widehat{\delta}$.

To establish a condition that ensures that firm 1's controller does not wish to deviate, note that if he does, he sets a price p_1^m (recall that $p_1^m \leq \widehat{p}$) and gets a one time profit of $\pi_1(p_1^m)$. From that point onward, firm 1's price will be slightly below c_2 and its per period profit will be $\pi_1(c_2)$. Therefore, firm 1's controllers does not wish to deviate from the collusive scheme provided that

$$\beta_1 \frac{\pi_1(\widehat{p})}{n(1-\delta)} \geq \beta_1 \left(\pi_1(p_1^m) + \frac{\delta \pi_1(c_2)}{1-\delta} \right), \tag{17}$$

or

$$\delta \geq \widehat{\delta}(\widehat{p}) \equiv \frac{\pi_1(p_1^m) - \frac{\pi_1(\widehat{p})}{n}}{\pi_1(p_1^m) - \pi_1(c_2)}. \tag{18}$$

Since $\pi_1(p_1^m) \geq \pi_1(\widehat{p})$, $\widehat{\delta}(\widehat{p}) > \widehat{\delta}$, firm 1 is the industry maverick. Hence the collusive scheme can be sustained as a subgame perfect equilibrium of the infinitely repeated game provided that condition (18) holds. Moreover, since $\widehat{p} \geq p_1^m$, it follows that $\pi_1'(\widehat{p}) \leq 0$ with strict inequality for $\widehat{p} > p_1^m$. Hence, $\widehat{\delta}(\widehat{p})$ increases with $\widehat{p}$, implying that firm 1's controller would like to set

$\widehat{p} = p_1^m$ as this maximizes his infinite discounted stream of profits under the collusive scheme and relaxes condition (18). Therefore, the collusive scheme is feasible provided that $\delta \geq \widehat{\delta}(p_1^m)$.

4.2 Tacit collusion with unilateral PCO

We now proceed by showing that when firms have asymmetric costs, even unilateral PCO can facilitate the collusive scheme characterized in Section 4.1. To this end, let us assume that only firm 1 invests in rivals and let $\alpha_1^2, ..., \alpha_1^n$ be its ownership stakes in firms $2, ..., n$. Since the collusive profit of each firm i is $\frac{\pi_i(\widehat{p})}{n}$, it follows that firm 1's infinite discounted stream of profits under collusion is $\frac{\pi_1(\widehat{p})+\sum_{j\neq 1}\alpha_1^j \pi_i(\widehat{p})}{n(1-\delta)}$. If firm 1's controller deviates, all rival firms make zero profits so firm 1's payoff is $\pi_1(p_1^m)+\frac{\delta\pi_1(c_2)}{1-\delta}$, exactly as in the absence of PCO. Consequently, to induce firm 1's controller to adhere to the collusive scheme it must be the case that

$$\beta_1 \left(\frac{\pi_1(\widehat{p}) + \sum_{j\neq 1} \alpha_1^j \pi_i(\widehat{p})}{n\,(1-\delta)} \right) \geq \beta_1 \left(\pi_1(p_1^m) + \frac{\delta\pi_1(c_2)}{1-\delta} \right), \tag{19}$$

or

$$\delta \geq \widehat{\delta}^{po}(\widehat{p}) \equiv \frac{\pi_1(p_1^m) - \frac{\pi_1(\widehat{p})+\sum_{j\neq 1}\alpha_1^j \pi_j(\widehat{p})}{n}}{\pi_1(p_1^m) - \pi_1(c_2)}. \tag{20}$$

Firm 1 will select $\widehat{p}$ in order to maximize the left-hand side of (19) subject to (20).

Proposition 3: *If firm 1 invests in rivals, the optimal collusive price from its perspective is $\widehat{p}^*$. This price is increasing with the investments of firm 1 in rivals and is above firm 1's monopoly price: $\widehat{p}^* > p_1^m$. Moreover, firm 1's investments in rivals facilitate tacit collusion by expanding the set of discount factors for which the optimal collusive scheme from firm 1's perspective becomes sustainable. PCO in an efficient rival raises the collusive price by less but facilitates collusion by more than an investment of similar size in a less efficient rival.*

Proof: Firm 1 chooses $\widehat{p}$ to maximize the left side of equation (19). Given Assumption 1, it follows that $\widehat{p}^*$ is increasing with each α_1^j and is above p_1^m. Moreover, $\widehat{p}^*$ minimizes $\widehat{\delta}^{po}(\widehat{p})$ and therefore relaxes condition (20) as much as possible.

To examine the effect of PCO on the ease of tacit collusion, note that given $\widehat{p}^*$, the collusive scheme is sustainable provided that $\delta \geq \widehat{\delta}^{po}(\widehat{p}^*)$. Since by revealed preferences, $\pi_1(\widehat{p}^*) + \sum_{j\neq 1} \alpha_1^j \pi_j(\widehat{p}^*) \geq \pi_1(p_1^m) + \sum_{j\neq 1} \alpha_1^j \pi_j(p_1^m)$, and since $\pi_1(p_1^m) + \sum_{j\neq 1} \alpha_1^j \pi_j(p_1^m) > \pi_1(p_1^m)$, it follows from (18) and (20) that $\widehat{\delta}^{po}(\widehat{p}^*) \leq \widehat{\delta}^{po}(p_1^m) < \widehat{\delta}(p_1^m)$. Hence, PCO by firm 1 expands the range of discount factors for which the collusive scheme can be sustained.

Finally, since $c_2 < ... < c_n$, it follows that $\pi_2(\widehat{p}^*) > ... > \pi_n(\widehat{p}^*)$, implying that PCO by firm 1 in an efficient rival raises $\widehat{p}^*$ by less and lowers $\widehat{\delta}^{po}(\widehat{p}^*)$ by more than does a similar investment in a less efficient rival. ■

Proposition 3 implies that investment by firm 1 in rival firms not only raises the collusive price but also makes it easier to sustain the collusive scheme. The proposition suggests that, to the extent that firm 1 invests in rivals, it will always prefer to invest in its most efficient rival since this will lead to a collusive price that is closer to firm 1's monopoly price and will expand the range of discount factors above which collusion can be sustained. Only if investment in the most efficient rival does not suffice to sustain the collusive scheme, will firm 1 begin to invest in the next efficient rival (note that any investment in rivals facilitates tacit collusion).

Finally, it is worth noting that firm 1 will have an incentive to minimize its investments in rivals subject to being able to facilitate tacit collusion. The reason for this is as follows: assuming that the capital market is perfectly competitive and the shares of firm 1's rivals are held by atomistic shareholders, firm 1 will pay a fair price for its rivals' shares, and will therefore just breaks even on these shares. Hence the change in the payoff of firm 1's shareholders from investing in rivals is simply equal to the resulting change in firm 1's direct profit (i.e., excluding its share in rivals' profits). But since $\widehat{p} > p_1^m$, the direct profit of firm 1 decreases following the investment, thereby implying that firm 1 will prefer to invest as little as possible in rivals subject to ensuring that the collusive scheme can be sustained.

5 Conclusion

Acquisitions of one firm's stock by a rival firm have traditionally been treated under Section 7 of the Clayton Act which condemns such acquisitions when their effect "may be substantially to lessen competition." However, the third paragraph of this section effectively exempts passive investments made "solely for investment." As argued in Gilo (2000), antitrust agencies and courts have applied this exemption in leading cases and did not conduct full-blown examinations as to whether such passive investments may substantially lessen competition.[21]

We showed that an across the board lenient approach towards passive investments in rivals may be misguided since even completely passive investments in rivals may facilitate tacit collusion. In particular, we showed that passive investments in rivals are especially likely to facilitate tacit collusion when the investments are multilateral, when they are not in industry mavericks, and when they are made by the most efficient firm in its most efficient rivals. In addition, we showed that direct investments by firms' controllers in rivals may either substitute investments by the firms themselves or may facilitate collusion further, especially when the controllers have small stakes in their own firms. We believe that it is important for antitrust courts and agencies to take account of these factors when considering cases involving passive investments among rivals.

Finally, our paper has examined the effects of PCO on tacit collusion taking the level of PCO in the industry as exogenously given. In a sense then our analysis is done from the perspective of antitrust authorities: when can you allow a firm to acquire a passive stake in a rival firm and when should you disallow such acquisition. In future research we wish to also look at PCO from the perspective of firms: that is, we wish to endogenize the PCO configuration of PCO in the industry and examine when should a firm try to acquire a passive stake in rivals and when shouldn't it.

6 Appendix

Following are the proofs of Corollaries 5 and 6 and a complete characterization of the effect of PCO on tacit collusion when there are 4 identical firms.

The case where there are 4 identical firms: Let $n = 4$ and suppose that firm 1 raises its ownership stake in firm 2 by Δ, where $\alpha_1^2 + \alpha_1^3 + \alpha_1^4 + \Delta < 1$. We show that this increase in

[21] There are only two cases in which the ability of passive investments to lessen competition was acknowledged: the FTC's decision in *Golden Grain Macaroni Co.* (78 F.T.C. 63, 1971), and the consent decree reached with the DOJ regarding US West's acquisition of Continental Cablevision (this decree was approved by the distric court in United states v. US West Inc., 1997-1 Trade cases (CCH), ¶71,767, D.C., 1997).

firm 1's stake in firm 2 facilitates tacit collusion if firm 2 is not the industry maverick but has no effect on tacit collusion if firm 2 is the industry maverick.

Using (8), tedious calculations show that when $n = 4$,

$$
\begin{aligned}
\widehat{\delta}_1 = 1 &- \frac{1 + (\alpha_1^2 + \Delta)\left(1 + \alpha_2^3\left(1 + \alpha_3^4\right) + \alpha_2^4\left(1 + \alpha_4^3\right) - \alpha_3^4\alpha_4^3\right)}{4\left(1 - \alpha_2^3\left(\alpha_3^2 + \alpha_3^4\alpha_4^2\right) - \alpha_2^4\left(\alpha_4^2 + \alpha_4^3\alpha_3^2\right) - \alpha_3^4\alpha_4^3\right)} \\
&+ \frac{\alpha_1^3\left(1 + \alpha_3^2\left(1 + \alpha_2^4\right) + \alpha_3^4\left(1 + \alpha_4^2\right) - \alpha_2^4\alpha_4^2\right)}{4\left(1 - \alpha_2^3\left(\alpha_3^2 + \alpha_3^4\alpha_4^2\right) - \alpha_2^4\left(\alpha_4^2 + \alpha_4^3\alpha_3^2\right) - \alpha_3^4\alpha_4^3\right)} \\
&+ \frac{\alpha_1^4\left(1 + \alpha_4^2\left(1 + \alpha_2^3\right) + \alpha_4^3\left(1 + \alpha_3^2\right) - \alpha_2^3\alpha_3^2\right)}{4\left(1 - \alpha_2^3\left(\alpha_3^2 + \alpha_3^4\alpha_4^2\right) - \alpha_2^4\left(\alpha_4^2 + \alpha_4^3\alpha_3^2\right) - \alpha_3^4\alpha_4^3\right)} \\
&- \frac{\alpha_2^3\left(\alpha_3^2 + \alpha_3^4\alpha_4^2\right) + \alpha_2^4\left(\alpha_4^2 + \alpha_3^2\alpha_4^3\right) + \alpha_3^4\alpha_4^3}{4\left(1 - \alpha_2^3\left(\alpha_3^2 + \alpha_3^4\alpha_4^2\right) - \alpha_2^4\left(\alpha_4^2 + \alpha_4^3\alpha_3^2\right) - \alpha_3^4\alpha_4^3\right)},
\end{aligned}
$$

$$
\begin{aligned}
\widehat{\delta}_2 = 1 &- \frac{1 + \alpha_2^1\left(1 + \alpha_1^3\left(1 + \alpha_3^4\right) + \alpha_1^4\left(1 + \alpha_4^3\right) - \alpha_3^4\alpha_4^3\right)}{4\left(1 - \alpha_1^3\left(\alpha_3^1 + \alpha_3^4\alpha_4^1\right) - \alpha_1^4\left(\alpha_4^1 + \alpha_3^1\alpha_4^3\right) - \alpha_3^4\alpha_4^3\right)} \\
&+ \frac{\alpha_2^3\left(1 + \alpha_3^1\left(1 + \alpha_1^4\right) + \alpha_3^4\left(1 + \alpha_4^1\right) - \alpha_1^4\alpha_4^1\right)}{4\left(1 - \alpha_1^3\left(\alpha_3^1 + \alpha_3^4\alpha_4^1\right) - \alpha_1^4\left(\alpha_4^1 + \alpha_3^1\alpha_4^3\right) - \alpha_3^4\alpha_4^3\right)} \\
&+ \frac{\alpha_2^4\left(1 + \alpha_4^1\left(1 + \alpha_1^3\right) + \alpha_4^3\left(1 + \alpha_3^1\right) - \alpha_1^3\alpha_3^1\right)}{4\left(1 - \alpha_1^3\left(\alpha_3^1 + \alpha_3^4\alpha_4^1\right) - \alpha_1^4\left(\alpha_4^1 + \alpha_3^1\alpha_4^3\right) - \alpha_3^4\alpha_4^3\right)} \\
&- \frac{\alpha_1^3\left(\alpha_3^1 + \alpha_3^4\alpha_4^1\right) + \alpha_1^4\left(\alpha_4^1 + \alpha_3^1\alpha_4^3\right) + \alpha_3^4\alpha_4^3}{4\left(1 - \alpha_1^3\left(\alpha_3^1 + \alpha_3^4\alpha_4^1\right) - \alpha_1^4\left(\alpha_4^1 + \alpha_3^1\alpha_4^3\right) - \alpha_3^4\alpha_4^3\right)},
\end{aligned}
$$

$$
\begin{aligned}
\widehat{\delta}_3 = 1 &- \frac{1 + \alpha_3^1\left(1 + (\alpha_1^2 + \Delta)\left(1 + \alpha_2^4\right) + \alpha_1^4\left(1 + \alpha_4^2\right) - \alpha_2^4\alpha_4^2\right)}{4\left(1 - (\alpha_1^2 + \Delta)\left(\alpha_2^1 + \alpha_2^4\alpha_4^1\right) - \alpha_1^4\left(\alpha_4^1 + \alpha_2^1\alpha_4^2\right) - \alpha_2^4\alpha_4^2\right)} \\
&+ \frac{\alpha_3^2\left(1 + \alpha_2^1\left(1 + \alpha_1^4\right) + \alpha_2^4\left(1 + \alpha_4^1\right) - \alpha_1^4\alpha_4^1\right)}{4\left(1 - (\alpha_1^2 + \Delta)\left(\alpha_2^1 + \alpha_2^4\alpha_4^1\right) - \alpha_1^4\left(\alpha_4^1 + \alpha_2^1\alpha_4^2\right) - \alpha_2^4\alpha_4^2\right)} \\
&+ \frac{\alpha_3^4\left(1 + \alpha_4^1\left(1 + (\alpha_1^2 + \Delta)\right) + \alpha_4^2\left(1 + \alpha_2^1\right) - (\alpha_1^2 + \Delta)\,\alpha_2^1\right)}{4\left(1 - (\alpha_1^2 + \Delta)\left(\alpha_2^1 + \alpha_2^4\alpha_4^1\right) - \alpha_1^4\left(\alpha_4^1 + \alpha_2^1\alpha_4^2\right) - \alpha_2^4\alpha_4^2\right)} \\
&- \frac{(\alpha_1^2 + \Delta)\left(\alpha_2^1 + \alpha_2^4\alpha_4^1\right) + \alpha_1^4\left(\alpha_4^1 + \alpha_2^1\alpha_4^2\right) + \alpha_2^4\alpha_4^2}{4\left(1 - (\alpha_1^2 + \Delta)\left(\alpha_2^1 + \alpha_2^4\alpha_4^1\right) - \alpha_1^4\left(\alpha_4^1 + \alpha_2^1\alpha_4^2\right) - \alpha_2^4\alpha_4^2\right)},
\end{aligned}
$$

and,

$$\begin{aligned}
\widehat{\delta}_4 &= 1-\frac{1+\alpha_4^1\left(1+\left(\alpha_1^2+\Delta\right)\left(1+\alpha_2^3\right)+\alpha_1^3\left(1+\alpha_3^2\right)-\alpha_2^3\alpha_3^2\right)}{4\left(1-\left(\alpha_1^2+\Delta\right)\left(\alpha_2^1+\alpha_2^3\alpha_3^1\right)-\alpha_1^3\left(\alpha_3^1+\alpha_2^1\alpha_3^2\right)-\alpha_2^3\alpha_3^2\right)} \\
&\quad+\frac{\alpha_4^2\left(1+\alpha_2^1\left(1+\alpha_1^3\right)+\alpha_2^3\left(1+\alpha_3^1\right)-\alpha_1^3\alpha_3^1\right)}{4\left(1-\left(\alpha_1^2+\Delta\right)\left(\alpha_2^1+\alpha_2^3\alpha_3^1\right)-\alpha_1^3\left(\alpha_3^1+\alpha_2^1\alpha_3^2\right)-\alpha_2^3\alpha_3^2\right)} \\
&\quad+\frac{\alpha_4^3\left(1+\alpha_3^1\left(1+\left(\alpha_1^2+\Delta\right)\right)+\alpha_3^2\left(1+\alpha_2^1\right)-\left(\alpha_1^2+\Delta\right)\alpha_2^1\right)}{4\left(1-\left(\alpha_1^2+\Delta\right)\left(\alpha_2^1+\alpha_2^3\alpha_3^1\right)-\alpha_1^3\left(\alpha_3^1+\alpha_2^1\alpha_3^2\right)-\alpha_2^3\alpha_3^2\right)} \\
&\quad-\frac{\left(\alpha_1^2+\Delta\right)\left(\alpha_2^1+\alpha_2^3\alpha_3^1\right)+\alpha_1^3\left(\alpha_3^1+\alpha_2^1\alpha_3^2\right)+\alpha_2^3\alpha_3^2}{4\left(1-\left(\alpha_1^2+\Delta\right)\left(\alpha_2^1+\alpha_2^3\alpha_3^1\right)-\alpha_1^3\left(\alpha_3^1+\alpha_2^1\alpha_3^2\right)-\alpha_2^3\alpha_3^2\right)}.
\end{aligned}$$

Differentiating these expressions with respect to Δ reveals that $\widehat{\delta}_1, \widehat{\delta}_3$, and $\widehat{\delta}_4$ decrease with Δ, while $\widehat{\delta}_2$ is independent of Δ (the latter result is not surprising - the proof of Corollary 3 shows that in fact, $\widehat{\delta}_i$ will never be affected by changes in α_j^i for all i and all $j \neq i$). Hence, the increase in firm 1's stake in firm 2 facilitates tacit collusion if firms $1, 3$, or 4 are the industry mavericks but it has no effect on tacit collusion if firm 2 is the industry maverick. ■

Proof of Corollary 5: Given that $\alpha_i^j = \overline{\alpha}$ for all $i \neq 1$ and all $j \neq i$, system (2) can be written as

$$\begin{aligned}
\pi_1 &= \frac{\pi^m}{n} + \alpha_1^2\pi_2 + \alpha_1^3\pi_3 + \cdots + \alpha_1^n\pi_n, \\
\pi_2 &= \frac{\pi^m}{n} + \overline{\alpha}\pi_1 + \overline{\alpha}\pi_3 + \cdots + \overline{\alpha}\pi_n, \\
&\vdots \\
\pi_n &= \frac{\pi^m}{n} + \overline{\alpha}\pi_1 + \overline{\alpha}\pi_2 + \cdots + \overline{\alpha}\pi_{n-1},
\end{aligned}$$

where $\sum_{j\neq 1} \alpha_1^j = (n-1)\overline{\alpha} + \Delta$. By symmetry, $\pi_2 = ... = \pi_n$; hence, the solution of the system is

$$\begin{aligned}
\pi_1 &= \frac{\left(1+\overline{\alpha}+\Delta\right)\frac{\pi^m}{n}}{\left(1-(n-1)\overline{\alpha}\right)\left(1+\overline{\alpha}\right)-\overline{\alpha}\Delta}, \\
\pi_j &= \frac{\left(1+\overline{\alpha}\right)\frac{\pi^m}{n}}{\left(1-(n-1)\overline{\alpha}\right)\left(1+\overline{\alpha}\right)-\overline{\alpha}\Delta}, \qquad j = 2, ..., n.
\end{aligned} \tag{A-1}$$

We now need to compute the profit that each firm can obtain when its controller deviates from the fully collusive scheme. If firm 1's controller deviates, then system (2) can be written as

$$\pi_1^{d_1} = \pi^m + \left((n-1)\overline{\alpha} + \Delta\right)\pi_j^{d_1},$$

$$\pi_j^{d_1} = \overline{\alpha}\pi_1^{d_1} + (n-1)\,\overline{\alpha}\pi_j^{d_1}, \qquad j = 2, ..., n.$$

Solving for $\pi_1^{d_1}$ yields,

$$\pi_1^{d_1} = \frac{\left(1-(n-2)\overline{\alpha}\right)\pi^m}{\left(1-(n-1)\overline{\alpha}\right)\left(1+\overline{\alpha}\right) - \overline{\alpha}\Delta}. \tag{A-2}$$

From (A-1) and (A-2) it follows that

$$\widehat{\delta}_1 \equiv 1 - \frac{\pi_1}{\pi_1^{d_1}} = \widehat{\delta} - \frac{(n-1)\overline{\alpha} + \Delta}{n\left(1-(n-2)\overline{\alpha}\right)}. \tag{A-3}$$

If the controller of some firm $i \neq 1$ deviates from the fully collusive scheme, then system (2) can be written as

$$\pi_1^{d_i} = \alpha_1^i \pi_i^{d_i} + \left(\overline{\alpha}(n-1) + \Delta - \alpha_1^i\right)\pi_j^{d_i},$$

$$\pi_i^{d_i} = \pi^m + \overline{\alpha}\pi_1^{d_i} + (n-2)\,\overline{\alpha}\pi_j^{d_i},$$

$$\pi_j^{d_i} = \overline{\alpha}\pi_1^{d_i} + \overline{\alpha}\pi_i^{d_i} + (n-3)\,\overline{\alpha}\pi_j^{d_i}, \qquad j = 2, ..., n, \quad j \neq i.$$

Solving this system for $\pi_i^{d_i}$ yields,

$$\pi_i^{d_i} = \frac{\left(\left(1-(n-1)\overline{\alpha}\right)\left(1+\overline{\alpha}\right) + \overline{\alpha}\left(1+\alpha_1^i - \Delta\right)\right)\pi^m}{\left(1+\overline{\alpha}\right)\left(\left(1-(n-1)\overline{\alpha}\right)\left(1+\overline{\alpha}\right) - \overline{\alpha}\Delta\right)}, \qquad i \neq 1. \tag{A-4}$$

From (A-1) and (A-4) it follows that

$$\widehat{\delta}_i \equiv 1 - \frac{\pi_i}{\pi_i^{d_i}} = \widehat{\delta} - \frac{\overline{\alpha}\left((n-1+\overline{\alpha}n+\Delta-\alpha_1^i\right)}{n\left((1-(n-1)\overline{\alpha})\left(1+\overline{\alpha}\right)+\overline{\alpha}\left(1+\alpha_1^i-\Delta\right)\right)}. \tag{A-5}$$

To compare (A-3) and (A-5), note that holding Δ constant, $\widehat{\delta}_i$ is increasing with α_1^i and hence is minimized at $\alpha_1^i = \overline{\alpha}$, i.e., when the increase in firm 1's PCOs is in firms other than i. But since $\Delta > 0$, then for all $i \neq 1$,

$$\widehat{\delta}_i\Big|_{\alpha_1^i=\overline{\alpha}} - \widehat{\delta}_1 = \frac{\Delta\left((1-(n-1)\overline{\alpha})\left(1+\overline{\alpha}\right)-\overline{\alpha}\Delta\right)}{n\left(1-(n-2)\overline{\alpha}\right)\left((1-(n-1)\overline{\alpha})\left(1+\overline{\alpha}\right)+\overline{\alpha}\left(1+\overline{\alpha}-\Delta\right)\right)} > 0.$$

Hence, $\widehat{\delta}_i > \widehat{\delta}_1$ for all values of α_1^i and all $i \neq 1$. Now suppose that firm 1's largest PCO is in firm i so that $\alpha_1^i \geq \alpha_1^j$ for all $j \neq 1$. Since $\widehat{\delta}_i$ is increasing with α_1^i, $\max\left\{\widehat{\delta}_2, \widehat{\delta}_3, ..., \widehat{\delta}_n\right\} = \widehat{\delta}_i$. That is, firm i is the industry maverick. Hence, by (7), the critical discount factor above which the fully collusive outcome can be sustained as a subgame perfect equilibrium of the infinitely repeated game is $\widehat{\delta}_i$.

When either $\Delta = 0$ (in which case $\alpha_1^i = \overline{\alpha}$ so that we are back in the symmetric case) or $\alpha_1^i = \overline{\alpha} + \Delta$ (firm 1 increases its ownership stake only in firm j), $\widehat{\delta}_i$ coincides with the expression in equation (12). Otherwise, since $\widehat{\delta}_i$ decreases with Δ, tacit collusion is facilitated when firm 1 increases its aggregate stake in rivals. Since $\widehat{\delta}_i$ increases with α_1^i, tacit collusion is particularly facilitated when Δ is spread equally among all of its rivals in which case, for every Δ, α_1^i is minimal and equal to $\overline{\alpha} + \frac{\Delta}{n-1}$. ■

The effect of a unilateral decrease in firm 1's aggregate ownership stake in rivals: Suppose that firm 1 lowers its aggregate ownership stake in rivals by Δ. Since $\Delta < 0$, (A-7) implies that $\widehat{\delta}_i$ is maximized at $\alpha_1^i = \overline{\alpha}$, i.e., whenever firm 1 lowers its ownership stake in firms other than firm i. Moreover, since $\Delta < 0$, the proof of Corollary 5 implies that $\widehat{\delta}_i\Big|_{\alpha_1^i=\overline{\alpha}} < \widehat{\delta}_1$. This implies in turn that $\widehat{\delta}_i < \widehat{\delta}_1$ for all $i \neq 1$. Consequently, the critical discount factor above which the fully collusive outcome can be sustained as a subgame perfect equilibrium of the infinitely repeated game is $\widehat{\delta}_1$. From equation (A-3) it is easy to see that $\widehat{\delta}_1$ is increasing as Δ falls, implying that tacit collusion is hindered. ■

Proof of Corollary 6: Given the transfer of ownership stake in firm 3 from firm 2 to firm 1, system (2) becomes

$$\begin{aligned}
\pi_1 &= \frac{\pi^m}{n} + \overline{\alpha}\pi_2 + (\overline{\alpha}+\Delta)\,\pi_3 + \cdots + \overline{\alpha}\pi_n, \\
\pi_2 &= \frac{\pi^m}{n} + \overline{\alpha}\pi_1 + (\overline{\alpha}-\Delta)\,\pi_3 + \cdots + \overline{\alpha}\pi_n, \\
&\vdots \\
\pi_n &= \frac{\pi^m}{n} + \overline{\alpha}\pi_1 + \overline{\alpha}\pi_2 + \cdots + \overline{\alpha}\pi_{n-1}.
\end{aligned}$$

By symmetry, $\pi_3 = ... = \pi_n$; hence, the solution of the system is given by

$$\begin{aligned}
\pi_1 &= \frac{(1+\overline{\alpha}+\Delta)\,\pi^m}{n\,(1-(n-1)\overline{\alpha})\,(1+\overline{\alpha})}, \\
\pi_2 &= \frac{(1+\overline{\alpha}-\Delta)\,\pi^m}{n\,(1-(n-1)\,\overline{\alpha})\,(1+\overline{\alpha})} \qquad \text{(A-6)} \\
\pi_i &= \frac{\pi^m}{n\,(1-(n-1)\overline{\alpha})}, \qquad i = 3,...,n.
\end{aligned}$$

If the controller of firm 1 deviates from the fully collusive scheme, then system (2) needs to be modified by replacing $\frac{\pi^m}{n}$ with π^m in the first line of the system and replacing $\frac{\pi^m}{n}$ with 0 in all other lines. Solving the modified system for firm 1's profit in this case yields,

$$\pi_1^{d_1} = \frac{((1-(n-2)\,\overline{\alpha})\,(1+\overline{\alpha})+\overline{\alpha}\Delta)\,\pi^m}{(1-\overline{\alpha}(n-1))\,(1+\overline{\alpha})^2}. \qquad \text{(A-7)}$$

Using (A-6) and (A-7) yields

$$\widehat{\delta}_1(\Delta) \equiv 1 - \frac{\pi_1}{\pi_1^{d_1}} = \widehat{\delta} - \frac{\overline{\alpha}\,(1+\overline{\alpha})\,(n-1)+\Delta}{n\,((1-(n-2)\,\overline{\alpha})\,(1+\overline{\alpha})+\overline{\alpha}\Delta)}. \qquad \text{(A-8)}$$

Likewise, if the controller of firm 2 deviates, the solution to the modified system (2) is such that

$$\pi_2^{d_2} = \frac{((1-(n-2)\,\overline{\alpha})\,(1+\overline{\alpha})-\overline{\alpha}\Delta)\,\pi^m}{(1-\overline{\alpha}(n-1))\,(1+\overline{\alpha})^2}. \qquad \text{(A-9)}$$

Using (A-6) and (A-9) yields

$$\widehat{\delta}_2(\Delta) \equiv 1 - \frac{\pi_2}{\pi_2^{d_2}} = \widehat{\delta} - \frac{\overline{\alpha}(1+\overline{\alpha})(n-1) - \Delta}{((1-(n-2)\overline{\alpha})(1+\overline{\alpha}) - \overline{\alpha}\Delta)}. \tag{A-10}$$

And, if the controller of some firm $i \neq 1, 2$ deviates, then the solution to the modified system (2) shows that its profit, $\pi_i^{d_i}$, is equal to the right-hand side of (11). Since the collusive profit of firm $i \neq 1, 2$ in (A-8) is equal to the right-hand side of (9), it follows that $\widehat{\delta}_i(\Delta) = \widehat{\delta}^{po}$ for all $i \neq 1, 2$, where $\widehat{\delta}^{po}$ is given by (12).

Now note that (i) $\widehat{\delta}_1(\Delta) = \widehat{\delta}_2(-\Delta)$, (ii) $\widehat{\delta}_1(0) = \widehat{\delta}_i(\Delta)$, and (iii) $\widehat{\delta}_1'(\Delta) < 0$. Since $\Delta > 0$, it follows that $\widehat{\delta}_2(\Delta) > \widehat{\delta}_i(\Delta) > \widehat{\delta}_1(\Delta)$. Hence, the critical discount factor above which the fully collusive outcome can be sustained as a subgame perfect equilibrium of the infinitely repeated game is $\widehat{\delta}_2(\Delta)$. Since $\widehat{\delta}_2(\Delta) > \widehat{\delta}_i(\Delta) = \widehat{\delta}^{po}$, it follows that tacit collusion is hindered. ■

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